MICRO-461
Low-power Radio Design for the IoT

6. Noise Modeling at RF

Christian Enz

Integrated Circuits Lab (ICLAB), Institute of Microengineering (IMT), School of Engineering (STI)

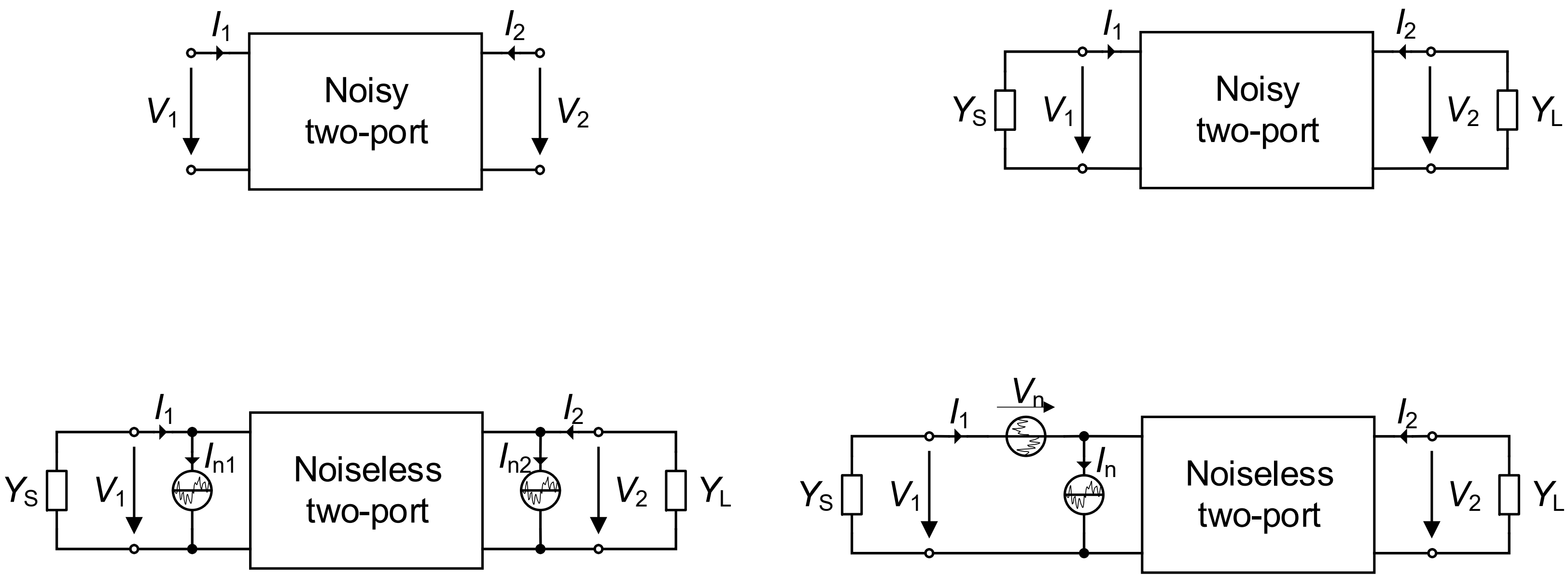
Swiss Federal Institute of Technology, Lausanne (EPFL), Switzerland

EPFL

Outline

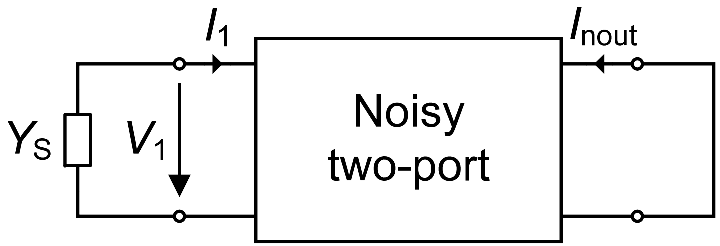
- **Noise in Two-port Networks**
- Noise in the MOS Transistor at RF

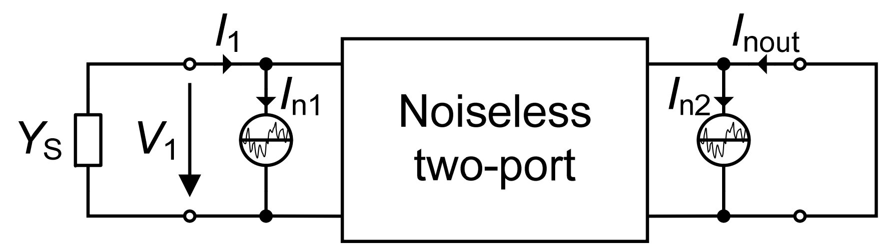
Noisy Two-port – Equivalent Two-port Circuits



- Model the noisy two-port such that the two circuit are equivalent

Noisy Two-port – Y-parameter Representation





- The output noise current I_{nout} depends on the two-port internal noise sources and on the source admittance Y_S
- A model of the noisy two-port that is independent of the source admittance Y_S requires at least **two noise sources** (either two current sources or two voltage sources or a combination)
- Using the Y-parameter representation, the noisy two-port can then be modeled by

$$I_1 = Y_{11} \cdot V_1 + Y_{12} \cdot V_2 + I_{n1} \qquad I_{n1} \triangleq I_1\big|_{V_1=V_2=0}$$

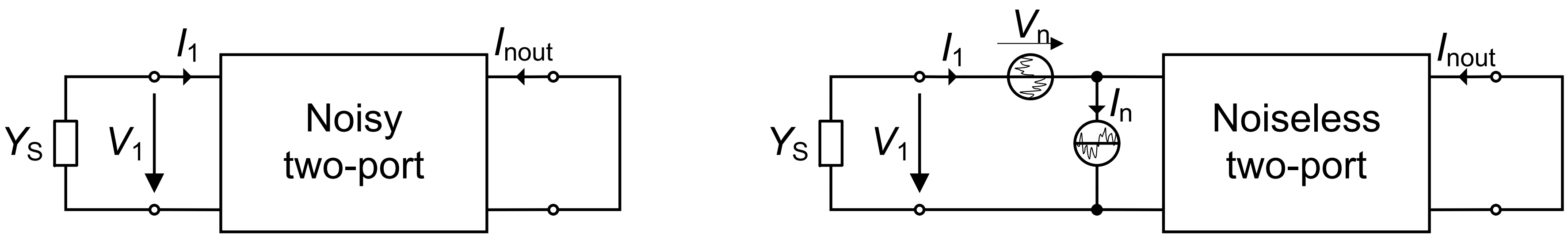
$$I_2 = Y_{21} \cdot V_1 + Y_{22} \cdot V_2 + I_{n2} \qquad I_{n2} \triangleq I_2\big|_{V_1=V_2=0}$$

where I_{n1} and I_{n2} represent all the noise sources within the two-port and are defined as the input and output currents when the input and output are short-circuited

G. Gonzales, *Microwave Transistor Amplifiers – Analysis and Design*, 2nd ed., Prentice-Hall, 1997, Appendix L.

Noisy Two-port – ABCD Matrix Representation

- The noisy two-port can also be modelled by referring all the noise sources at the input using the ABCD-parameters as shown below



$$V_1 = A \cdot V_2 - B \cdot I_2 + V_n$$
$$I_1 = C \cdot V_2 - D \cdot I_2 + I_n$$

$$V_n \triangleq B \cdot I_2\big|_{V_1=V_2=0} = -\frac{1}{Y_{21}} \cdot I_2\big|_{V_1=V_2=0}$$

$$I_n \triangleq D \cdot I_2\big|_{I_1=V_2=0} = -\frac{Y_{11}}{Y_{21}} \cdot I_2\big|_{I_1=V_2=0}$$

where V_n is a noise voltage source that represents all the noise of the device referred to the input when the **source impedance is zero** (input short-circuited) and I_n is a noise current source that represents all noise of the device referred to the input when the **source admittance is zero** (input open circuited)

G. Gonzales, *Microwave Transistor Amplifiers – Analysis and Design*, 2nd ed., Prentice-Hall, 1997, Appendix L.

Relation Between Noise Sources

- Noise sources of both representations I_{n1}, I_{n2} and V_n, I_n are related to each other by

$$V_n = -\frac{I_{n2}}{Y_{21}} \qquad I_{n1} = I_n - Y_{11} \cdot V_n$$

$$I_n = I_{n1} - \frac{Y_{11}}{Y_{21}} \cdot I_{n2} \qquad I_{n2} = -Y_{21} \cdot V_n$$

- Since both of these sources (V_n and I_n or I_{n1} and I_{n2}) are due to the same physical noise sources within the device, they are usually **correlated**. The mean-square values of sources V_n and I_n can be written in terms of the mean square values of sources I_{n1} and I_{n2} according to

$$\overline{|V_n|^2} = \frac{\overline{|I_{n2}|^2}}{|Y_{21}|^2} \quad \text{and} \quad \overline{|I_n|^2} = \overline{|I_{n1}|^2} + \left|\frac{Y_{11}}{Y_{21}}\right|^2 \cdot \overline{|I_{n2}|^2} - 2\Re\left\{\frac{Y_{11}^*}{Y_{21}^*} \cdot \overline{I_{n1} \cdot I_{n2}^*}\right\}$$

where the last term accounts for the **correlation** existing between source I_{n1} and I_{n2}. The latter has to be evaluated from the internal noise sources.

G. Gonzales, *Microwave Transistor Amplifiers – Analysis and Design*, 2nd ed., Prentice-Hall, 1997, Appendix L.

Correlation Admittance

- To account for the **correlation** usually existing between noise sources V_n and I_n, noise source I_n can be written as

$$I_n = I_{nu} + I_{nc} = I_{nu} + Y_c \cdot V_n$$

where I_{nu} stands for the part of I_n **uncorrelated** to V_n and I_{nc} represents the part of I_n that is **fully correlated** to V_n

- The **correlation admittance** Y_c is then defined as

$$Y_c \triangleq \frac{\overline{I_n \cdot V_n^*}}{\overline{V_n^2}}$$

and the **mean-square value** of source I_n is then given by

$$\overline{|I_n|^2} = \overline{|I_{nu}|^2} + |Y_c|^2 \cdot \overline{|V_n|^2} + Y_c^* \cdot \underbrace{\overline{I_{nu} \cdot V_n^*}}_{=0} + Y_c \cdot \underbrace{\overline{V_n \cdot I_{nu}^*}}_{=0} = \overline{|I_{nu}|^2} + \underbrace{|Y_c|^2 \cdot \overline{|V_n|^2}}_{=\overline{|I_{nc}|^2}}$$

G. Gonzales, *Microwave Transistor Amplifiers – Analysis and Design*, 2nd ed., Prentice-Hall, 1997, Appendix L.

Correlation Coefficient

- The mean square values of the correlated and uncorrelated sources can also be written as

$$\overline{|I_{nc}|^2} = |Y_c|^2 \cdot \overline{|V_n|^2} = |c|^2 \cdot \overline{|I_n|^2}$$
$$\overline{|I_{nu}|^2} = \overline{|I_n|^2} - |Y_c|^2 \cdot \overline{|V_n|^2} = \left(1 - |c|^2\right) \cdot \overline{|I_n|^2}$$

