MICRO-461
Low-power Radio Design for the IoT

5. Modeling of active and passive devices at RF

Passive Devices

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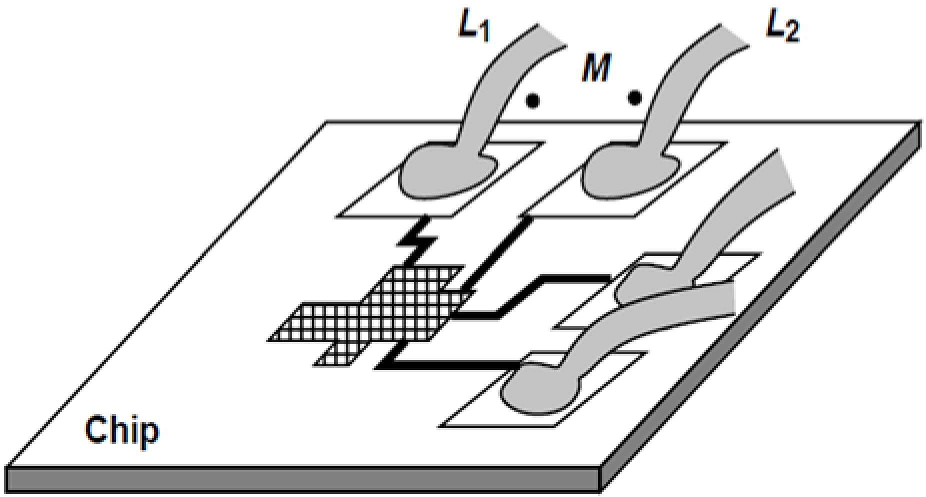
Outline

- **Introduction**
- Inductors
- Transformers
- Varactors

Introduction



- Reduction of off-chip components translates into a reduction of system cost
- Modeling issues of off-chip inductors
- The bond wires and package pins connecting chip to outside world may experience significant coupling

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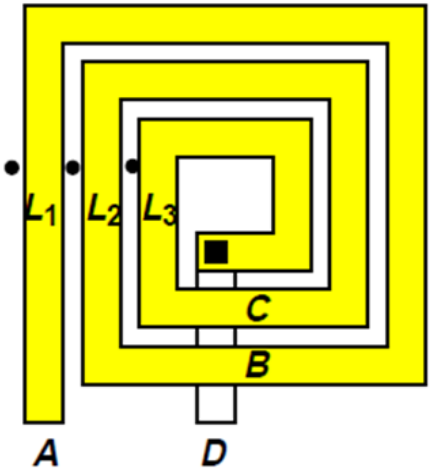
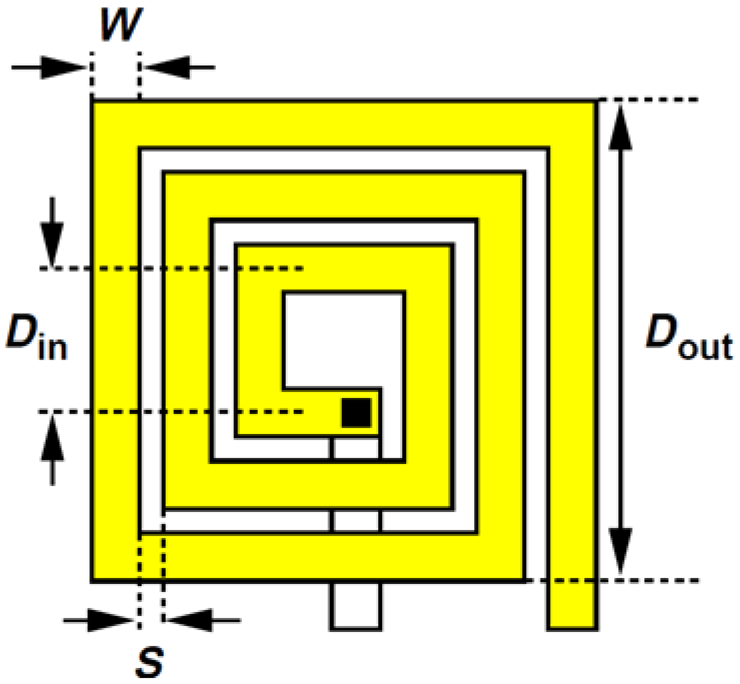
Basic Planar Inductor Structure



$$L_{tot} = L_1 + L_2 + L_3 + M_{12} + M_{13} + M_{23}$$

- Has mutual coupling between every two turns and larger inductance than straight wire
- Spiral is implemented on top metal layer to minimize parasitic resistance and capacitance
- Inductance of an N-turn planar spiral structure inductor has $N(N+1)/2$ terms
- Factors that limit the growth rate of an inductance of spiral inductor as function of N:
  - Due to planar geometry the inner turns have smaller size and exhibit smaller inductance.
  - The mutual coupling factor is about 0.7 for adjacent turns hence contributing to lower inductance.

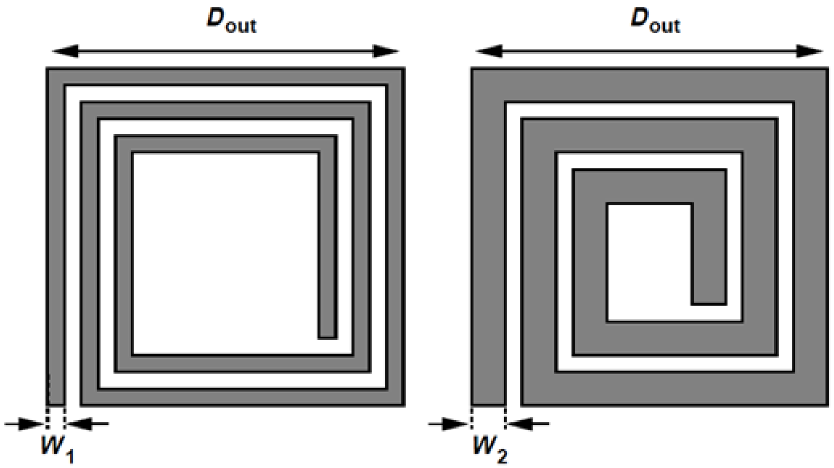
Geometry of Inductor Effects Inductance



Various dimensions of a spiral inductor

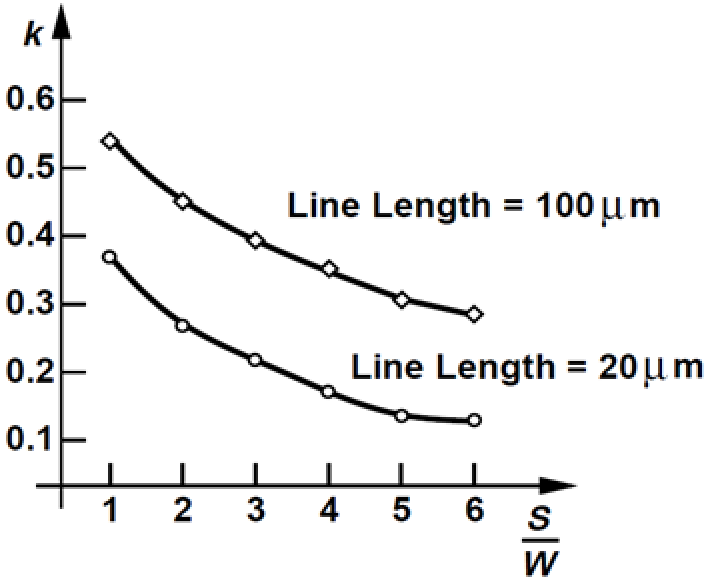
- A two dimensional square spiral inductor is fully specified by following four quantities:
  - Outer dimension, D_{out}
  - Line width, W
  - Line spacing, S
  - Number of turns, N

