MICRO-461
Low-power Radio Design for the IoT

10. Oscillators
10.2. Phase Noise

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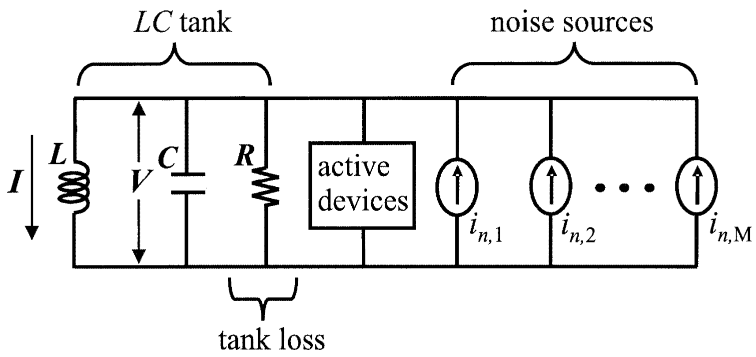
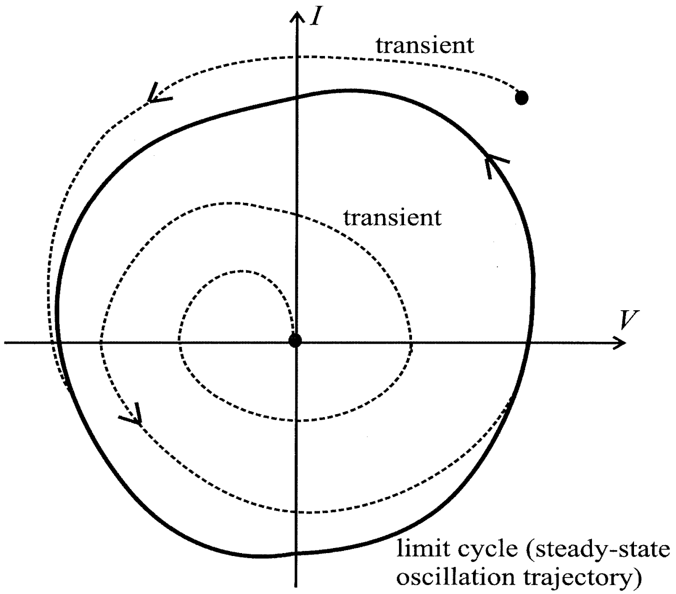
Outline