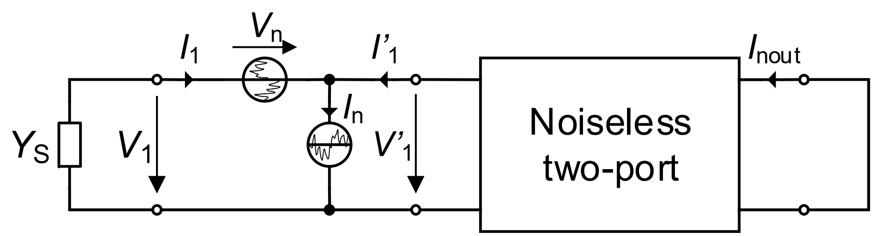
where c is the normalized **correlation factor** defined as

$$c \triangleq \frac{\overline{I_n \cdot V_n^*}}{\sqrt{\overline{|I_n|^2} \cdot \overline{|V_n|^2}}} = Y_c \cdot \sqrt{\frac{\overline{|V_n|^2}}{\overline{|I_n|^2}}}$$

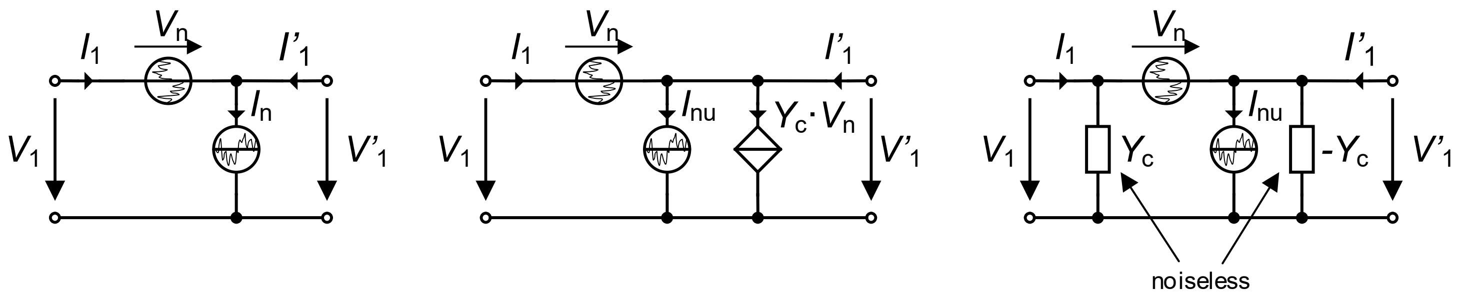
where the following relation has been used to derive the last equation

$$\overline{I_n \cdot V_n^*} = \underbrace{\overline{I_{nu} \cdot V_n^*}}_{=0} + Y_c \cdot \overline{|V_n|^2} = Y_c \cdot \overline{|V_n|^2}$$

Equivalent Circuits of the Noisy Two-port



- The noisy two-port can be modelled by a noiseless two-port in cascade with a noise two-port that includes noise sources V_n and I_n and hence models all the noise of the noisy two-port referred to the input
- The noise two-port can be modelled accounting for the correlation between the current and voltage noise source in the following equivalent ways



H. Rothe and W. Dahlke, "Theory of Noisy Fourpoles," Proc. of the IRE, Vol. 44, No. 6, June 1956.
H. Beneking, *High Speed Semiconductor Devices*, Chapman-Hall, 1994.

Power Spectral Densities

- Noise sources V_n and I_n are described by their noise power (or rms values) considering a common noise bandwidth
- For narrow-band systems, sources V_n and I_n can also be described by their power spectral densities S_v and S_i
- The noise current PSD can be splitted into a fully correlated term S_{ic} and an uncorrelated term S_{uc} according to

$$S_{ic} = |Y_c|^2 \cdot S_v = |c|^2 \cdot S_i \quad S_{iu} = S_i - |Y_c|^2 \cdot S_v = \left(1 - |c|^2\right) \cdot S_i$$

- where the correlation factor c is given by

$$c = Y_c \cdot \sqrt{\frac{S_v}{S_i}}$$

Equivalent Noise Resistance and Conductance

- Considering that the noise sources V_n, I_{nu} and I_{nc} are treated as thermal noise (even though they usually are not) produced by an equivalent resistance or conductance, the PSD can be rewritten as

$$S_v \triangleq 4kT \cdot R_v \quad S_i \triangleq 4kT \cdot G_i \quad S_{iu} \triangleq 4kT \cdot G_{iu} \quad S_{ic} \triangleq 4kT \cdot G_{ic}$$

resulting in

$$G_{iu} = G_i - |Y_c|^2 \cdot R_v = \left(1 - |c|^2\right) \cdot G_i \quad G_{ic} = |Y_c|^2 \cdot R_v = |c|^2 \cdot G_i$$

and the correlation factor is then given by

$$c = Y_c \cdot \sqrt{R_v / G_i} \quad \text{and} \quad |c|^2 = \left(G_c^2 + B_c^2\right) \cdot R_v / G_i$$

- Note that R_v, G_i, G_{iu} and G_{ic} are usually **frequency dependent**

G. Gonzales, *Microwave Transistor Amplifiers – Analysis and Design*, 2nd ed., Prentice-Hall, 1997, Appendix L.
T. Lee, *The Design of CMOS Radio-Frequency Integrated Circuits*, 2nd ed., Cambridge University Press, 2004.

Noise Factor Definition

- In many circuits and systems we are actually interested in the **signal-to-noise ratio** SNR defined as the ratio of the signal power to the noise power
- As the signal is amplified along the signal path, it also accumulates more noise
- The **noise factor** evaluates how the SNR is degraded along the path

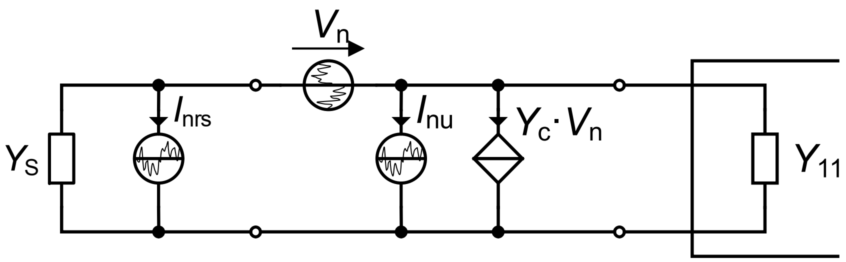
$$F \triangleq \frac{SNR_{in}}{SNR_{out}} > 1$$

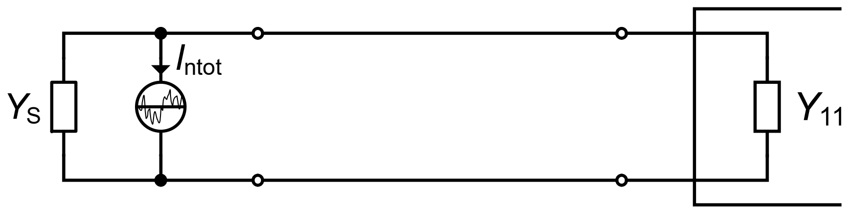
- For an amplifier having a power gain G we can write

$$F = \frac{S_i/N_i}{G \cdot S_i/\left(G \cdot \left(N_i + N_a\right)\right)} = \frac{N_i + N_a}{N_i} = 1 + \frac{N_a}{N_i}$$

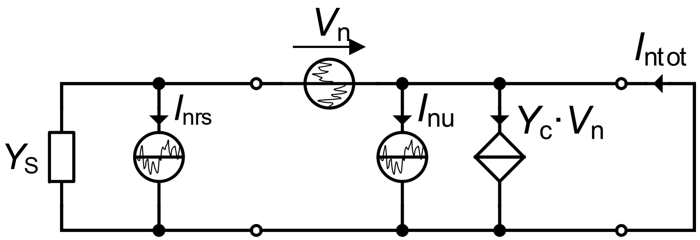
where $N_i + N_a$ is the total noise power referred to the amplifier input, N_a is the input-referred noise power added by the amplifier to the noise power already present at the input of the amplifier N_i

Equivalent Input-referred Total Noise Source





- The noise coming from the source I_{nrs} adds to the input-referred noise sources V_n and I_n of the two-port network to generate the total noise at the input of the noiseless two-port
- The total noise can hence be modelled by the equivalent Norton noise current source I_{ntot} which corresponds to the short-circuit current of the above left schematic



$$I_{ntot} = I_{nrs} + I_{nu} + (Y_s + Y_c) \cdot V_n$$

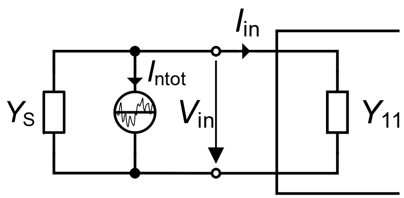
H. Beneking, *High Speed Semiconductor Devices*, Chapman-Hall, 1994.

Noise Factor of the Noisy Two-port Network

- The current delivered at the two-port input is given by

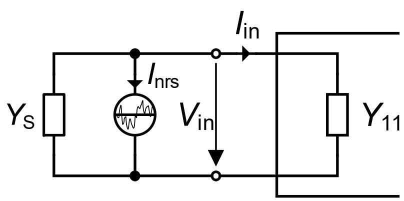
$$I_{in} = -\frac{Y_{11}}{Y_s + Y_{11}} \cdot I_{ntot}$$

- The total noise power delivered at the two-port input is then given by



$$N_{in}\big|_{tot} = \Re\{Z_{in}\} \cdot \overline{|I_{in}|^2} = \Re\{Z_{in}\} \cdot \frac{|Y_{11}|^2}{|Y_s + Y_{11}|^2} \cdot \overline{|I_{ntot}|^2}$$

- Whereas the rms noise power delivered at the input coming from the source is



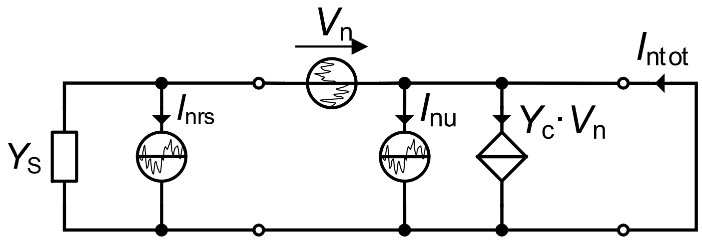
$$N_{in}\big|_{V_n=0, I_n=0} = \Re\{Z_{in}\} \cdot \frac{|Y_{11}|^2}{|Y_s + Y_{11}|^2} \cdot \overline{|I_{nrs}|^2}$$

- The noise factor is then simply given by

$$F = \frac{N_{in}\big|_{tot}}{N_{in}\big|_{V_n=0, I_n=0}} = \frac{\overline{|I_{ntot}|^2}}{\overline{|I_{nrs}|^2}}$$

Noise Factor of the Noisy Two-port Network

$$Y_s = G_s + jB_s$$



$$I_{ntot} = I_{nrs} + I_{nu} + (Y_s + Y_c) \cdot V_n$$

- Accounting for the fact that the noise source I_{nrs} is uncorrelated to the other noise sources V_n and I_{nu} and that the noise current source I_{nu} is by definition uncorrelated with the voltage noise source V_n, the mean-square value of the total noise current is then simply given by

$$\overline{|I_{ntot}|^2} = \overline{|I_{nrs}|^2} + \overline{|I_{nu}|^2} + |Y_c + Y_s|^2 \cdot \overline{|V_n|^2}$$