Effect of Doubling Line Width of Inductor



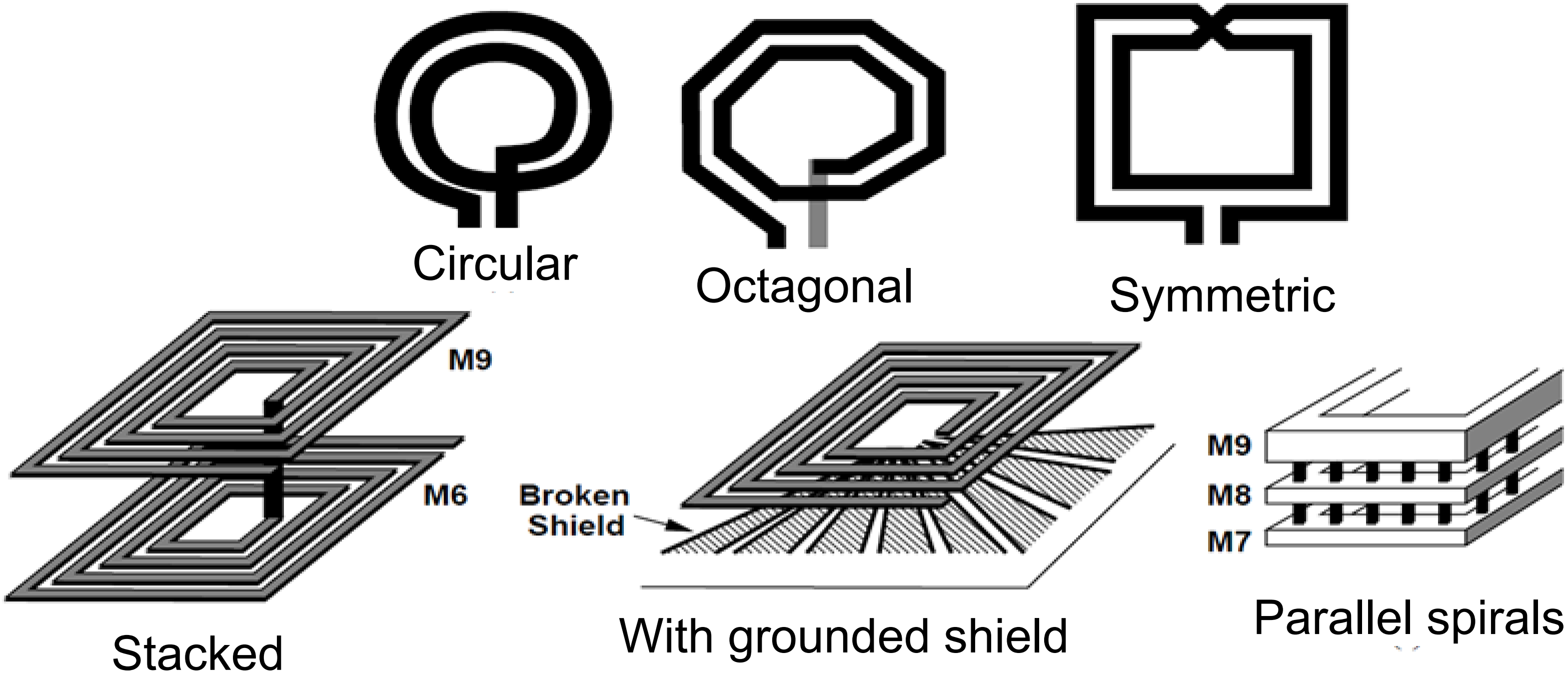
- Doubling the width inevitably decreases the diameter of inner turn, thus lowering their inductance
- The spacing between the legs reduces, hence their mutual inductance also decrease

Magnetic Coupling Factor Plot



- Coupling factor between 2 straight metal lines as a function of their normalized spacing S/W
- Obtained from electromagnetic field simulations

Inductor Structures Encountered in RFIC Design



- Various inductor geometries shown above are result of improving the trade-offs in inductor design, specifically those between:
  - The quality factor and the capacitance
  - The inductance and the dimensions
- Note that these various inductor geometries provide additional degrees of freedom but also complicate the modeling task

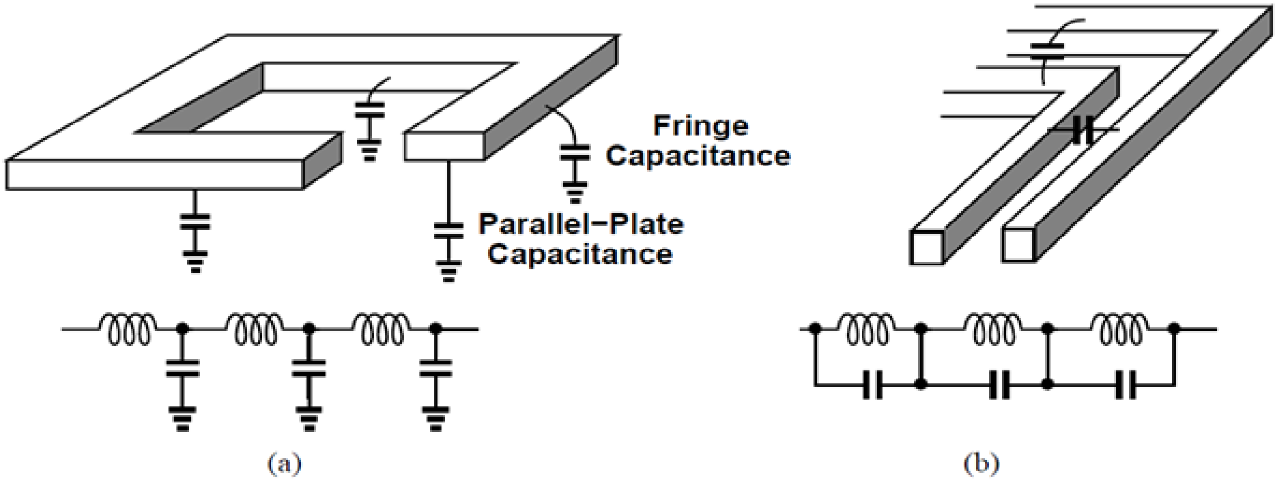
Inductance Equations

- Closed form inductance equations can be found based on
  - Curve fitting methods
  - Physical properties of inductors
- Various expressions have been reported in literature [1,2,3]. For example, an empirical formula that has less than 10% error for inductors in the range of 5 to 50 nH is given in [1] and can be reduced to the following form for a square spiral

$$L \approx 1.3 \times 10^{-7} \frac{A_m^{5/3}}{A_{tot}^{1/6} W^{1.75} (W + S)^{0.25}},$$