- Fundamentals
- Linear analysis
- Nonlinear analysis

Limit Cycle in the V-I State Space

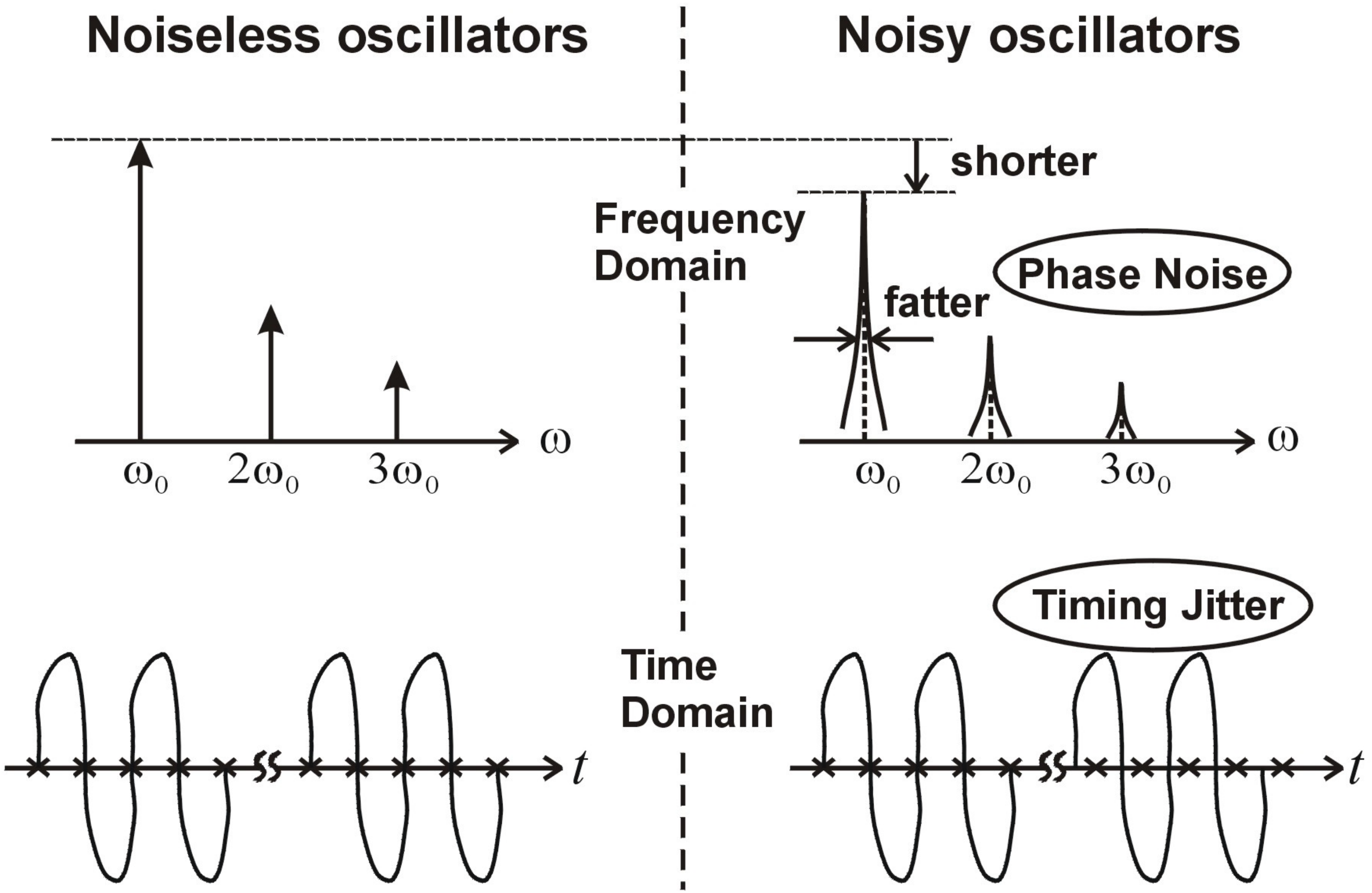




$$V = A \cdot \cos\left(\omega_0 t + \phi(t)\right)$$

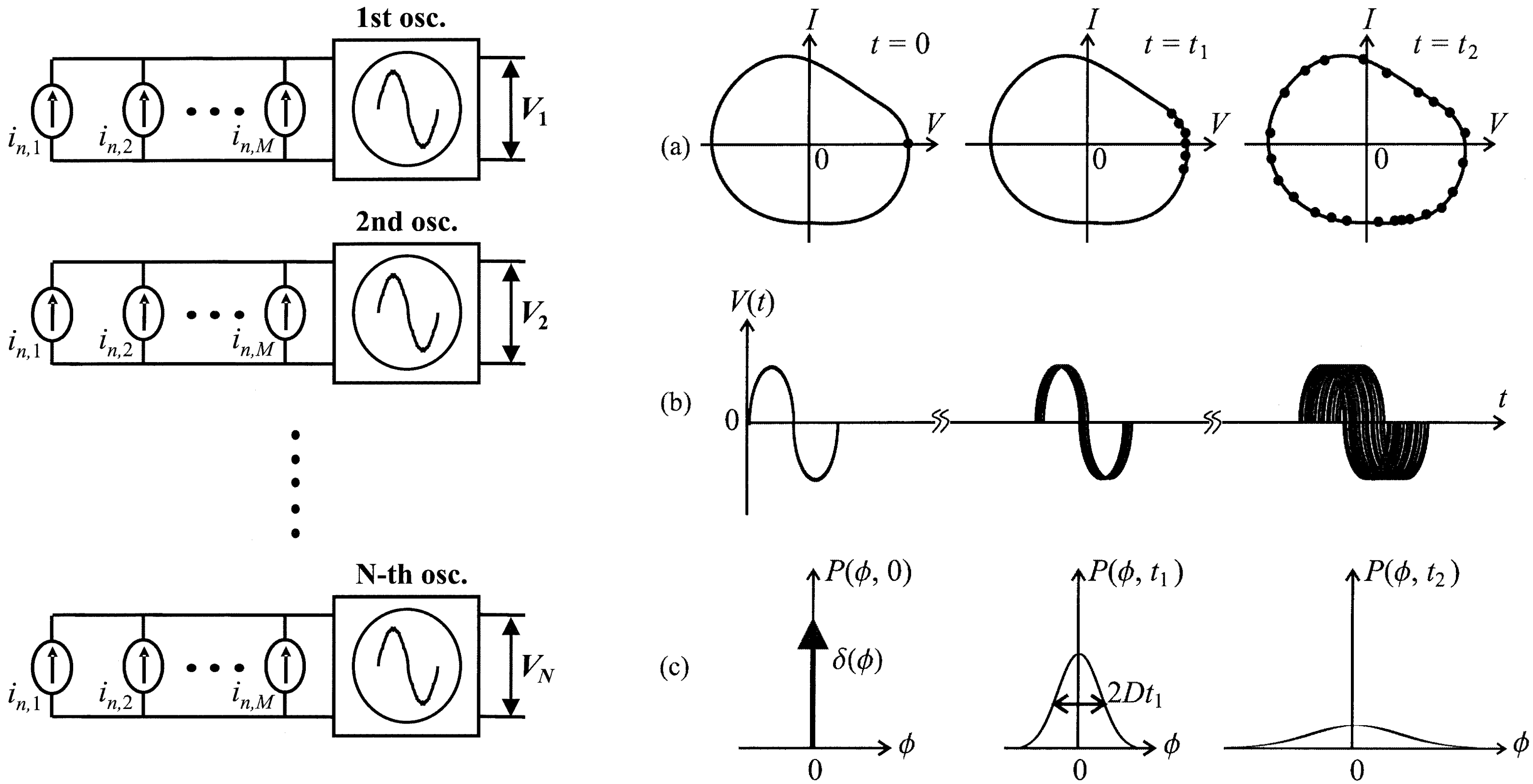
D. Ham and A. Hajimiri, "Virtual damping and Einstein relation in oscillators," JSSC, vol. 38, No. 3, pp. 407-418, March 2003.

Manifestation of Phase Noise



Courtesy D. Ham, Tutorial on Phase Noise, ESSCIRC 2014.

Phase Diffusion Seen through an Ensemble of Oscillators



D. Ham and A. Hajimiri, "Virtual damping and Einstein relation in oscillators," JSSC, vol. 38, No. 3, pp. 407-418, March 2003.