- From the above definition, the **noise factor** is then given by

$$F = \frac{\overline{|I_{ntot}|^2}}{\overline{|I_{nrs}|^2}} = 1 + \frac{\overline{|I_{nu}|^2} + |Y_c + Y_s|^2 \cdot \overline{|V_n|^2}}{\overline{|I_{nrs}|^2}}$$

$$\overline{|I_{nrs}|^2} = 4kTB \cdot G_s$$

G. Gonzales, *Microwave Transistor Amplifiers – Analysis and Design*, 2nd ed., Prentice-Hall, 1997, Appendix L.

Spot Noise Factor

- The **spot noise factor** is then obtained by replacing the mean square values by the PSD

$$F = 1 + \frac{S_{iu} + \left|Y_c + Y_s\right|^2 \cdot S_v}{4kT \cdot G_s} = 1 + \frac{G_{iu} + \left|Y_c + Y_s\right|^2 \cdot R_v}{G_s} =$$

$$= 1 + \frac{G_{iu}}{G_s} + \left[\left(G_s + G_c\right)^2 + \left(B_s + B_c\right)^2\right] \cdot \frac{R_v}{G_s}$$

where G_s, B_s, G_c and B_c are defined as

$$Y_s = G_s + jB_s \quad \text{and} \quad Y_c = G_c + jB_c$$

- Note that the spot noise factor is usually frequency dependent, but for narrow-band systems, it is about equal to the noise factor
- F increases with B_s but for a given B_s it has a **minimum** wrt G_s

G. Gonzales, *Microwave Transistor Amplifiers – Analysis and Design*, 2nd ed., Prentice-Hall, 1997, Appendix L.
T. Lee, *The Design of CMOS Radio-Frequency Integrated Circuits*, 2nd ed., Cambridge University Press, 2004.

Minimum Noise Figure

- The noise factor F reaches a minimum F_{min} for a particular value of the source admittance $Y_{opt} = G_{opt} + j \cdot B_{opt}$
- The optimum source conductance G_{opt} and susceptance B_{opt} can be expressed in terms of the **four circuit noise parameters** R_v, G_{iu}, G_c and B_c according to

$$G_{opt} = \sqrt{\frac{G_{iu}}{R_v} + G_c^2} = \sqrt{\frac{G_i}{R_v} - B_c^2} \quad \text{and} \quad B_{opt} = -B_c$$

- The **minimum noise factor** F_{min} is then given by

$$F_{\min} = 1 + 2R_v \cdot \left(G_{opt} + G_c\right) = 1 + 2R_v \cdot \left(\sqrt{\frac{G_{iu}}{R_v} + G_c^2} + G_c\right)$$

📖 G. Gonzales, *Microwave Transistor Amplifiers – Analysis and Design*, 2nd ed., Prentice-Hall, 1997, Appendix L.
📖 T. Lee, *The Design of CMOS Radio-Frequency Integrated Circuits*, 2nd ed., Cambridge University Press, 2004.

Noise Factor and Noise Parameters

- The actual noise factor F may also be written in terms of the four **noise parameters** F_{min}, R_v, G_{opt} and B_{opt} and the source admittance as

$$F = F_{\min} + \frac{R_v}{G_s} \cdot \left[\left(G_s - G_{opt} \right)^2 + \left(B_s - B_{opt} \right)^2 \right]$$

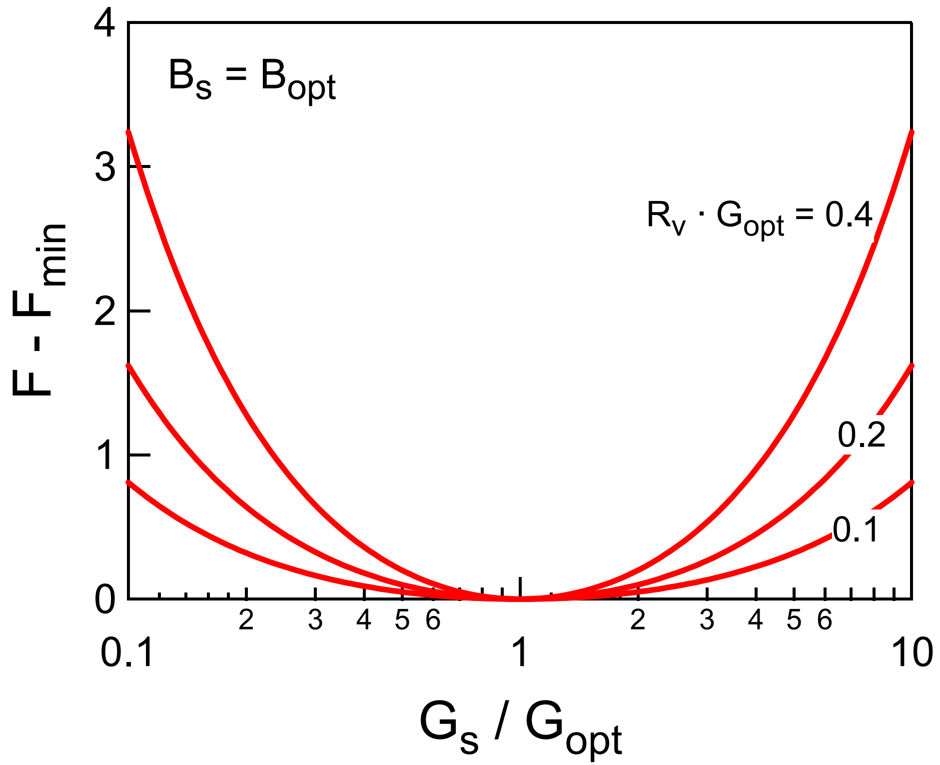
- In the same way power gain can be maximized by **impedance matching**, the noise can be minimized by setting the source admittance to Y_{opt}
- The situation $F = F_{min}$ obtained for $G_s = G_{opt}$ **AND** $B_s = B_{opt}$ corresponds to **noise matching**
- Noise matching usually does not coincide with **gain matching**
- The R_v/G_s tells us something about the relative sensitivity of the noise factor to departures from the optimum conditions

📖 G. Gonzales, *Microwave Transistor Amplifiers – Analysis and Design*, 2nd ed., Prentice-Hall, 1997, Appendix L.
📖 T. Lee, *The Design of CMOS Radio-Frequency Integrated Circuits*, 2nd ed., Cambridge University Press, 2004.

F_{min} versus Source Conductance G_s

- Assuming that $B_s = B_{opt}$, the noise increase due to deviation from the noise matching condition is given by

$$F - F_{\min}\big|_{B_s = B_{opt}} = R_v \cdot G_{opt} \cdot \frac{G_{opt}}{G_s} \cdot \left(\frac{G_s}{G_{opt}} - 1 \right)^2$$



Noisy Two-port Parameters

- The four noise parameters F_{min}, R_v, G_{opt} and B_{opt} are usually **obtained from measurements**
- They can then be used to derive the circuit noise parameters R_v, G_i, G_c and B_c of the noisy two-port circuit according to

$$G_i = \left|Y_{opt}\right|^2 \cdot R_v = \left(G_{opt}^2 + B_{opt}^2\right) \cdot R_v$$

$$G_c = \frac{F_{\min} - 2R_v G_{opt} - 1}{2R_v} \qquad B_c = -B_{opt}$$

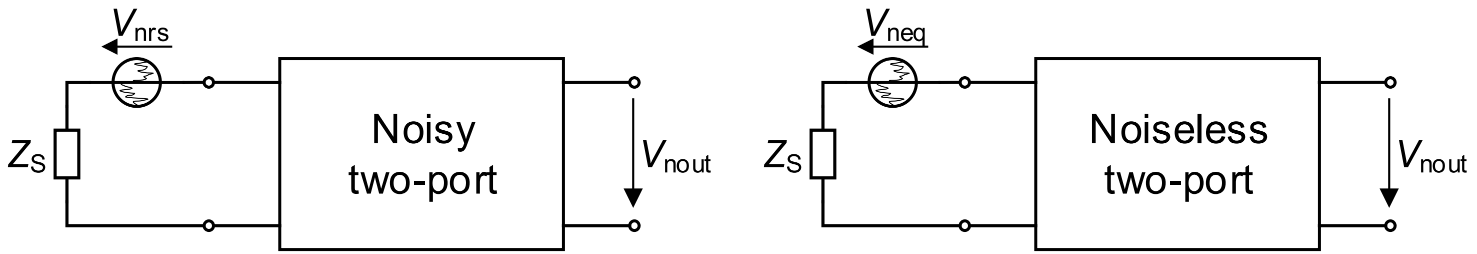
- G_{iu}, G_{ic} can then be calculated as

$$G_{iu} = \left(1 - |c|^2\right) \cdot G_i \qquad G_{ic} = |c|^2 \cdot G_i$$

with the correlation factor c given by

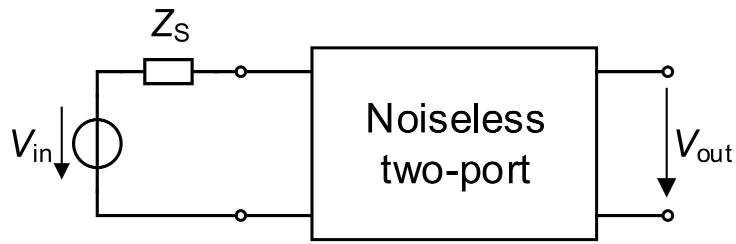
$$c = Y_c \cdot \sqrt{\frac{R_v}{G_i}} = \frac{G_c + jB_c}{\sqrt{G_{opt}^2 + B_{opt}^2}} \quad \text{and} \quad |c|^2 = \frac{G_c^2 + B_c^2}{G_{opt}^2 + B_{opt}^2}$$

Simplified Calculation of the Actual Noise Factor



- If we are only interested in the **actual noise factor** F, the latter can be derived without calculating F_{min}, R_v, G_{opt} and B_{opt}
- It can be calculated from the above circuits where $Z_S = R_S + jX_S$ and where V_{nrs} corresponds to the noise coming from the real part of the source impedance with a PSD given by $S_{V_{nrs}} = 4kTR_S$
- First, calculate the output noise voltage V_{nout} and related PSD $S_{V_{nout}}$ accounting for all the noise sources inside the two-port and the noise source V_{nrs} with PSD

Simplified Calculation of the Actual Noise Factor



- Calculate the voltage gain $A_v = V_{out}/V_{in}$ which is in general complex, depends on frequency and on the source impedance Z_S
- Calculate the input-referred noise PSD

$$S_{V_{neq}} = \frac{S_{V_{nout}}}{|A_v|^2}$$

- The noise factor is then given by

$$F = \frac{S_{V_{neq}}}{S_{V_{nrs}}} = \frac{S_{V_{neq}}}{4kTR_S} = \frac{R_{neq}}{R_S} = 1 + \frac{R_{namp}}{R_S}$$