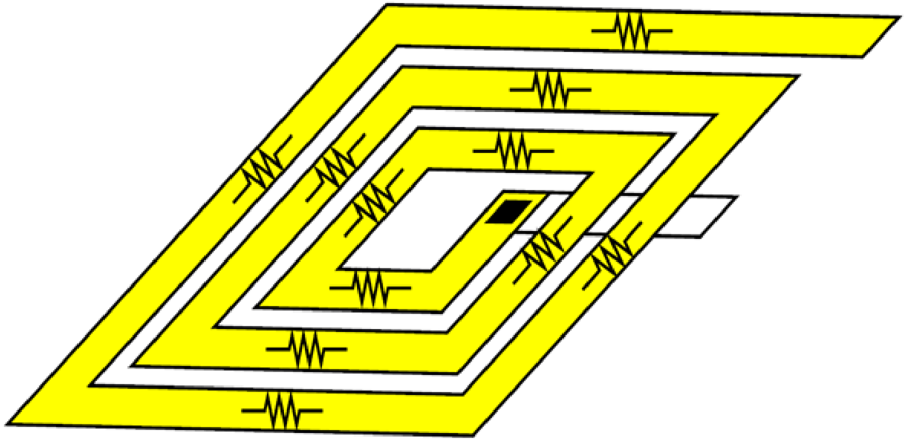
- Where A_m is the metal area (the shaded area) and $A_{tot} \cong D_{out}^2$ is the total inductor area
- All units are metric

Parasitic Capacitance of Integrated Inductors



- Planar spiral inductor suffers from parasitic capacitance because the metal lines of the inductor exhibit parallel plate capacitance and adjacent turns bear fring capacitance

Loss Mechanisms: Metal Resistance



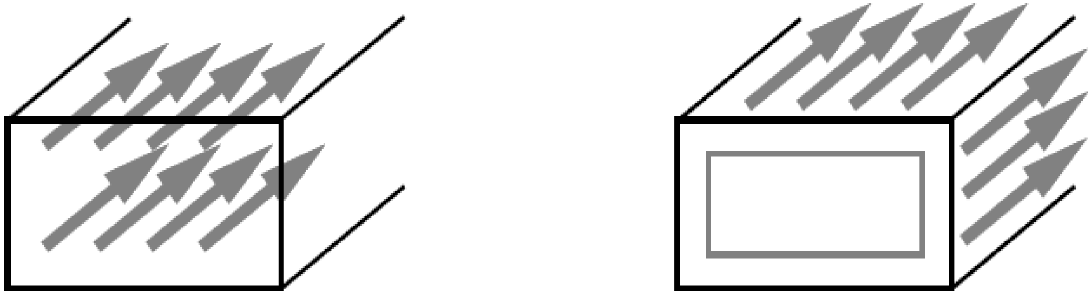
- Suppose the metal line forming an inductor exhibits a series resistance, R_S
- The Q may be defined as the ratio of the desirable impedance, $\omega_0 L_1$, and the undesirable impedance, R_S:

$$Q = \frac{L_1 \omega_0}{R_S}$$

- For example, a 5-nH inductor operating at 5 GHz with an R_S of 15.7Ω has a Q of 10

Loss Mechanisms – Skin Effect

Current distribution in a conductor



(a) At low frequency (b) At high frequency

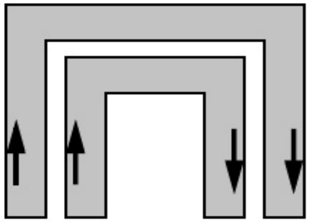
- The skin depth δ is given by

$$\delta = \frac{1}{\sqrt{\pi \cdot f \cdot \mu \cdot \sigma}}$$

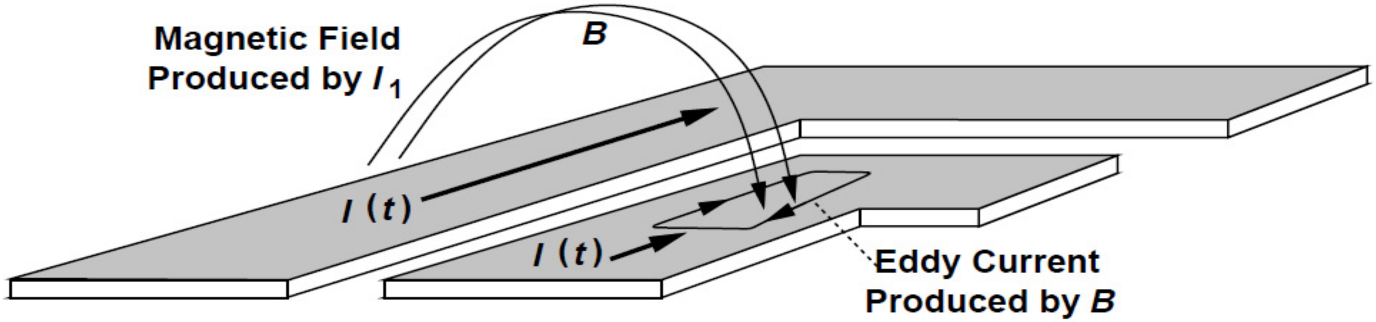
- where f denotes the frequency, μ the permeability, and σ the conductivity. For example, $\delta \approx 1.4\mu m$ at 10 GHz for aluminum. The extra resistance of a conductor due to the skin effect is equal to

$$R_{skin} = \frac{1}{\sigma \cdot \delta} \propto \sqrt{f}$$

Skin Effect – Current Crowding Effect



(a) Current distribution in adjacent turns



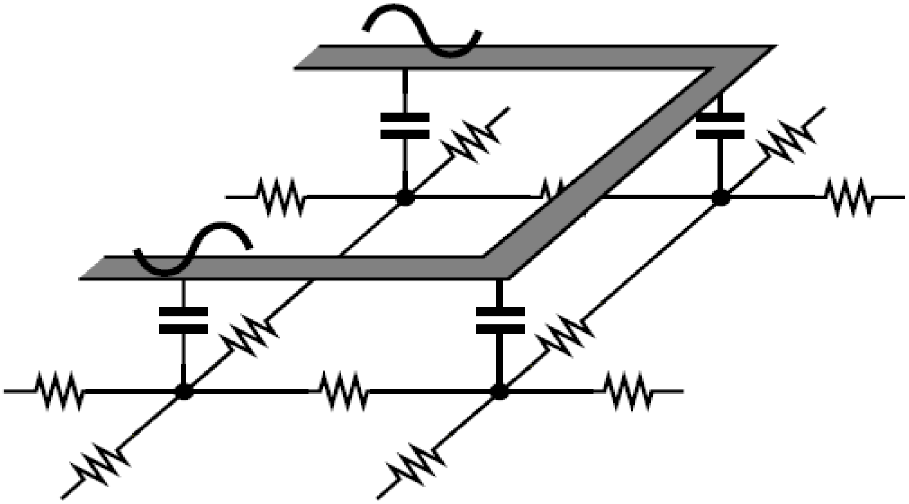
(b) Detailed view of (a)

- For $f \geq f_{crit}$, the magnetic field produced by adjacent turn induces eddy current, causing unequal distribution of current across the conductor width, hence altering the effective resistance of the turn
- For $f \geq f_{crit}$, the effective resistance R_{eff} therefore increases according to

$$R_{eff} \cong R_0 \left[1 + \frac{1}{10} \left(\frac{f}{f_{crit}} \right)^2 \right] \quad \text{with} \quad f_{crit} \cong \frac{3.1}{2\pi\mu} \frac{W+S}{W^2} R_{\square}$$