Phase Diffusion

- Neglecting the amplitude fluctuations, the output voltage of an oscillator is given by

$$v(t) = A \cdot \cos\left(\omega_0 t + \phi(t)\right)$$

- Where $\phi(t)$ represents the phase fluctuation and is a random process
- The instantaneous noise frequency is then given

$$\Delta\omega(t) = \frac{d\phi(t)}{dt}$$

- and hence

$$\phi(t) = \int_0^t \Delta\omega(\tau) \cdot d\tau + \phi(0)$$

📖 F. Herzel and B. Razavi, “A Study of Oscillator Jitter Due to Supply and Substrate Noise,” TCAS II, Vol. 46, No. 1, pp. 56-62, Jan. 1999.
📖 H. Hegazi, J. Rael and A. Abidi, *The Designer's Guide to High-Purity Oscillators*, Springer, 2005.

Oscillator Output Power Spectral Density

- If it is assumed that the instantaneous frequency noise fluctuation $\Delta\omega$ is a white noise with constant PSD and ACF given by

$$R_{\Delta\omega}(\tau) = 2D \cdot \delta(\tau)$$

- where D is the diffusivity, then $\phi(t)$ is a Wiener process having a Gaussian distribution centered around $\phi(0)$ and a variance that increases linearly with time

$$\sigma_{\phi}^2(t) = 2D \cdot t$$

- The ACF of the sine wave is then obtained as

$$R_v(\tau) = \mathrm{E}\left[v(t+\tau) \cdot v(t)\right] = \frac{A^2}{2} \cdot \cos(\omega_0 \tau) \cdot \exp\left[-D \cdot |\tau|\right]$$

- The single-sided PSD of the sine wave is then obtained by taking the Fourier transform, resulting in

$$S_v(\omega) = A^2 \cdot \frac{D}{(\omega - \omega_0)^2 + D^2}$$

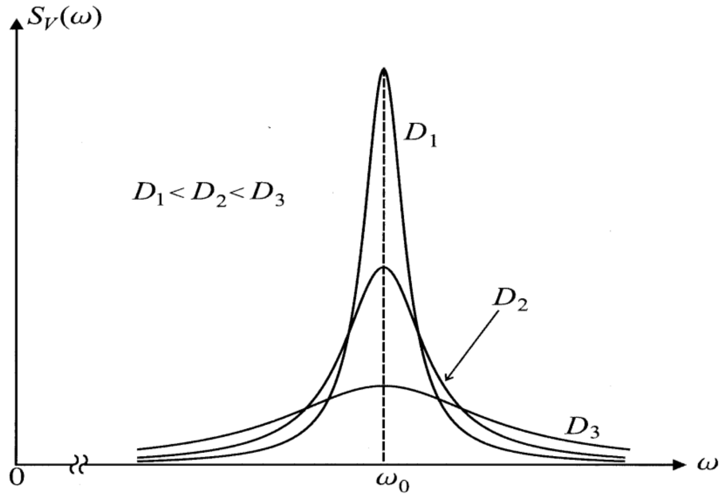
F. Herzel and B. Razavi, “A Study of Oscillator Jitter Due to Supply and Substrate Noise,” TCAS II, Vol. 46, No. 1, pp. 56-62, Jan. 1999.

H. Hegazi, J. Rael and A. Abidi, *The Designer's Guide to High-Purity Oscillators*, Springer, 2005.

Linewidth Broadening and Phase Noise

- It can be shown that the resulting PSD of $v(t)$ is a Lorentzian

$$S_v(\omega) = A^2 \cdot \frac{D}{(\omega - \omega_0)^2 + D^2}$$



- The corresponding phase noise at a given offset $\Delta\omega$ is defined as the ratio of the PSD at $\omega_0 + \Delta\omega$ to the total carrier power $A^2/2$

$$\mathcal{L}(\Delta\omega) = \frac{S_v(\omega)}{\frac{A^2}{2}} = \frac{2D}{\Delta\omega^2 + D^2} \cong \frac{2D}{\Delta\omega^2} \quad \text{for} \quad \Delta\omega \gg D$$

📖 F. Herzel and B. Razavi, "A Study of Oscillator Jitter Due to Supply and Substrate Noise, TCAS II, Vol. 46, No. 1, pp. 56-62, Jan. 1999.
📖 H. Hegazi, J. Rael and A. Abidi, *The Designer's Guide to High-Purity Oscillators*, Springer, 2005.

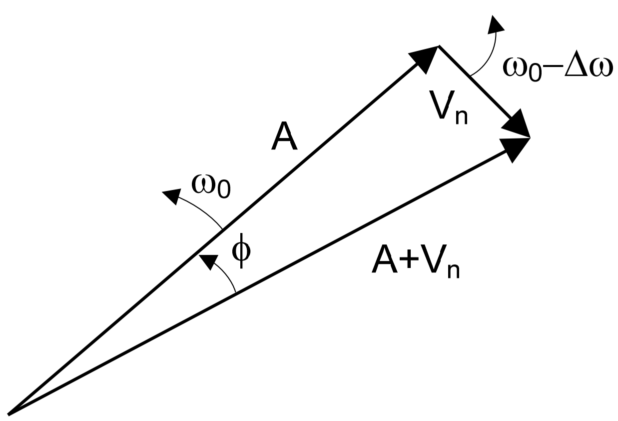
Additive Noise and Noisy Phasor

- An harmonic oscillator delivers a noisy signal typically given by

$$V(t) = A \cdot \left(1 + \varepsilon(t)\right) \cdot \sin\left(\omega_0 t + \phi(t)\right) \quad \text{with} \quad \varepsilon(t) \triangleq \frac{\Delta A(t)}{A}$$

where A and ω_0 are the amplitude and frequency of the carrier without noise and $\varepsilon(t)$ and $\phi(t)$ represent the variations of amplitude and phase induced by the noise sources having a bandwidth much smaller than ω_0. Note that if the waveform is not sinusoidal, then $V(t)$ represents the fundamental.