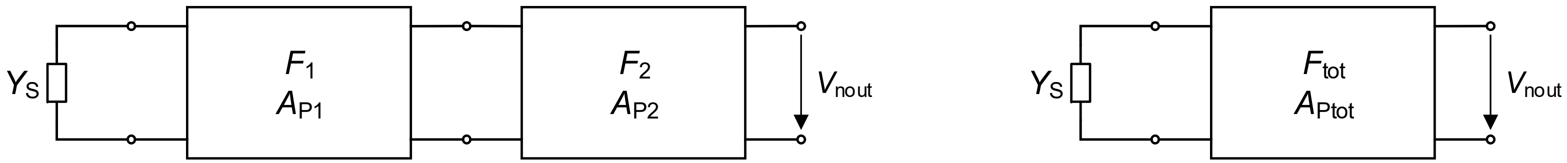
- where $R_{neq} = S_{V_{neq}}/(4kT)$ and $R_{namp} = R_{neq}\big|_{R_S=0}$ is the input-referred noise resistance due only to the two-port noise sources

Simplified Calculation of the Actual Noise Factor

- This method is usually simpler than deriving F_{min}, R_v, G_{opt} and B_{opt} particularly when the source impedance is reduced to a resistance which is usually the case for 50Ω systems, but it does not provide any information on the minimum achievable noise factor
- However F_{min} could be obtained from the above calculation but for this, the source impedance has to remain complex $Z_S = R_S + jX_S$
- F_{min} can then be obtained by differentiating F with respect to R_S and X_S
- R_{opt} and X_{opt} are obtained by solving

$$\left.\frac{\partial F}{\partial R_S}\right|_{X_S=const} = 0 \quad \text{and} \quad \left.\frac{\partial F}{\partial X_S}\right|_{R_S=const} = 0$$

The Friis Formula



- The noise factor F_{tot} of a cascade of two amplifiers each characterized by their noise factors F_1 and F_2 and their available power gains A_{P1} and A_{P2} is given by

$$F_{tot} = F_1 + \frac{F_2 - 1}{A_{P1}}$$

- The available power gain A_{P1} is defined as the available power at its output (the power that it would deliver to a matched load) divided by the available source power (the power that the source would deliver to a matched load)

$$A_{P1} = \frac{R_{in1}^2}{(R_S + R_{in1})^2} \cdot A_{v1}^2 \cdot \frac{R_S}{R_{out1}}$$

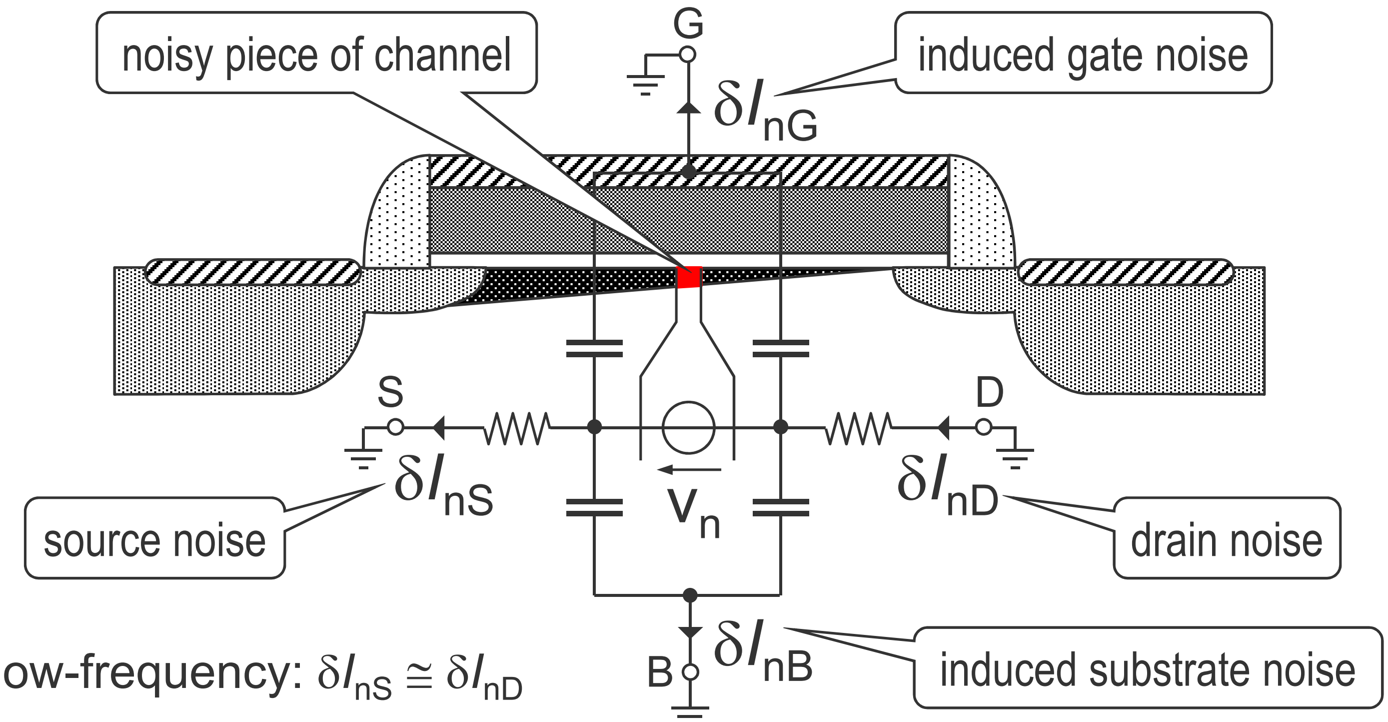
- Can be generalized to the cascade of m stages

$$F_{tot} = 1 + (F_1 - 1) + \frac{F_2 - 1}{A_{P1}} + \ldots + \frac{F_m - 1}{A_{P1} \cdot A_{P2} \cdots A_{Pm-1}}$$

Outline

- Noise in Two-port Networks
- **Noise in the MOS Transistor at RF**

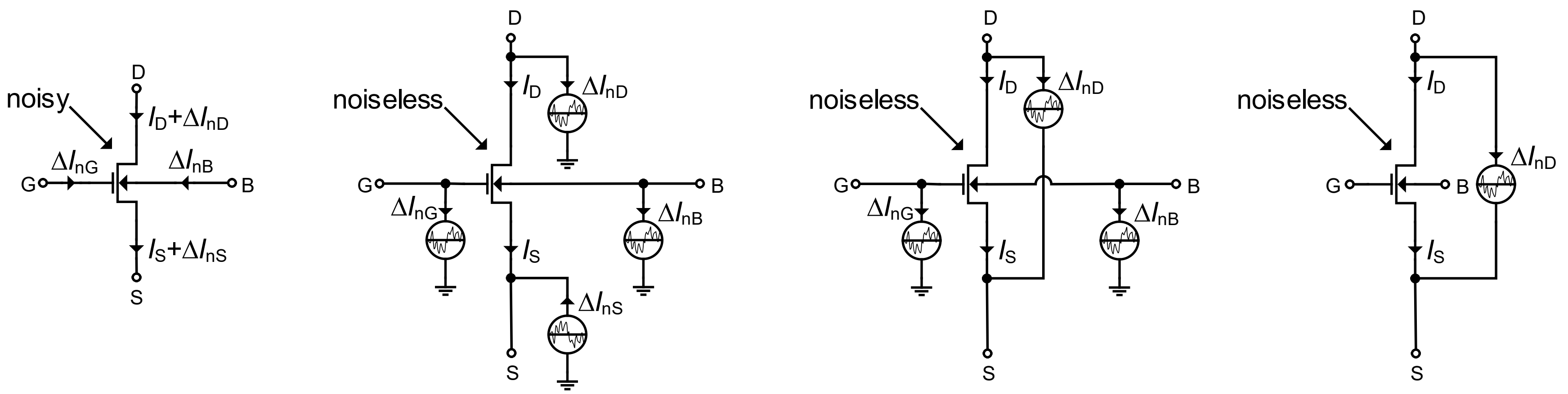
Channel Noise and Terminal Noise Currents



At low-frequency: $\delta I_{nS} \cong \delta I_{nD}$

- The thermal noise generated by voltage fluctuations in the channel appears at the drain, source but also at the gate and bulk as terminal current fluctuations
- The channel voltage fluctuations are transferred to the drain and source through the (trans)conductances and to the gate and bulk by capacitive coupling

General and Simplified HF Thermal Noise Model



Noisy transistor | NQS noise model | QS noise model | LF noise model

- Normally requires one noise source per terminal
- Under **quasi-static** assumption, the source and drain noise PSD and the drain-source cross-PSD are approximately equal
- At low-frequency ($\omega \ll \omega_{qs}$), the induced gate and substrate noise can be ignored
- The thermal noise model then reduces to the single noise source between drain and source

A.-S. Porret and C. Enz, IEE Proc.-Circuits Devices Syst., April 2004.

Thermal Noise at the Drain (long-channel)

- Channel thermal noise power spectral density (PSD)

$$S_{\Delta I_{nD}^2} = 4kT \cdot G_{nD} \quad \text{with} \quad G_{nD} = \delta_{nD} \cdot G_{ms} = \gamma_{nD} \cdot G_m$$

where δ_{nD} is the **drain thermal noise parameter** and $\gamma_{nD} = n \cdot \delta_{nD}$ the **thermal noise excess factor**

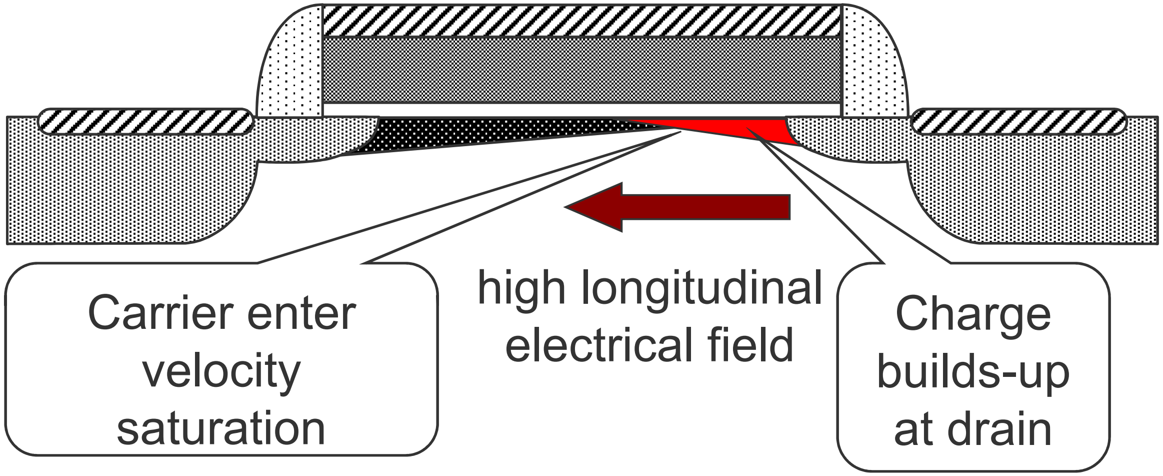
- The drain conductance G_{nD} is **bias dependent** according to

$$\frac{G_{nD}}{G_{spec}} \cong \frac{2}{3} \cdot \frac{q_s^2 + \frac{3}{4} q_s + q_s q_d + \frac{3}{4} q_d + q_d^2}{q_s + q_d + 1} = \begin{cases} \frac{2}{3} \cdot \frac{q_s^2 + q_s q_d + q_d^2}{q_s + q_d} & SI \\ \frac{1}{2} \cdot (q_s + q_d) & WI \end{cases}$$