- Where $R_{\square}$ represents the dc sheet resistance of the metal

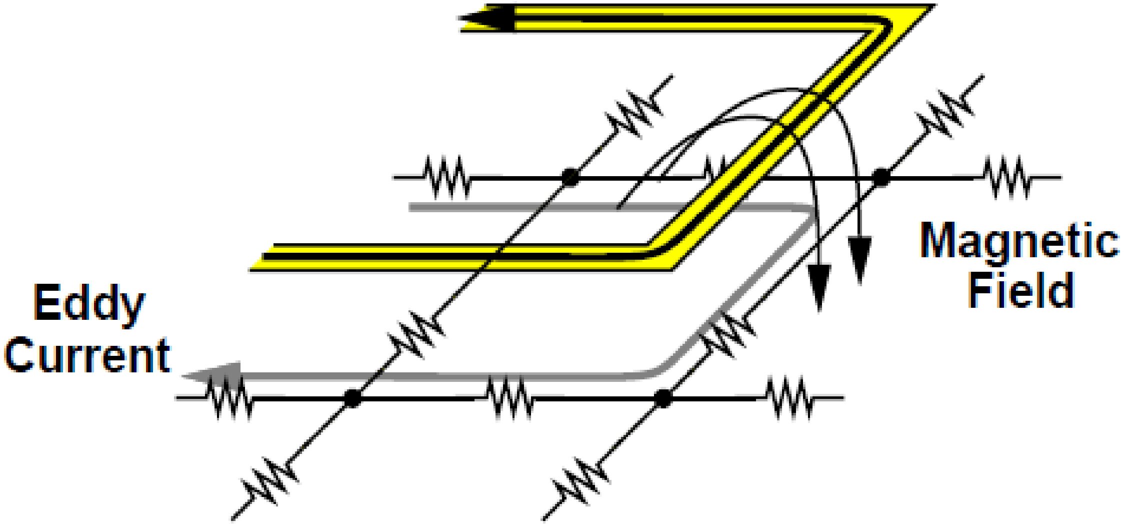
Capacitive Coupling to Substrate



Substrate loss due to capacitive coupling

- Voltage at each point of the spiral rise and fall with time causing displacement current flow between this capacitance and substrate
- This current causes loss and reduces the Q of the inductor

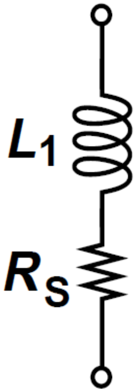
Magnetic Coupling to Substrate

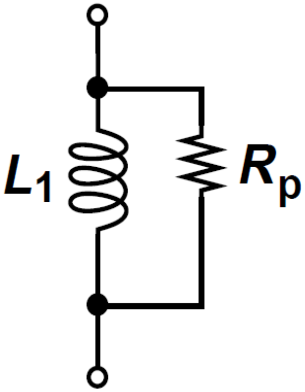


- The time varying inductor current generates eddy current in the substrate
- Lenz's law states that this current flows in the opposite direction
- The induction of eddy currents in the substrate can be viewed as transformer coupling

Modeling Loss by Series or Parallel Resistor

$$Q = \frac{\omega L_1}{R_S}$$

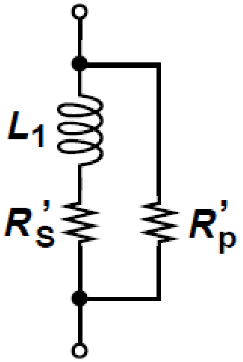




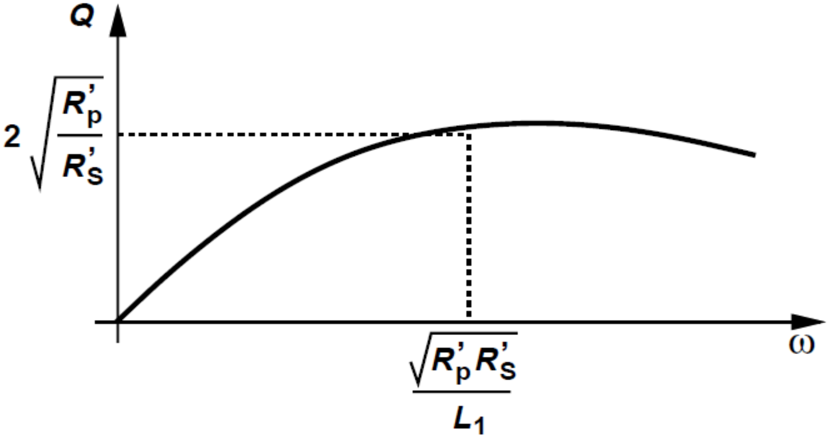
$$Q = \frac{R_p}{\omega L_1}$$

- A constant series resistance R_S model inductor loss for limited range of frequencies
- A constant parallel resistance R_p model inductor loss for narrow range of frequencies
- Note that the behavior of inductor Q predicted by above two models has suggested opposite trends of Q with frequency

Modeling Loss by Both Series and Parallel Resistors



Modeling loss by both parallel and series resistances



Resulting behavior of Q

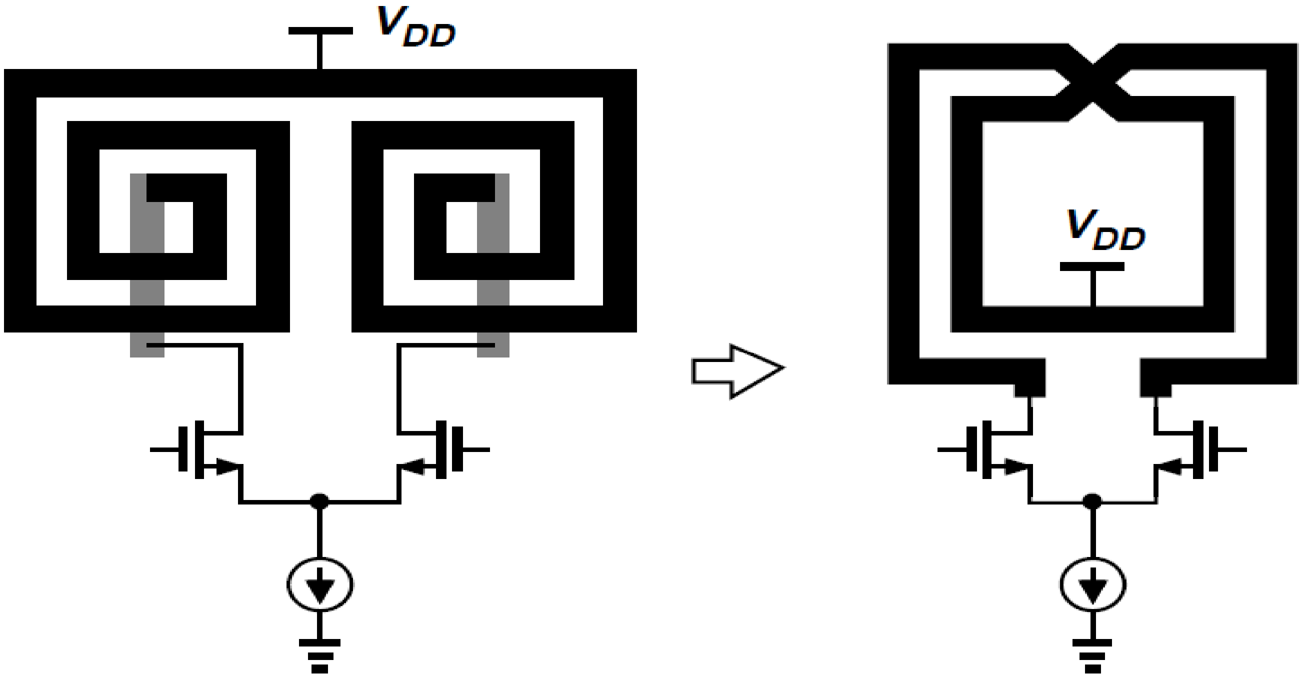
$$R'_S = \frac{\omega L_1}{2Q} \quad \text{and} \quad R'_p = 2Q\omega L_1$$