- Can be viewed as a noisy phasor $\vec{V}(t)$



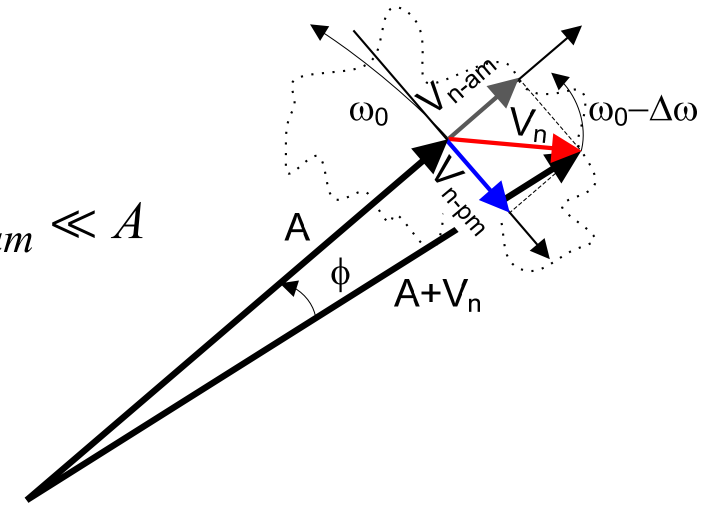
$$\vec{\mathbf{V}}(t) \triangleq A \cdot \left(1 + \varepsilon(t)\right) \cdot e^{j\omega_0 t} \cdot e^{j\phi(t)}$$

with

$$V(t) = \Re\left\{\vec{\mathbf{V}}(t)\right\}$$

Close-in Phase Noise

- The additive noise can be decomposed into AM (or in-phase I) and PM (or quadrature Q) components
- The phase angle fluctuation is then given by



$$\phi = \arctan\left(\frac{V_{n\text{-}pm}}{A + V_{n\text{-}am}}\right) \cong \frac{V_{n\text{-}pm}}{A} \quad \text{since} \quad V_{n\text{-}am} \ll A$$

- The (unilateral) PSD of this angle fluctuation is then given by the ratio of the PSD of the PM component to the power of the carrier (A is the peak value!)

$$S_\phi(\Delta\omega) = \frac{S_{V_{n\text{-}pm}}}{A^2/2}$$

Single Sideband Phase Noise

- Close-in phase noise is usually dominated by the PM component leading to

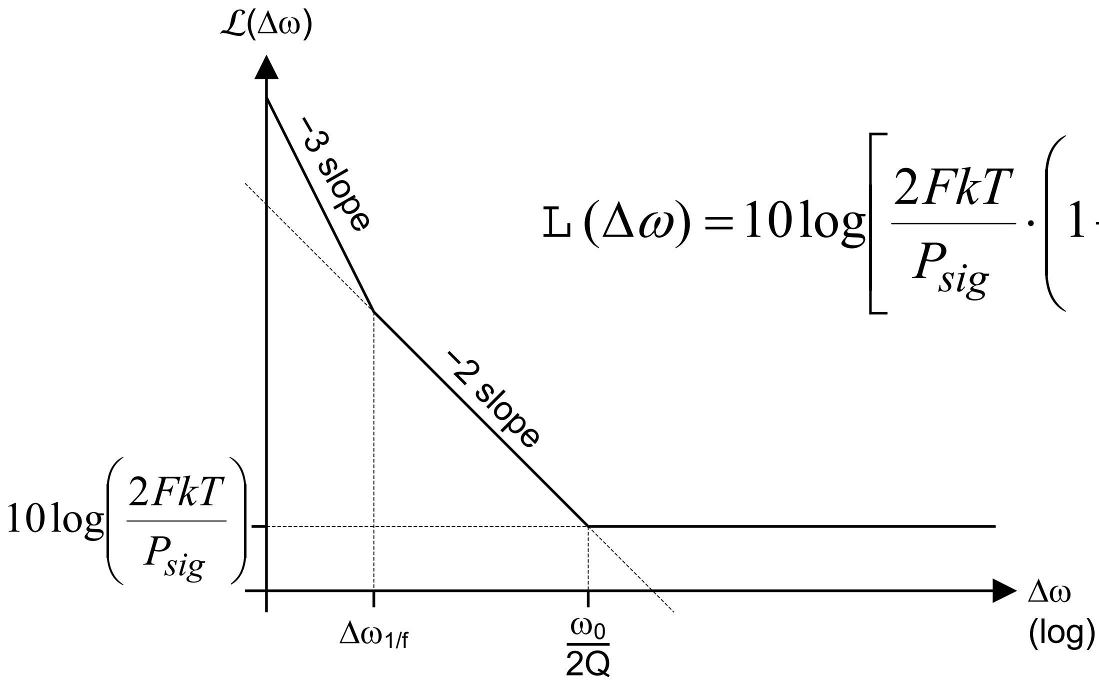
$$S_\phi(\Delta\omega) \cong \frac{S_{V_n}}{A^2/2}$$