- In saturation

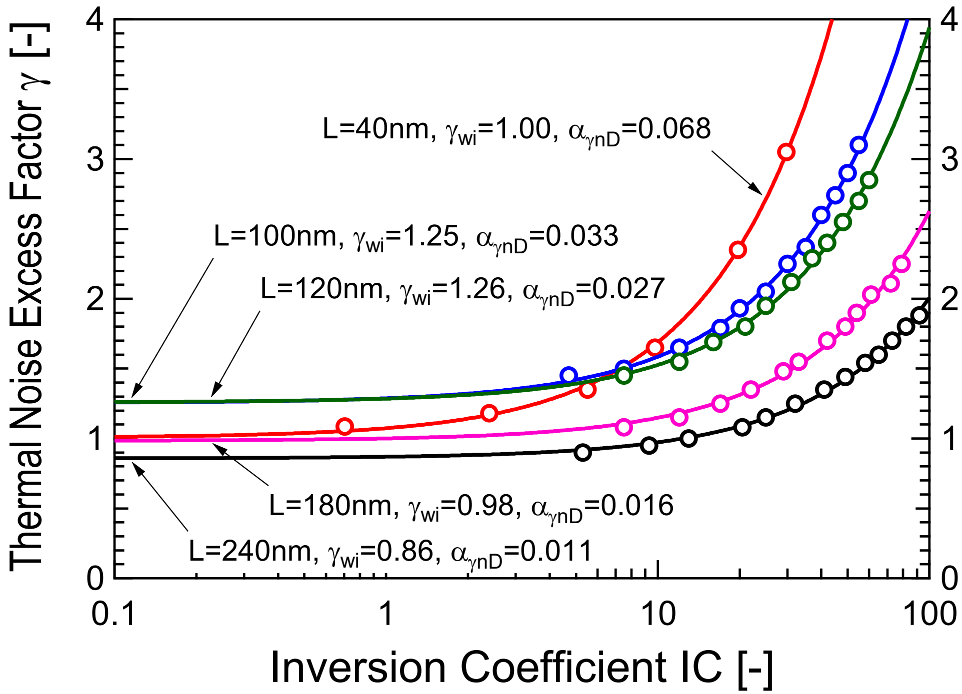
$$\delta_{nD} = \frac{2}{3} \cdot \frac{q_s + 3/4}{q_s + 1} = \begin{cases} \frac{1}{2} & WI \\ \frac{2}{3} & SI \end{cases} \quad \text{and} \quad \gamma_{nD} = n \cdot \delta_{nD} = \begin{cases} \frac{n}{2} \cong 0.8 & WI \\ n \cdot \frac{2}{3} \cong 1 & SI \end{cases}$$

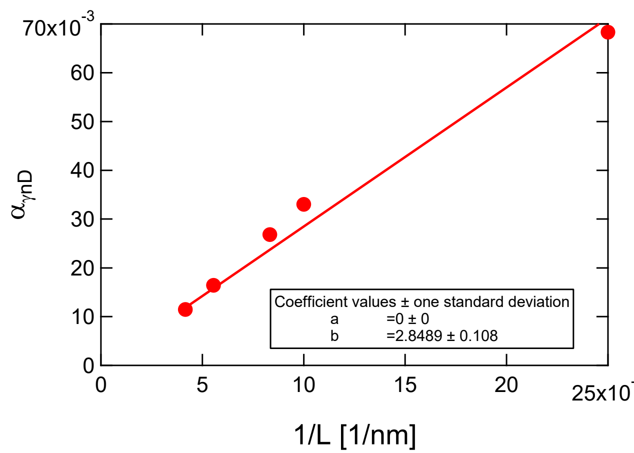
Effect of Velocity Saturation



- For short-channel devices in SI and saturation → lateral electrical field larger than critical field → carrier **velocity saturation**
- Carrier velocity limited → additional charge builds up close to the drain → additional thermal noise without increase of G_m → **increase** of δ_{nDsat} compared to the long-channel value 2/3

Short-channel Effects on γ_{nD} (in saturation)





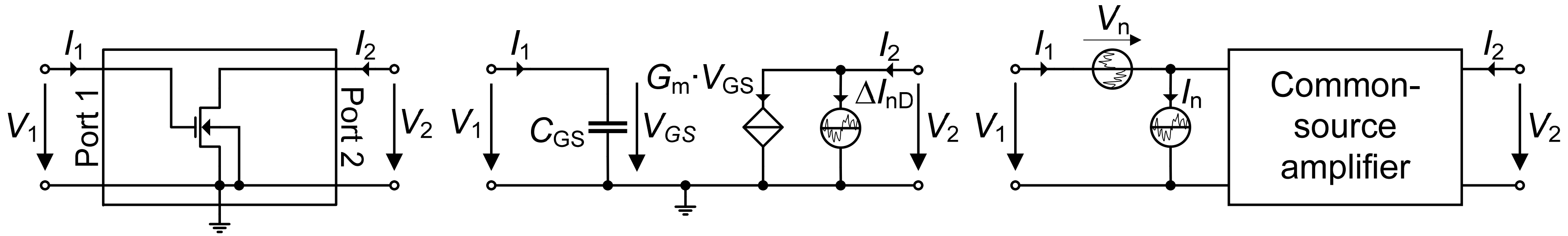
- The noise excess factor γ_{nD} can be modelled versus IC as

$$\gamma_{nD} \cong \gamma_{wi} + \alpha_{\gamma_{nD}} \cdot IC$$

- Where γ_{wi} and $\alpha_{\gamma_{nD}}$ are empirical factors
- $\alpha_{\gamma_{nD}}$ scales approximatively as $\alpha_{\gamma_{nD}} \cong 2.85/L$ where L is in nm

📖 A. Antonopoulos et al., “CMOS Small-Signal and Thermal Noise Modeling at High Frequencies,” TED, vol. 60, No. 11, Nov. 2013.
📖 M. Chalkiadaki, PhD Thesis 2016.

Example 1: Channel Thermal Noise Only



- The input referred noise sources and the correlation admittance are calculated as

$$R_v = \gamma_{nD}/G_m \quad G_i = \gamma_{nD}/G_m \cdot (\omega C_{GS})^2 \quad G_c = 0 \quad B_c = \omega C_{GS}$$

- V_n and I_n are fully correlated since there is only one noise source I_{nD}

$$\rho_{GD} = Y_c \cdot \sqrt{R_v/G_i} = j$$

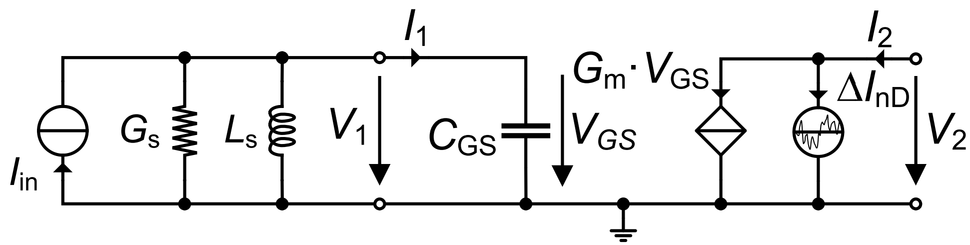
- The optimum source admittance and the minimum noise figure are given by

$$G_{opt} = \sqrt{\frac{G_i}{R_v} - B_c^2} = 0 \quad B_{opt} = -B_c = -\omega C_{GS} \quad F_{\min} = 1 + 2R_v \cdot (G_{opt} + G_c) = 1$$

- The optimum source admittance is simply equal to the conjugate match for maximum gain

Example 1: Channel Thermal Noise Only

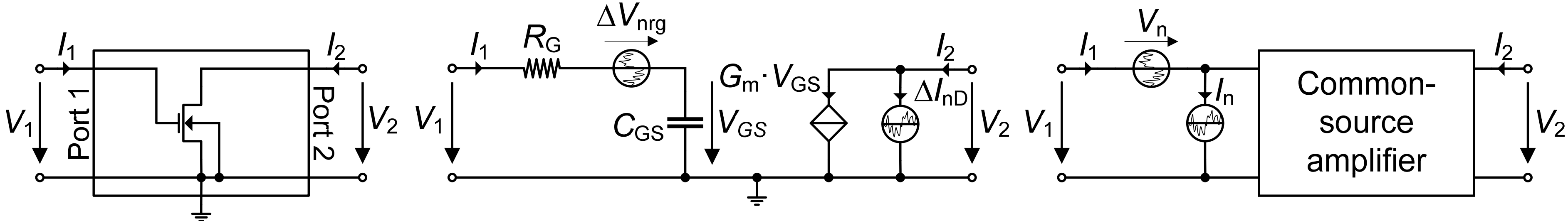
- The reason why $F_{min} = 1$ can be explained as follows



$$Z_{in} \triangleq \frac{V_1}{I_{in}} = \frac{1}{Y_{in}} = \frac{1}{G_s + \frac{1}{j\omega L_s} + j\omega C_{GS}}$$

- For $G_s = G_{opt} = 0$ and L_s made to resonate with C_{GS} at the operating frequency, the quality factor of the input circuit becomes infinity and the gain at the input becomes infinity, leading to a minimum noise factor of 1!

Example 2: Effect of the Gate Resistance Noise



- For $\omega R_G C_{GS} \ll 1$, the noise parameters R_v, G_i and Y_c of the equivalent noisy two-port are given by

$$R_v = \frac{\gamma_{nD}}{G_m} + R_G = \frac{\gamma_{nD}}{G_m} \cdot (1 + \alpha_G) \quad G_i = \frac{\gamma_{nD}}{G_m} \cdot (\omega C_{GS})^2 \quad G_c = \frac{(\omega C_{GS})^2 R_G}{1 + \alpha_G} \quad B_c = \frac{\omega C_{GS}}{1 + \alpha_G}$$

- The gate resistance R_G directly adds to the input-referred resistance R_v
- α_G represents the ratio of the noise PSD of the gate resistance to the input referred channel noise

$$\alpha_G \triangleq \frac{4kTR_G}{4kT\,\gamma_{nD}/G_m} = \frac{G_m \cdot R_G}{\gamma_{nD}} \ll 1$$

- V_n and I_n are now partially correlated according to

$$\rho_{GD} = Y_c \cdot \sqrt{\frac{R_v}{G_i}} = \frac{\omega R_G C_{GS} + j}{\sqrt{1 + \alpha_G}} \cong \frac{j}{\sqrt{1 + \alpha_G}} \quad \text{for} \quad \omega R_G C_{GS} \ll 1$$

Example 2: Effect of the Gate Resistance Noise

- The optimum source admittance and the minimum noise factor are then given by

$$G_{opt} = \sqrt{\frac{G_i}{R_v} - B_c^2} = \omega C_{GS} \cdot \frac{\sqrt{\alpha_G}}{1+\alpha_G} \cong \omega C_{GS} \cdot \sqrt{\alpha_G} \quad \text{for} \quad \alpha_G \ll 1$$