- The overall Q of the inductor is then given by

$$Q = \frac{\omega R'_p L_1}{\omega^2 L_1^2 + R'_S \cdot \left(R'_S + R'_p\right)}$$

- Which shows a maximum at $\sqrt{R'_p R'_S / L_1}$

Symmetric Inductor



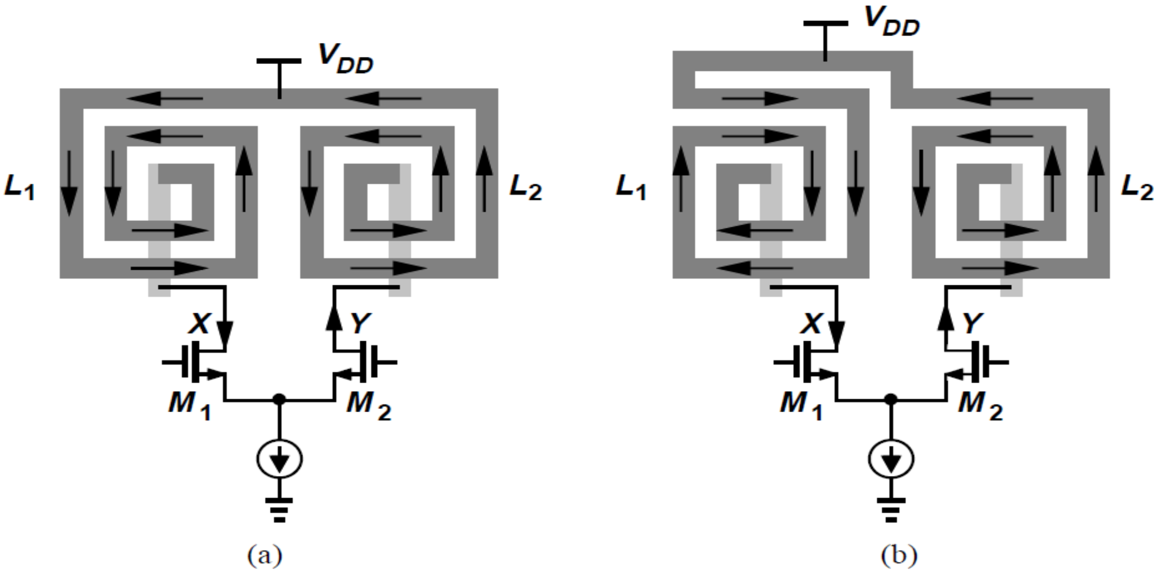
- Differential circuits can employ a single symmetric inductor instead of two asymmetric inductors
- It has two advantages:
  - Save area
  - Differential geometry also exhibit higher Q

Mirror/Step Symmetry of Single Ended Inductor

Load inductors in a differential pair

Mirror symmetry

Step symmetry

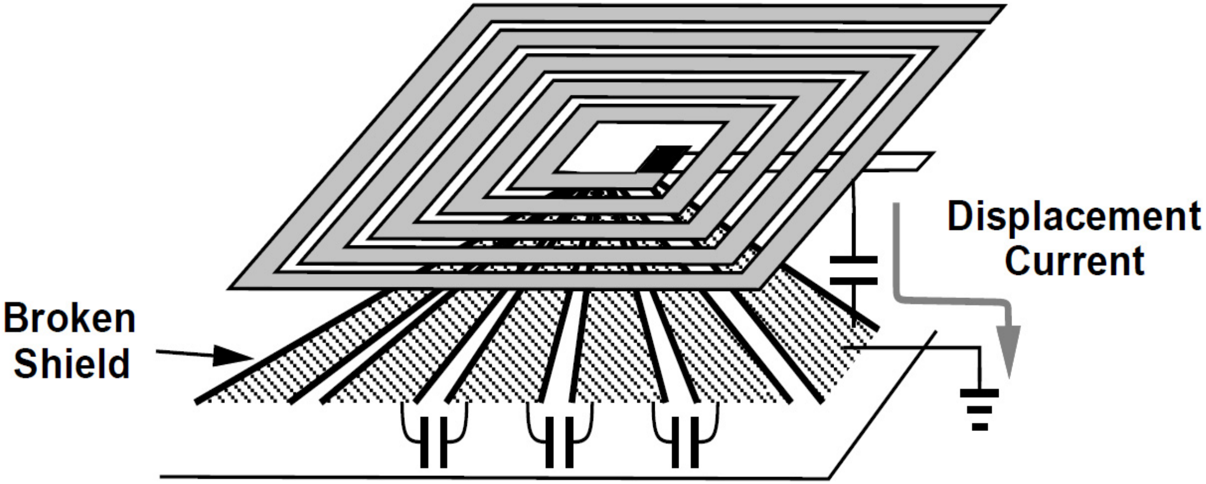


$$L_{eq} = L_1 + L_2 - 2M$$

$$L_{eq} = L_1 + L_2 + 2M$$

- Lower Q
- Higher Q

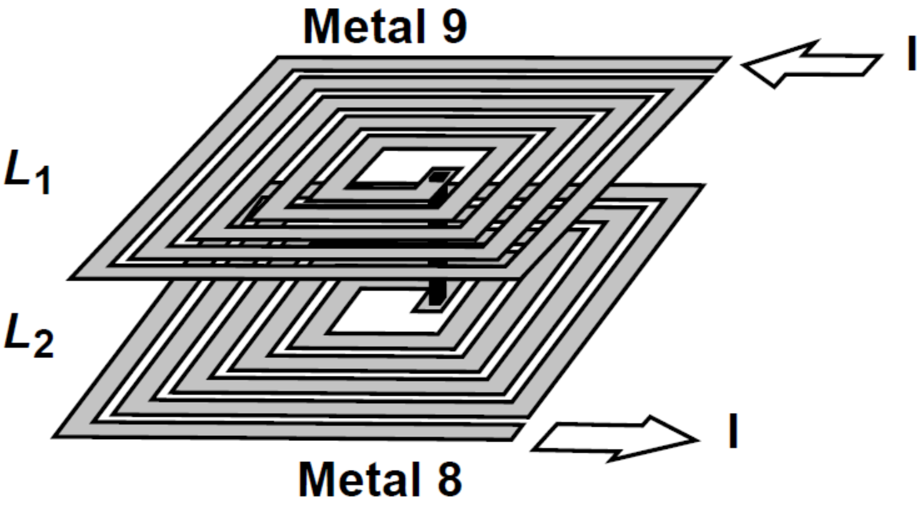
Inductors with Ground Shield



- This structure allows the displacement current to flow through the low resistance path to ground to avoid electrical loss through substrate
- Eddy currents through a continuous shield drastically reduce inductance and Q, so a “patterned” shield is used
- This shield reduces the effect of capacitive coupling to substrate
- Eddy currents of magnetic coupling still flows through substrate

Stacked Inductors

$$L_{tot} = L_1 + L_2 + 2M$$
$$M = L_1 = L_2$$
$$L_{tot} = 4L$$



- Similarly, N stacked spiral inductor operating in series raises total inductance by a factor of N^2