- The standardized single sideband (SSB) phase noise $\mathcal{L}$ as measured by the phase noise analyzer is defined as

$$\mathcal{L}(\Delta\omega) \triangleq \frac{S_\phi(\Delta\omega)}{2} = \frac{1}{2}\frac{S_{V_n}}{A^2/2} = \frac{S_{V_n}}{A^2}$$

- The SSB phase noise $\mathcal{L}$ is measured in dBc/Hz

Leeson’s Empirical Model



$$\mathcal{L}(\Delta\omega) = 10\log\left[\frac{2FkT}{P_{sig}} \cdot \left(1 + \left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right) \cdot \left(1 + \frac{\Delta\omega_{1/f}}{|\Delta\omega|}\right)\right]$$

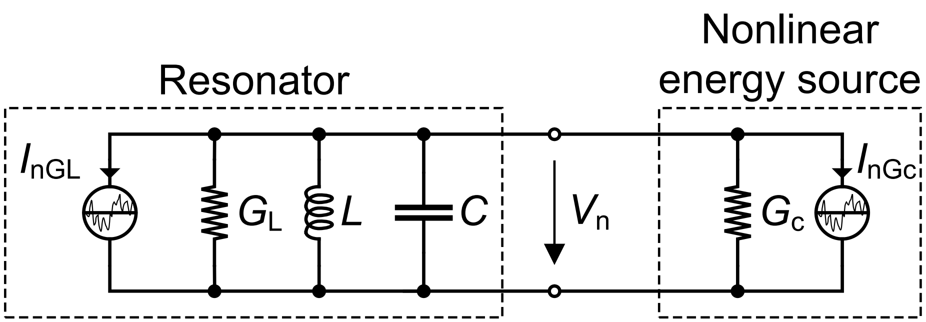
- Empirical model
- Slope −3 comes from up-converted 1/f noise, slope −2 from thermal noise
- Close to the carrier, the phase noise is determined by the up-converted 1/f noise
- Parameters F and $\Delta\omega_{1/f}$ are not easy to obtain analytically and are hence extracted from measurements

Outline

- Fundamentals
- Linear analysis
- Nonlinear analysis

Linear Phase Noise Analysis – Parallel LC Oscillator

- The small-signal equivalent circuit of a generic parallel LC oscillator including the noise sources due to the resistive losses in the tank G_L (actually its parallel equivalent conductance given below) and the noise coming from the active nonlinear circuit (usually a transconductor) is shown below



$$Z_{res}(\omega) = \frac{j\frac{\omega}{\omega_0}Z_0}{1-\left(\frac{\omega}{\omega_0}\right)^2 + j\frac{\omega}{\omega_0}\frac{1}{Q}}$$

$$Z_0 = \sqrt{\frac{L}{C}} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{1}{G_L \cdot Z_0} = \frac{\omega_0 C}{G_L} = \frac{1}{G_L \omega_0 L}$$

$$G_L = -G_c = G$$

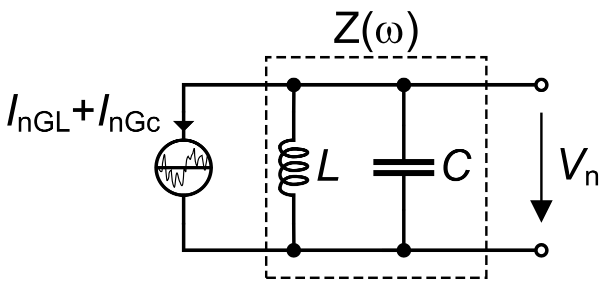
- The (unilateral) PSD of the noise sources are given by

$$S_{I_{nGL}} = 4kT \cdot G_L \quad \text{and} \quad S_{I_{nGc}} = 4kT \cdot \gamma G_c$$

where γ is the transconductor excess noise factor

Parallel LC Oscillator – Voltage Noise

- In steady-state condition, the losses are compensated by the negative conductance provided by the circuit and hence the circuit reduces to



$$Z = \frac{V_n}{I_n} = \frac{Z_0}{j \cdot \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)}$$

with $Z_0 = \sqrt{\frac{L}{C}}$ $\omega_0 = \frac{1}{\sqrt{LC}}$

- Close to the carrier, we have

$$\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \cong 2\frac{\Delta\omega}{\omega_0} \quad \text{with} \quad \Delta\omega = \omega - \omega_0 \quad \text{and hence} \quad Z \cong \frac{Z_0}{j2\frac{\Delta\omega}{\omega_0}} = -\frac{j\omega_0 Z_0}{2\Delta\omega}$$