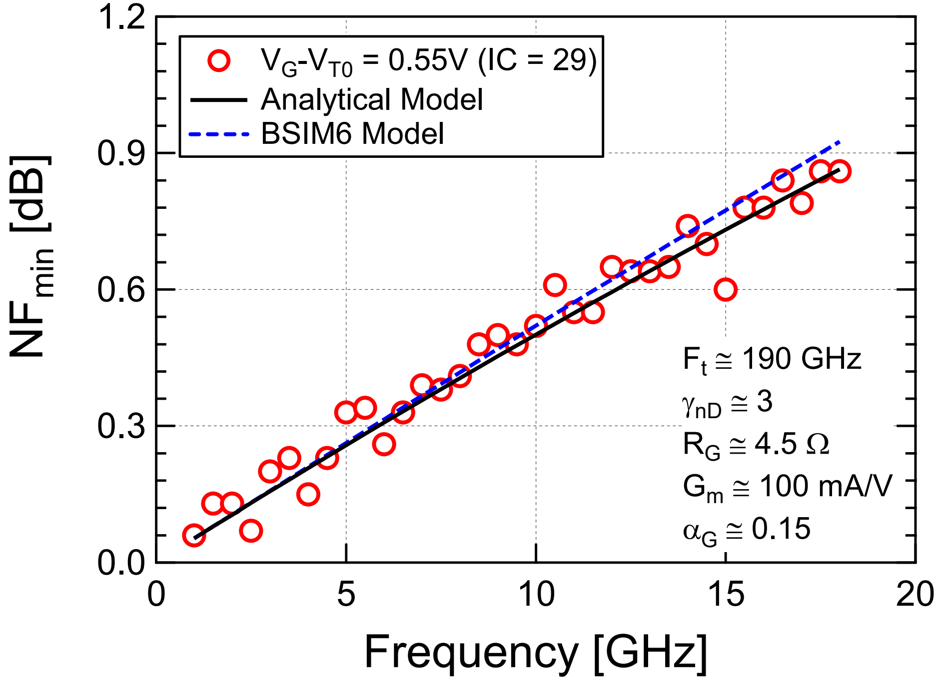
$$B_{opt} = -B_c = -\frac{\omega C_{GS}}{1+\alpha_G} \cong -\omega C_{GS} \quad \text{for} \quad \alpha_G \ll 1$$

$$F_{\min} = 1 + 2R_v \cdot \left(G_{opt} + G_c\right) \cong 1 + 2\frac{\gamma_{nD}}{G_m} \cdot \omega C_{GS} \cdot \left(\sqrt{\alpha_G} + \omega R_G C_{GS}\right)$$

$$\cong 1 + 2\gamma_{nD} \cdot \sqrt{\alpha_G} \cdot \frac{\omega}{\omega_t} \quad \text{with} \quad \alpha_G = \frac{G_m \cdot R_G}{\gamma_{nD}}$$

where $\omega_t = G_m / C_{GS}$

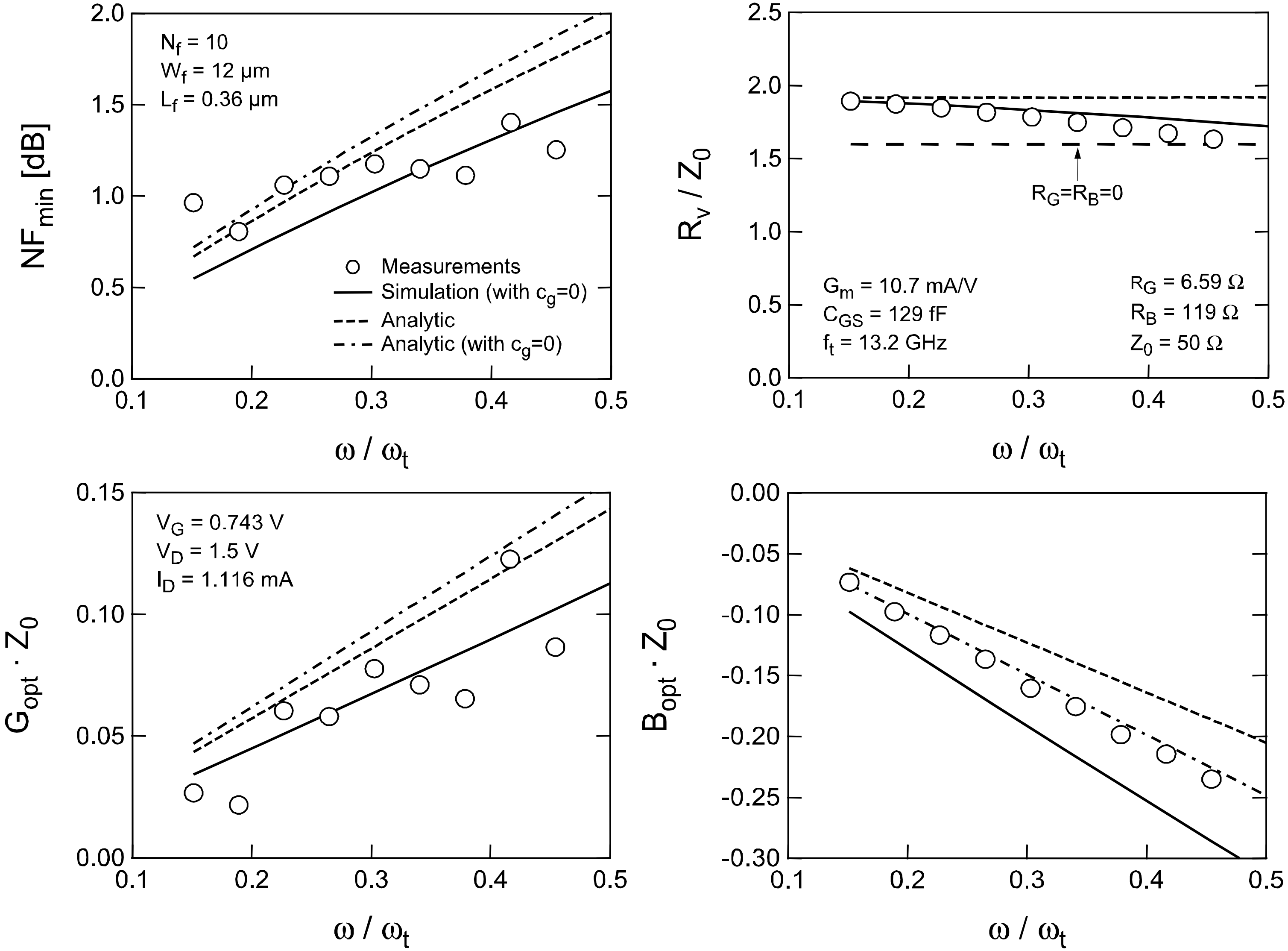
Comparison to Measurements on 40 nm Device



$$F_{\min} \cong 1 + 2\gamma_{nD} \cdot \sqrt{\alpha_G} \cdot \frac{\omega}{\omega_t} \quad \text{with} \quad \alpha_G \triangleq \frac{G_m \cdot R_G}{\gamma_{nD}}$$

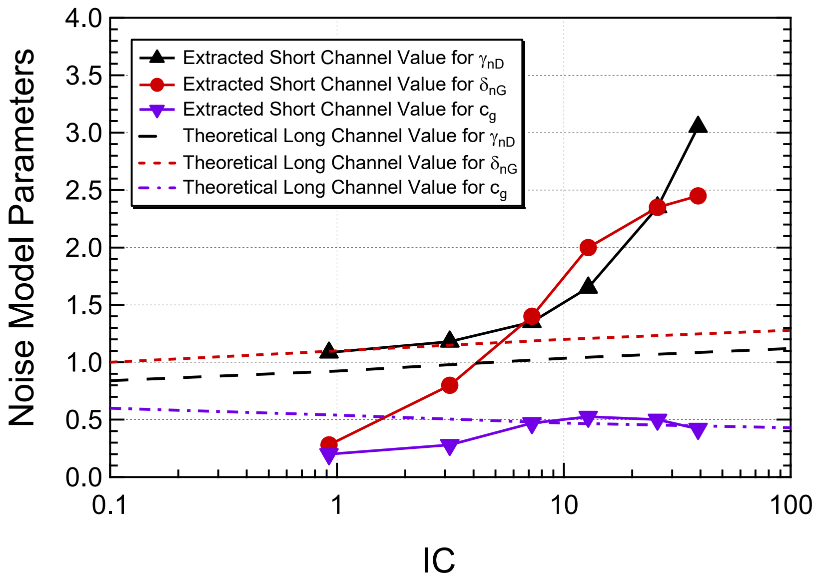
- This simple model works very well for $\omega \ll \omega_t$

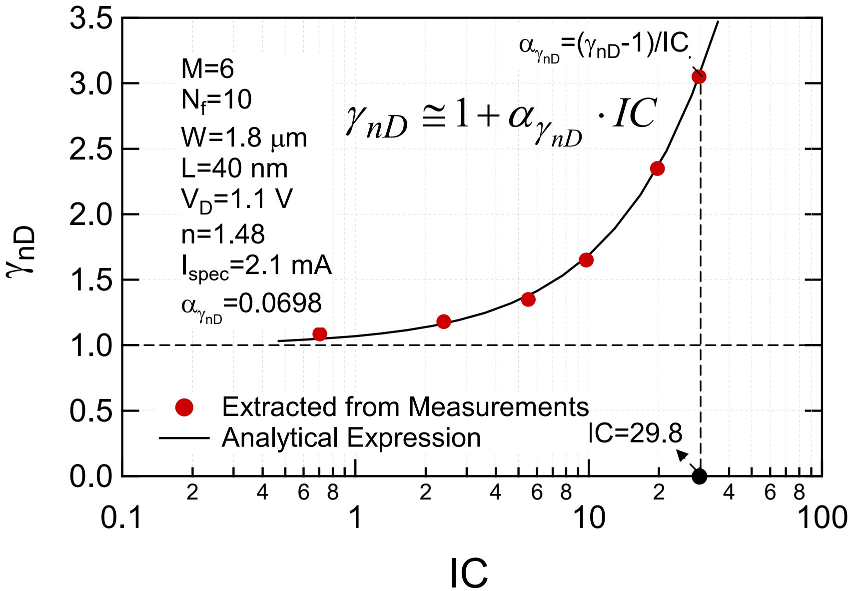
Measured and Simulated Noise Parameters



📖 C. C. Enz and E. A. Vittoz, *Charge-Based MOS Transistor Modeling – The EKV model for low-power and RF IC design*, Wiley, 2006.