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- Varactors

Transformers

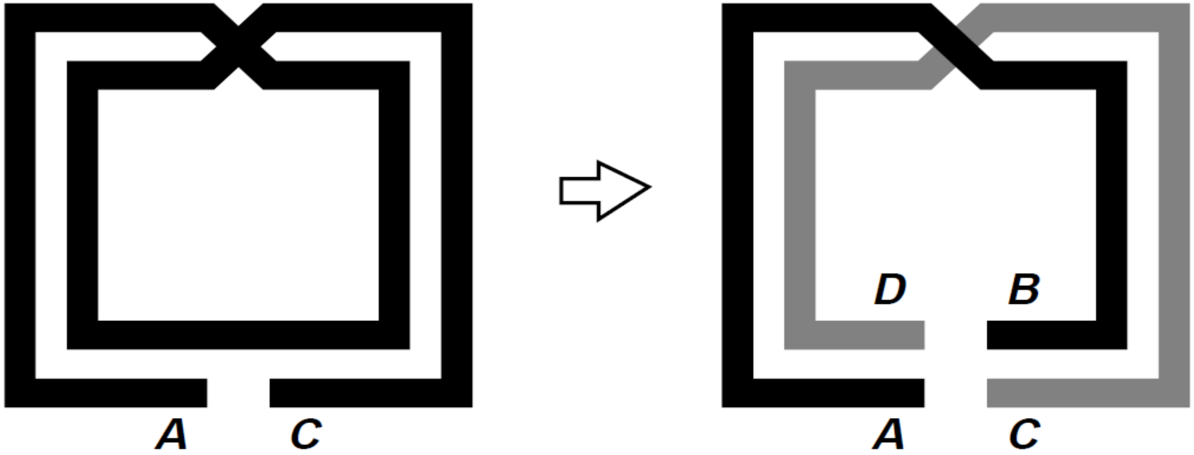
Useful function of transformer in RF Design:

- Impedance matching
- Feedback and feedforward with positive and negative polarity
- Single ended to differential conversion and vice-verse
- AC coupling between stages

Characteristics of Well-designed Transformers

- Low series resistance in primary and secondary windings
- High magnetic coupling between primary and secondary windings
- Low capacitive coupling between primary and secondary windings
- Low parasitic capacitance to the substrate

Transformer Structures

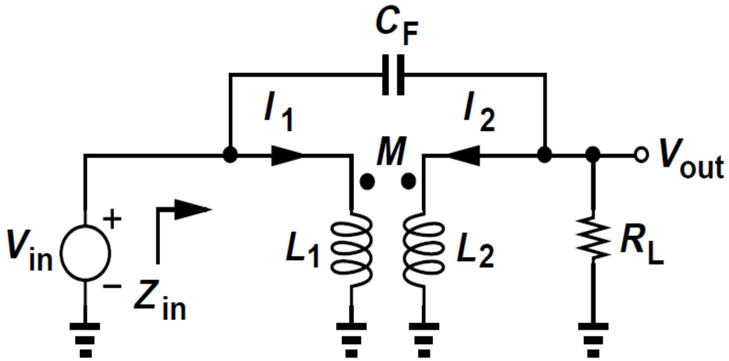


Transformer derived from a symmetric inductor

$$L_{AC} = 2L_{AB} + 2M$$

- Segments AB and CD are mutually coupled inductors
- Primary and secondary are identical so this is 1:1 transformer

Simple Transformer Model and its Transfer Function



- The transformer action gives

$$V_{in} = sL_1 \cdot I_1 + sM \cdot I_2$$

$$V_{out} = sM \cdot I_1 + sL_2 \cdot I_2$$

- Finding I_1 from 1st equation and replacing in the 2nd equation leads to

$$I_2 = \frac{V_{out}}{sL_2} - \frac{M\left(V_{in} - sM \cdot I_2\right)}{sL_1 L_2}$$

- KCL at output node yields

$$sC_F \cdot \left(V_{in} - V_{out}\right) - I_2 = \frac{V_{out}}{R_L}$$

Simple Transformer Model and its Transfer Function

- Replacing I_2 in above equation and simplifying the result, we obtain

$$\frac{V_{out}}{V_{in}} = \frac{s^2 L_1 L_2 C_F \cdot \left(1 - \frac{M^2}{L_1 L_2}\right) + M}{s^2 L_1 L_2 C_F \cdot \left(1 - \frac{M^2}{L_1 L_2}\right) + s\frac{L_1 L_2}{R_L} \cdot \left(1 - \frac{M^2}{L_1 L_2}\right) + L_1}$$

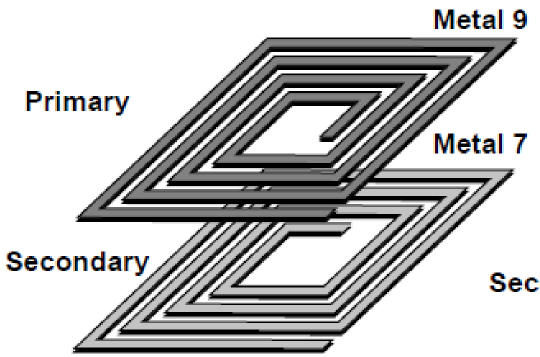
- Setting $C_F = 0$ in the above equation leads to the input to output transfer function

$$\frac{V_{out}}{V_{in}} = \frac{M}{s\frac{L_1 L_2}{R_L} \cdot \left(1 - \frac{M^2}{L_1 L_2}\right) + L_1}$$

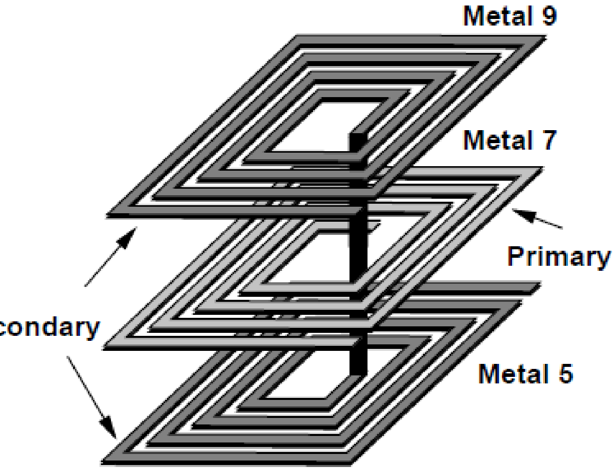
- The input impedance is given by

$$Z_{in} = sL_1 - \frac{s^2 M^2}{R_L + sL_2}$$

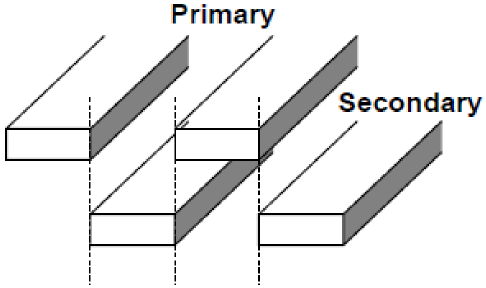
Stacked Transformers



One to one Stack transformer



One to two Stack transformer



Staggering of turns to reduce capacitive coupling

- Higher magnetic coupling
- Unlike planar structures, primary and secondary can be identical and symmetrical
- Overall area is less than planar structure
- Larger capacitive coupling compared to planar structure

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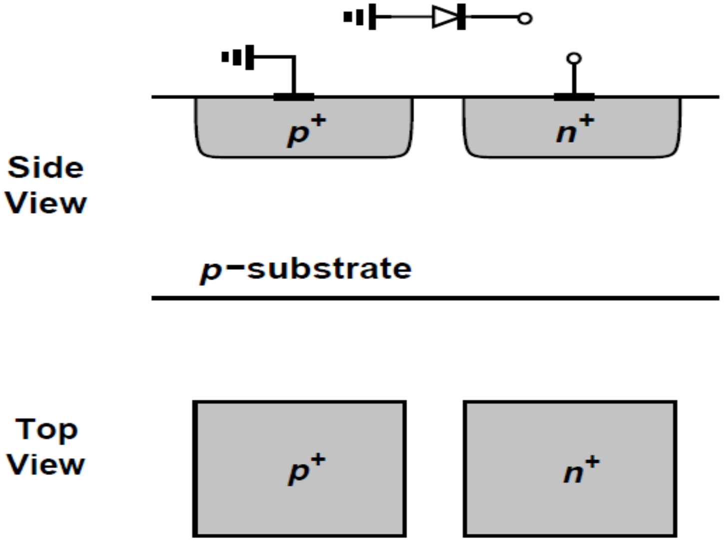
Varactors