- The PSD of the noise voltage fluctuations is then given by

$$S_{V_n} = (1+\gamma)\cdot 4kTG \cdot \left(\frac{\omega_0 Z_0}{2\Delta\omega}\right)^2 = \frac{(1+\gamma)\cdot 4kT}{G}\cdot\left(\frac{\omega_0}{2Q\Delta\omega}\right)^2 = \frac{(1+\gamma)\cdot kT}{G\cdot Q^2}\cdot\left(\frac{\omega_0}{\Delta\omega}\right)^2$$

where $Z_0 = 1/(G \cdot Q)$ has been used

Parallel LC Oscillator – SSB Phase Noise

- The standardized single sideband (SSB) phase noise $\mathcal{L}$ is then given by

$$\mathcal{L}(\Delta\omega) = \frac{S_\phi(\Delta\omega)}{2} = \frac{S_{V_n}}{A^2} = \frac{(1+\gamma)\cdot 4kT}{G\cdot A^2}\cdot\left(\frac{\omega_0}{2Q\Delta\omega}\right)^2 = \frac{(1+\gamma)\cdot kT}{G\cdot Q^2\cdot A^2}\cdot\left(\frac{\omega_0}{\Delta\omega}\right)^2$$

  where A is the carrier peak amplitude

- Phase noise $\mathcal{L}$ is inversely proportional to the **square of the offset frequency $\Delta\omega$** and the **square of the amplitude A**
- It is also inversely proportional to Q^2, but this is assuming that G is constant and independent of Q (which is actually not always the case as we will see later)
- The noise factor used in the Leeson expression of the $1/\omega^2$ portion of the spectrum can then easily be identified as

$$F = 1+\gamma$$

Parallel LC Oscillator – Alternative Expressions of PN

- Losses in the LC tank are usually dominated by losses in the inductor which are represented by a series resistor r related to the loss conductance G_L by

$$G_L = G = \frac{1}{r \cdot \left(1 + Q_L^2\right)} \cong \frac{1}{r \cdot Q_L^2} \quad \text{for} \quad Q_L \gg 1$$

- Assuming an ideal capacitor, the Q of the parallel circuit is equal to the inductor Q_L and hence

$$S_{V_n} = (1+\gamma) \cdot kT \cdot r \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2$$

which does not depend on Q^2 anymore

- It can also be written in terms of Q and the impedance level $1/(\omega_0 C) = Z_0$ as

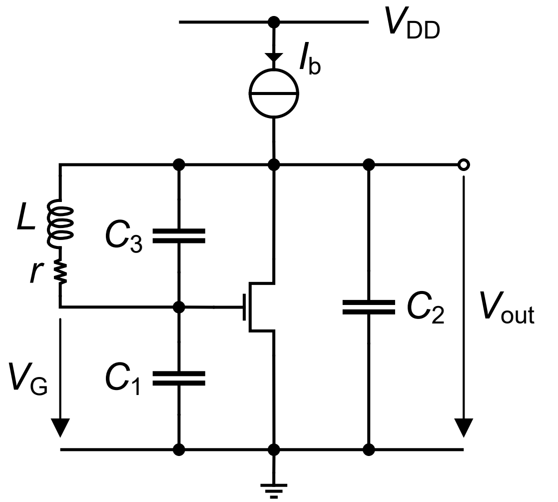
$$S_{V_n} = \frac{(1+\gamma) \cdot kT}{Q \cdot \omega_0 C} \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2 = \frac{(1+\gamma) \cdot kT \cdot Z_0}{Q} \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2$$

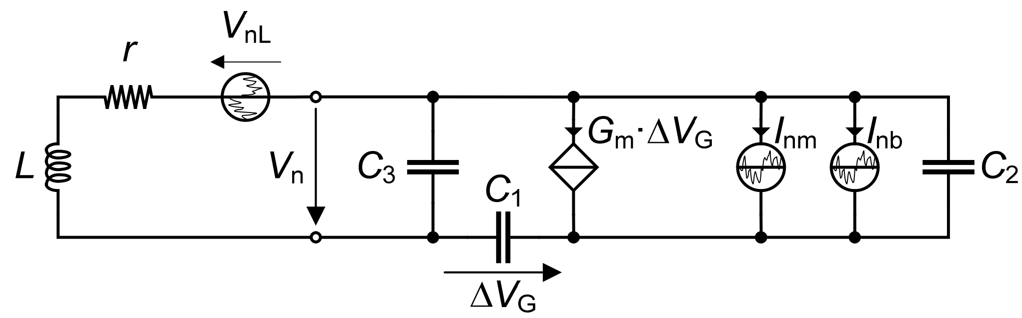
- Which leads to the following equivalent expressions of the standardized single sideband (SSB) phase noise $\mathcal{L}$

$$\mathcal{L}(\Delta\omega) = \frac{(1+\gamma) \cdot kT}{G \cdot Q^2 \cdot A^2} \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2 = \frac{(1+\gamma) \cdot kT \cdot r}{A^2} \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2 = \frac{(1+\gamma) \cdot kT}{Q \cdot \omega_0 C \cdot A^2} \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2 = \frac{(1+\gamma) \cdot kT \cdot Z_0}{Q \cdot A^2} \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2$$

Linear Phase Noise Analysis – Pierce Oscillator

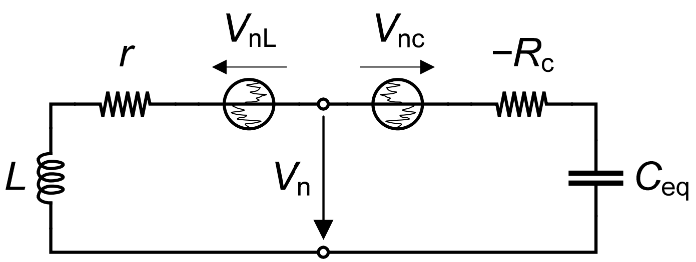
- The equivalent small-signal circuit including the noise sources from the inductor V_{nL}, MOS transistor I_{nm} and bias current source I_{nb} is given below





Note that V_n is not the voltage at the output but the voltage across L

- The active circuit, including its noise sources, can be replaced by its Thévenin source