Extracted Noise Factors for a 40nm Bulk CMOS Process



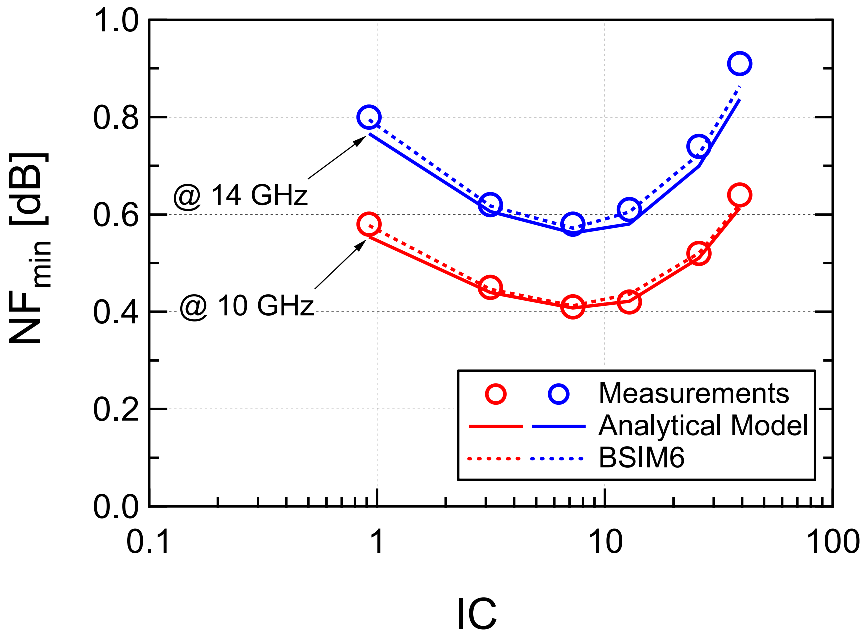


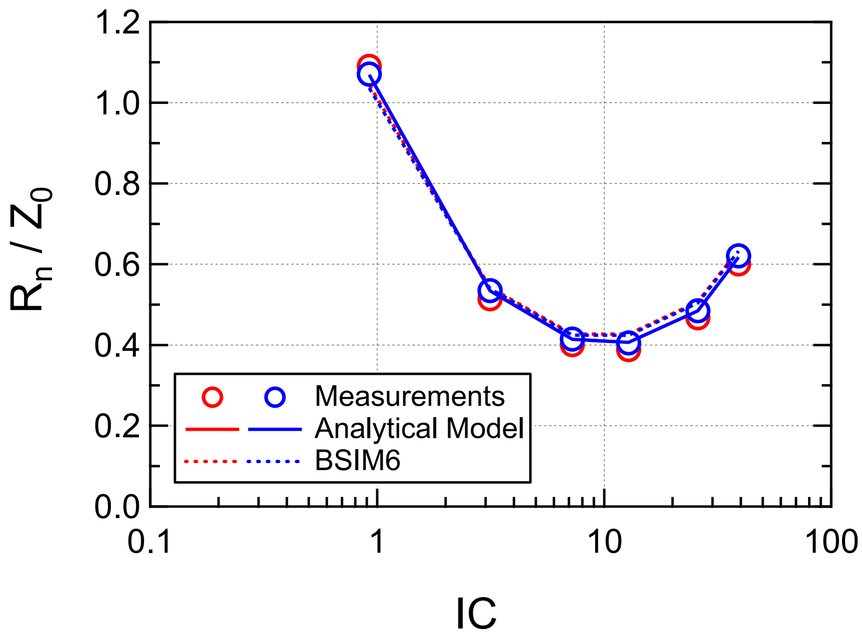
- The previous values of the noise factors γ_{nD}, δ_{nG} and c_G are given for long-channel transistor ignoring the short-channel effects (SCE) such as velocity saturation (VS)
- SCE and particularly VS tend to degrade the noise performance resulting in an increase of γ_{nD} and δ_{nG} particularly in SI where VS is predominant
- This is confirmed by the values shown below extracted from noise measurements made on a 40 nm bulk CMOS device

📖 M. Chalkiadaki and C. Enz, TMTT, 2015.
📖 M. Chalkiadaki, PhD Thesis No. 7030, 2016

NF_{min} and R_n versus IC for 40nm Bulk CMOS Process

- The minimum noise figure NF_{min} and input-referred noise resistance R_n show a minimum in MI due to the sharp increase of γ_{nD} at high IC





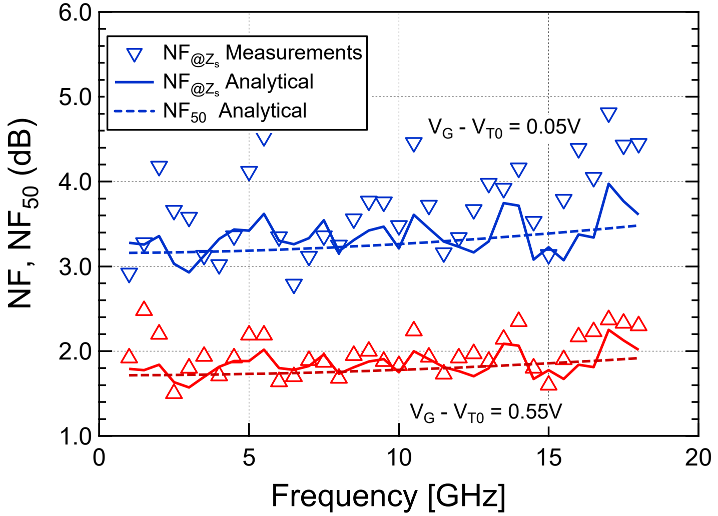
$$F_{\min} \cong 1 + 2\omega C_{GS} \cdot \frac{\gamma_{nD}}{G_m} \cdot \sqrt{\frac{\beta_{nG}}{\gamma_{nD}} \cdot \left(1 - c_g^2\right)}$$

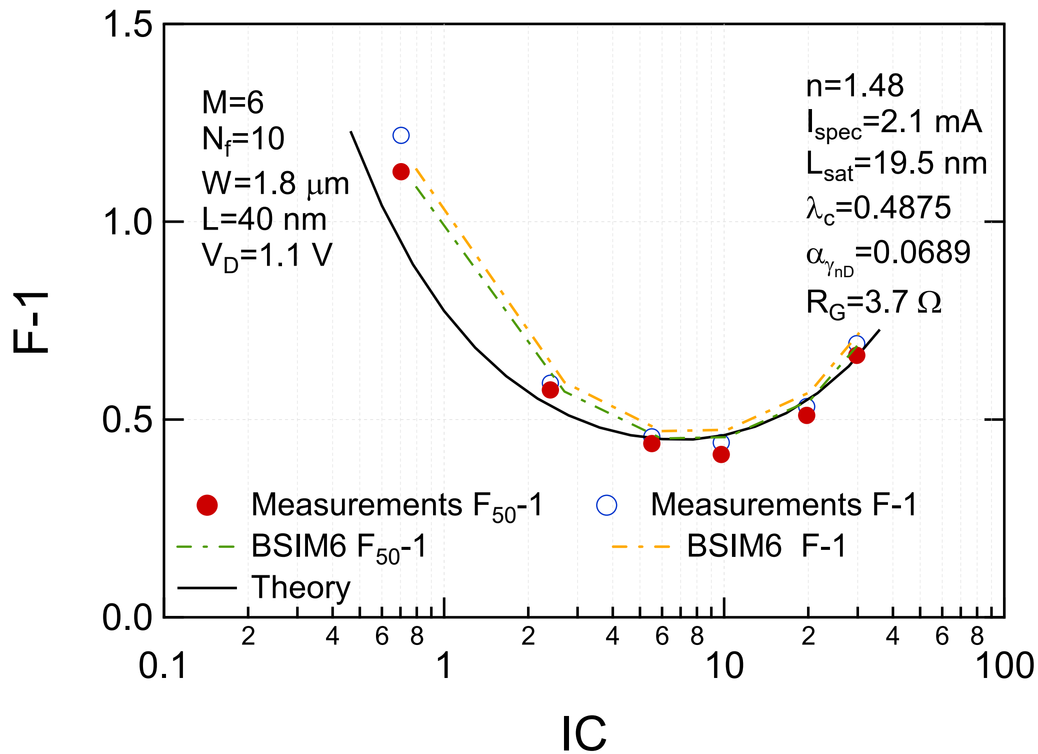
$$R_n \cong \frac{\gamma_{nD}}{G_m} + R_G$$