- Varactor is a voltage-dependent capacitor
- Two important attributes of varactor design become critical in oscillator design
  - The capacitance range i.e. ratio of maximum to minimum capacitance that varactor can provide
  - The quality factor of the varactor

PN Junction Varactor

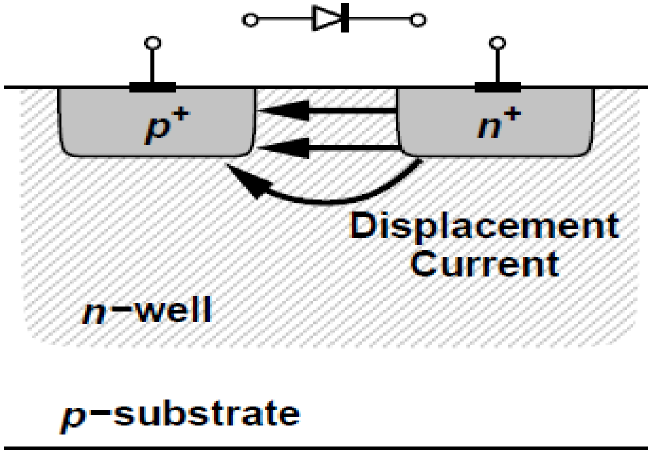


$$C_j = \frac{C_{j0}}{\left(1 + \frac{V_D}{V_0}\right)^m}$$

where C_{j0} is the capacitance at zero bias voltage, V_0 the built-in potential and m is an exponent around 0.3 in integrated structure

- Note that junction varactor have a weak dependence of C_j upon V_D, because for $V_{D,max} = 1V$, then $C_{j,max}/C_{j,min} \approx 1.23$ (Low range)

Varactor Q Calculation Issues



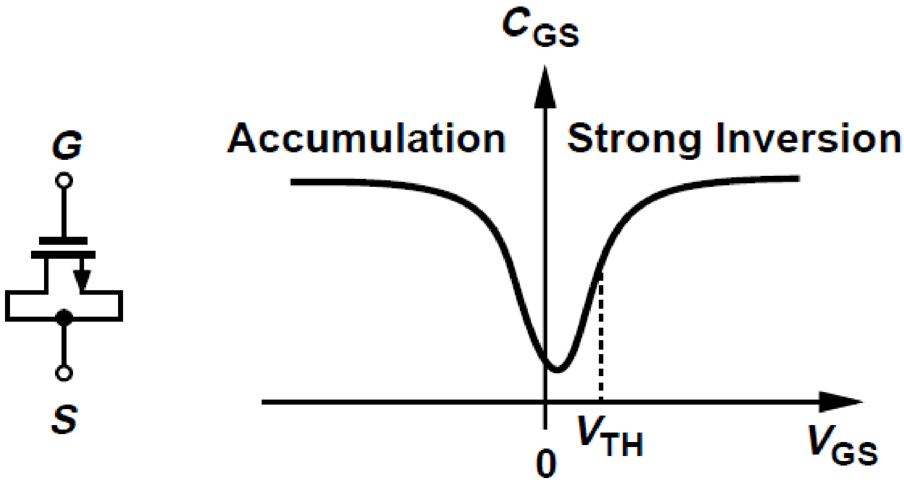
Current distribution in varactor

Q of varactor is obtained by measurement on fabricated structures

Difficult to calculate it because of the 2D current distribution

- As shown above, due to the two dimensional flow of current it is difficult to compute the equivalent series resistance of the structure
- N-well sheet resistance can not be directly applied to calculation of varactor series resistance

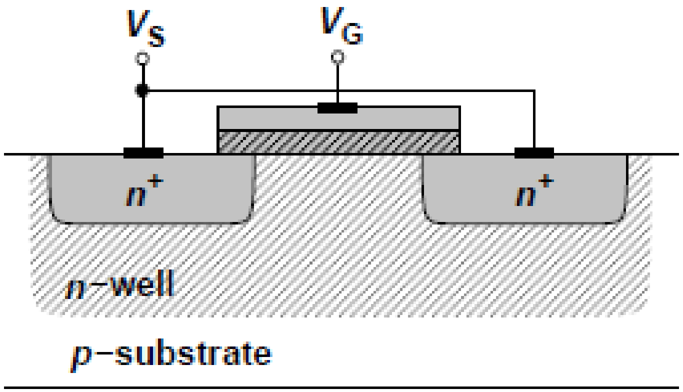
MOS Varactor

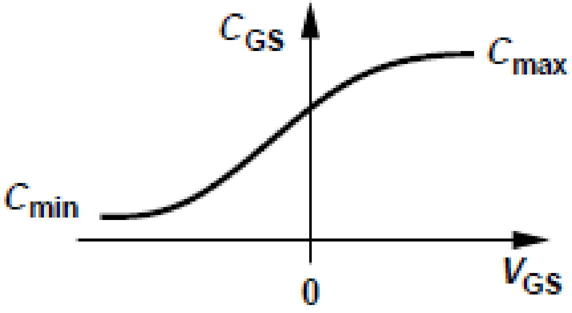


Variation of gate capacitance with V_{GS} for a regular MOS device

- A regular MOSFET exhibits a voltage dependent gate capacitance
- The non-monotonic behavior with respect to gate voltage limits the design flexibility

Accumulation Mode MOS Varactor

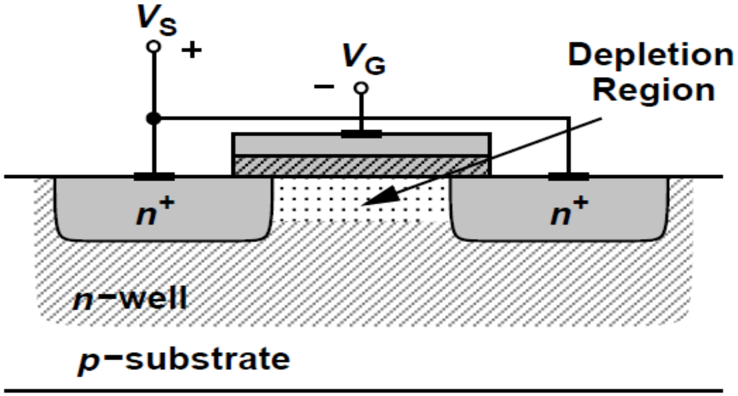




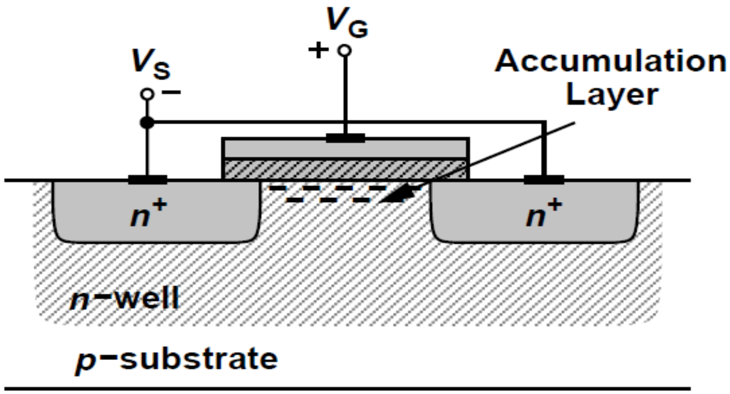
C/V characteristics of varactor

- Accumulation-mode MOS varactor is obtained by placing an NMOS inside an nwell
- The variation of capacitance with V_{GS} is monotonic
- The C/V characteristics scale well with scaling in technology
- Unlike PN junction varactor this structure can operate with positive and negative bias so as to provide maximum tuning range