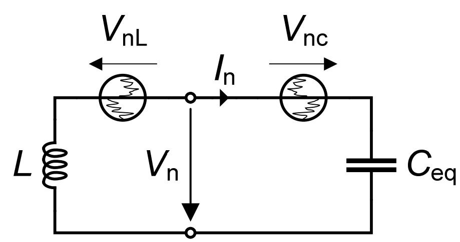


$$R_c \cong \frac{G_m C_1 C_2}{\omega_0^2 \left(C_1 C_2 + C_1 C_3 + C_2 C_3\right)^2} = \frac{G_m}{\left(\omega_0 C_{eq}\right)^2} \cdot \frac{C_1 C_2}{\left(C_1 + C_2\right)^2}$$

$$C_{eq} = C_3 // C_{12} = C_3 + C_{12} = C_3 + \frac{C_1 C_2}{C_1 + C_2} = \frac{C_1 C_2 + C_1 C_3 + C_2 C_3}{C_1 + C_2}$$

Pierce Oscillator – Noise Transfer Functions

- At the resonance frequency the inductor loss r is compensated by the negative resistance $-R_c$ provided by the circuit. The latter then simplifies to



- The noise transfer function from sources V_{nL} and V_{nc} to V_n are given by

$$H_{nL}(\omega) = \frac{V_n}{V_{nL}} = \frac{1}{1-(\omega/\omega_0)^2} \quad \text{and} \quad H_{nc}(\omega) = \frac{V_n}{V_{nc}} = -\frac{(\omega/\omega_0)^2}{1-(\omega/\omega_0)^2} \quad \text{with} \quad \omega_0 = \frac{1}{\sqrt{LC_{eq}}}$$

- At an offset frequency $\Delta\omega \ll \omega_0$ from the carrier we have

$$\omega = \omega_0 + \Delta\omega \;\rightarrow\; \frac{\omega}{\omega_0} = 1 + \frac{\Delta\omega}{\omega_0} \;\rightarrow\; \left(\frac{\omega}{\omega_0}\right)^2 = \left(1 + \frac{\Delta\omega}{\omega_0}\right)^2 \cong 1 + 2\frac{\Delta\omega}{\omega_0}$$

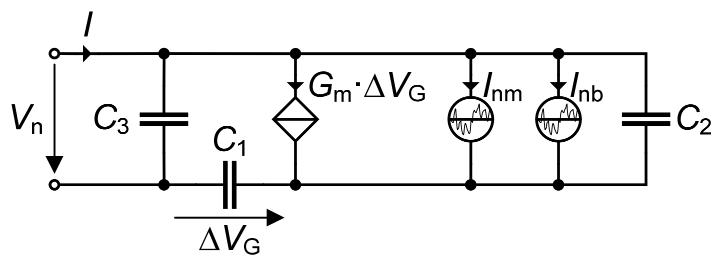
$$H_{nL}(\Delta\omega) \cong -\frac{\omega_0}{2\Delta\omega} \quad \text{and} \quad H_{nc}(\Delta\omega) \cong \frac{\omega_0}{2\Delta\omega}$$

Pierce Oscillator – Voltage Noise

- The noise voltage PSD is given by

$$S_{V_n} = |H_{nL}|^2 S_{V_{nL}} + |H_{nc}|^2 S_{V_{nc}} \cong \left(\frac{\omega_0}{2\Delta\omega}\right)^2 \left(S_{V_{nL}} + S_{V_{nc}}\right) \quad \text{with} \quad S_{V_{nL}} = 4kT \cdot r$$

where $S_{V_{nc}}$ has to be evaluated from the following circuit



$$V_{nc} = Z_{nm} \cdot (I_{nm} + I_{nb})$$

$$S_{V_{nc}} = |Z_{nm}|^2 \cdot \left(S_{I_{nm}} + S_{I_{nb}}\right)$$

$$S_{I_{nm}} = 4kT \cdot \gamma_{nm} G_{mm} \qquad S_{I_{nb}} = 4kT \cdot \gamma_{nb} G_{mb}$$

$$Z_{nm} = -\frac{C_1}{G_m C_3 + s(C_1 C_2 + C_1 C_3 + C_2 C_3)} \cong -\frac{C_1}{s(C_1 C_2 + C_1 C_3 + C_2 C_3)} = -\frac{C_1}{C_1 + C_2} \cdot \frac{1}{s C_{eq}}$$

$$S_{V_{nc}} = 4kT \cdot \left(\frac{C_1}{C_1 + C_2} \cdot \frac{1}{\omega_0 C_{eq}}\right)^2 \cdot (\gamma_{nm} G_{mm} + \gamma_{nb} G_{mb})$$

Pierce Oscillator – Voltage Noise across Tank

- Finally the noise voltage PSD is given by

$$S_{V_n} \cong kT \cdot r \cdot (1+\gamma) \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2$$

where γ is the noise excess factor representing the noise contribution of the circuit and given by

$$\gamma = \frac{1}{r} \cdot \left(\frac{C_1}{C_1+C_2} \cdot \frac{1}{\omega_0 C_{eq}}\right)^2 \cdot (\gamma_{nm} G_{mm} + \gamma_{nb} G_{mb}) = \frac{Q}{\omega_0 C_{eq}} \cdot \left(\frac{C_1}{C_1+C_2}\right)^2 \cdot (\gamma_{nm} G_{mm} + \gamma_{nb} G_{mb})$$

$$C_{eq} = C_3 // C_{12} = \frac{C_1 C_2 + C_1 C_3 + C_2 C_3}{C_1 + C_2}$$

- Since the critical transconductance is given by

$$G_{mcrit} = r \cdot \omega_0^2 \cdot \frac{(C_1 C_2 + C_1 C_3 + C_2 C_3)^2}{C_1 C_2} = r \cdot \omega_0^2 \cdot \frac{C_1}{C_2} \cdot \left(\frac{C_1 + C_2}{C_1}\right)^2 \cdot C_{eq}^2$$