M. Chalkiadaki and C. Enz, TMTT, July 2015.

Actual Noise Figure

- The actual noise figure also shows a minimum in MI



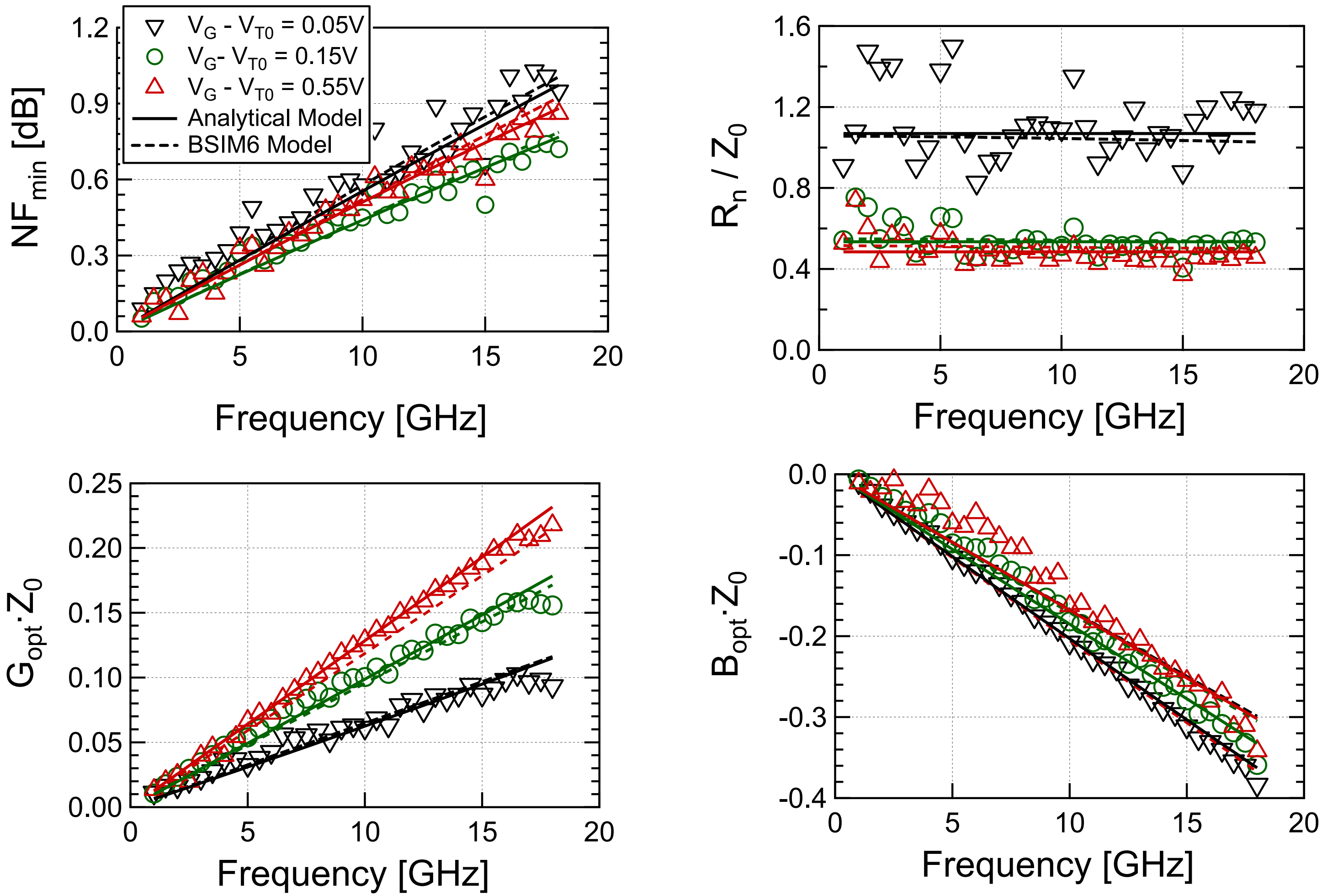


$$F-1=\frac{1}{50\Omega}\cdot\left(\frac{\gamma_{nD}}{G_m}+R_G\right)$$

M. Chalkiadaki, PhD Thesis 2016
M. Chalkiadaki and C. Enz, TMTT, July 2015.

Noise Parameters for a 40nm Bulk CMOS Process

N-channel, M=6, N_f = 10, W_f = 2 μm, L_f = 40 nm, V_S = 0 V, V_D = 1.1 V

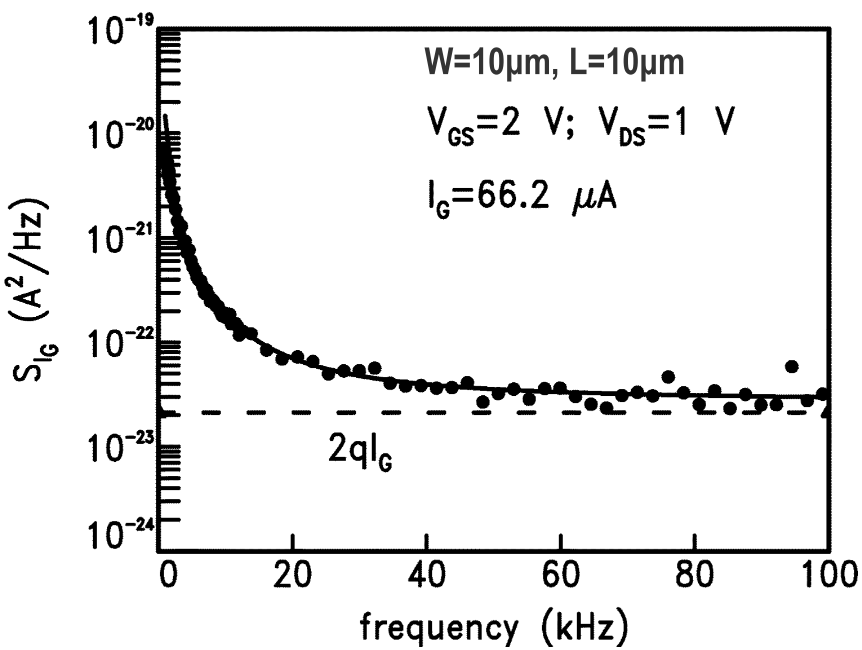


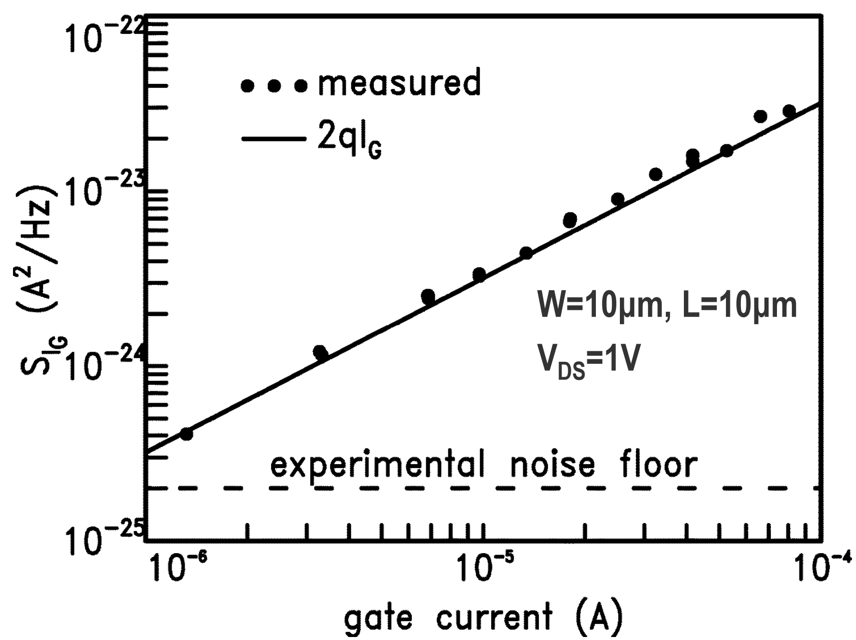
M. Chalkiadaki and C. Enz, TMTT, 2015.

Shot Noise of the Gate Leakage Current

- In addition to the IGN (thermal noise), there is also an additional component coming from the **gate leakage current** which shows **shot noise** and has a PSD given by

$$S_{\Delta I_G} = 2q \cdot I_G$$





A. J. Scholten *et al.*, TED, March 2003.

Effect of the Gate Tunneling Current Shot Noise

- The effect of the shot noise coming from the tunneling current can be added by redefining G_{nG} as

$$G_{nG}(\omega) = \beta_{nG} \cdot \frac{(\omega C_{GS})^2}{G_m} + \frac{2qI_G}{4kT} = \beta_{nG} \cdot \frac{(\omega C_{GS})^2}{G_m} + \frac{I_G}{2U_T}$$

- The minimum noise factor is then given by

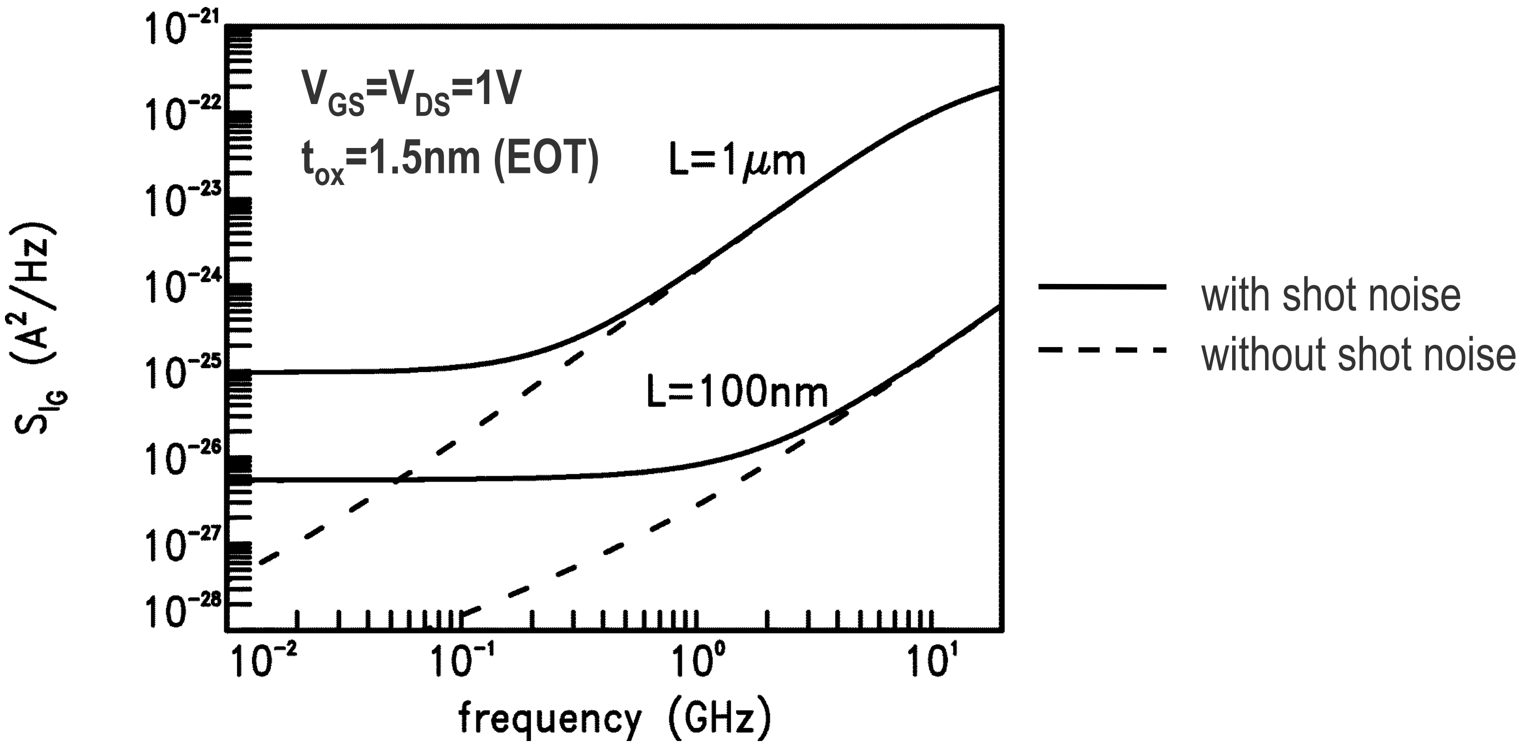
$$F_{\min} = 1 + \sqrt{\frac{2\gamma_{nD} \cdot I_G}{G_m \cdot U_T} \cdot \left(1 - c_g^2\right) \cdot \left(1 + \left(\frac{\omega}{\omega_c}\right)^2\right)} \quad \text{with} \quad \omega_c \triangleq \omega_t \cdot \sqrt{\frac{I_G}{2\beta_{nG} \cdot G_m \cdot U_T}}$$

- The gate tunneling current sets a non-zero minimum value of F_{min} at frequencies below ω_c

$$F_{\min} = \begin{cases} 1 + \sqrt{\dfrac{2\gamma_{nD} \cdot I_G}{G_m \cdot U_T} \cdot \left(1 - c_g^2\right)} & \text{for} \quad \omega \ll \omega_c \\ 1 + 2\dfrac{\omega}{\omega_t} \cdot \sqrt{\gamma_{nD} \cdot \beta_{nG} \cdot \left(1 - c_g^2\right)} & \text{for} \quad \omega \gg \omega_c \end{cases}$$

Shot Noise of the Gate Leakage Current

- The gate leakage shot noise is independent of frequency whereas the IGN and the noise coming from the gate resistance are proportional to ω^2
- The plot (simulations) below shows that the gate noise for L=100nm is dominated by shot noise for f<1GHz and by IGN and gate resistance noise for f>1GHz



A. J. Scholten *et al.*, TED, March 2003.

References

Most of this Chapter is based on Chapter 13 of Reference [1]

[1] C. Enz and E. A. Vittoz, *Charge-Based MOS Transistor Modeling - The EKV Model for Low-Power and RF IC Design*, 1st ed. Wiley, 2006.

[2] G. Gonzalez, *Microwave Transistor Amplifiers – Analysis and Design*, 2nd ed. Prentice-Hall, 1996.

[3] T. H. Lee, *The Design of CMOS Radio-Frequency Integrated Circuits*, 2nd ed. Cambridge University Press, 2004.

[4] G. D. Vendelin, A. M. Pavio and U. L. Rohde, *Microwave Circuit Design Using Linear and Nonlinear Techniques*, 2nd ed. Wiley, 2006.

[5] B. Razavi, *RF Microelectronics*, 2nd ed. Pearson, 2012.