Accumulation Mode MOS Varactor Operation

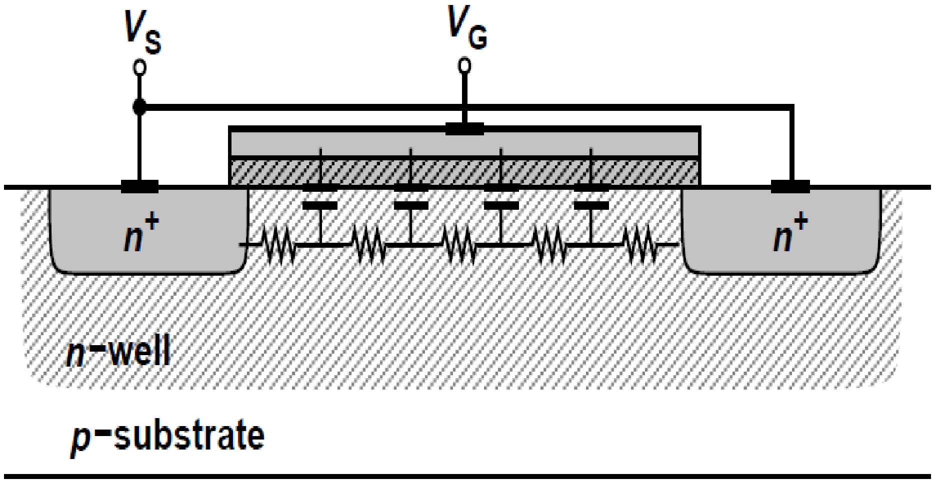


- $V_G < V_S$
- Depletion region is formed under gate oxide
- Equivalent capacitance is the series combination of gate capacitance and depletion capacitance

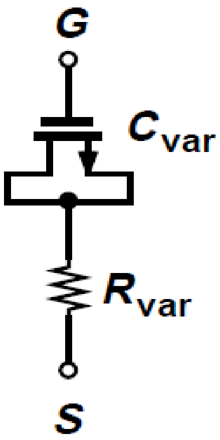


- $V_G > V_S$
- Formation of channel under gate oxide

Q of Accumulation mode MOS Varactor



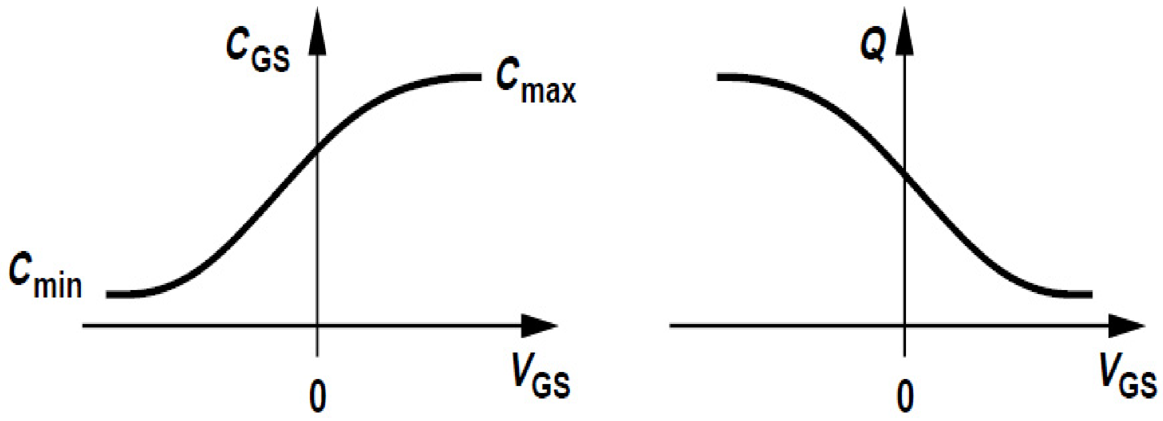
(a)



(b)

- The Q of the varactor is determined by the resistance between source and drain terminals
- Approximately calculated by lumped model shown in above

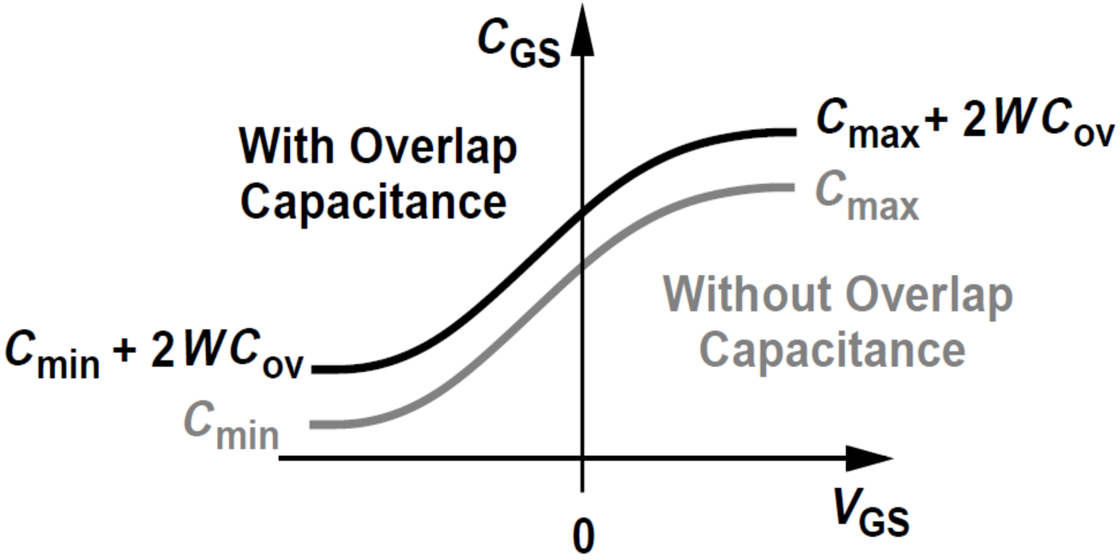
Variation of MOS Varactor Q with Capacitance



Variation of varactor Q with capacitance

- For C_{min}, the capacitance is small and resistance is large
- For C_{max}, the capacitance is large and resistance is small
- Above comments suggest that Q remains relatively constant
- In practice, Q drops as we increase capacitance from C_{min} to C_{max}, suggesting that relative rise in capacitance is greater than fall in resistance

Effect of Overlap Capacitance on Capacitance Range



- Overlap capacitance is relatively voltage independent.
- Overlap capacitance shifts the C/V characteristics up, yielding a ratio of

$$\frac{C_{\max} + 2WC_{ov}}{C_{\min} + 2WC_{ov}}$$

References

Most of this Chapter is based on Chapter 7 of Reference [1]

[1] B. Razavi, *RF Microelectronics*, 2nd ed. Pearson, 2012.