- The γ noise excess factor can also be written as

$$\gamma = \frac{C_1}{C_2} \cdot \frac{\gamma_{nm} G_{mm} + \gamma_{nb} G_{mb}}{G_{mcrit}}$$

Pierce Oscillator – Noise Excess Factor

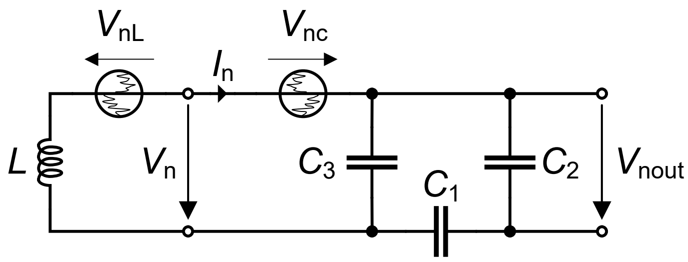
- Since the minimum G_{mcrit} is obtained for $C_1 = C_2$, the γ noise excess factor reduces to

$$\gamma = \frac{\gamma_{nm} G_{mm} + \gamma_{nb} G_{mb}}{G_{mcrit}} \quad \text{for} \quad C_1 = C_2$$

- Since $G_{mm}/G_{mcrit} > 3$, for ensuring start-up and reaching the desired amplitude, the noise can be slightly degraded by the active part of the oscillator

Pierce Oscillator – Noise at the Output

- V_n is the noise voltage across the resonator. Usually we are more interested in the noise at the oscillator output V_{nout}



$$V_{nout} = \frac{C_1}{C_1 + C_2} \cdot V_n$$

- And hence

$$S_{V_{nout}} = \left(\frac{C_1}{C_1 + C_2}\right)^2 \cdot S_{V_n} = \left(\frac{C_1}{C_1 + C_2}\right)^2 \cdot kT \cdot r \cdot (1+\gamma) \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2$$

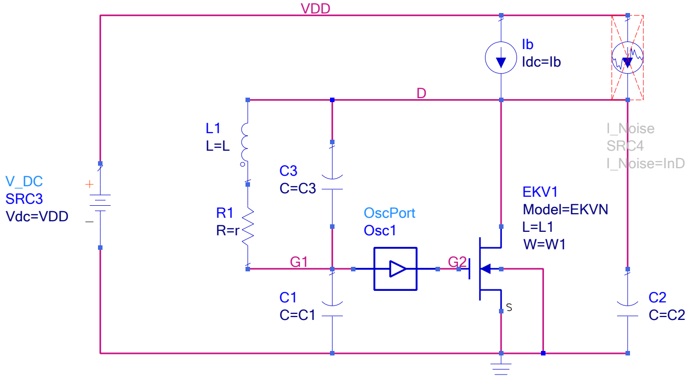
- The SSB phase noise at the output is then given by

$$\mathcal{L}(\Delta\omega) = \frac{S_{V_{nout}}}{A^2} = \frac{kT \cdot r \cdot (1+\gamma)}{A^2} \cdot \left(\frac{C_1}{C_1 + C_2}\right)^2 \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2 = \frac{kT \cdot (1+\gamma)}{A^2 \cdot Q \cdot \omega_0 C_{eq}} \cdot \left(\frac{C_1}{C_1 + C_2}\right)^2 \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2$$

with Q given by

$$Q = \frac{\omega_0 L}{r} = \frac{1}{\omega_0 C_{eq} r} \qquad C_{eq} = C_3 // C_{12} = \frac{C_1 C_2 + C_1 C_3 + C_2 C_3}{C_1 + C_2}$$

Pierce Oscillator – ADS HB Simulations (WI)



EKV version 2.6
EPFL

ekv_va_Model
EKVN

TYPE=1	E0=5.917e+7	HDIF=2e-07
L=10E-6	UCRIT=3.75e+6	AVTO=1E-6
W=10E-6	UCEX=1.76	AKP=1E-6
TNOM=27	LAMBDA=0.340	AGAMMA=1E-6
M=1.0	DL=-7.6e-8	AF=0.8265
NS=1.0	DW=3.9e-8	KF=0
COX=8.46e-3	WETA=0.0	AllParams=
XJ=1.6e-7	LETA=220.0e-3	
VTO=VT0n	Q0=0.000420	
TCV=6.03e-4	LK=3.80e-7	
GAMMA=0.540	IBA=0.0	
PHI=990.0e-3	IBB=270.0e+6	
KP=420.0e-6	IBBT=0.0	
BEX=-1.569	IBN=1.0	
THETA=0.0	RSH=600.0	

Simulation using the full EKV 2.6 model for a 180nm CMOS generic process.

Transistor biased in WI with $IC = 0.1$

VAR
Technology
T0=273
UT=0.025875
kT=qelectron*UT
T=kT/boltzmann
Tcelsius=T-T0
n=1.271
nUT=n*UT
VT0n=0.455
Ispecn=0.715E-6
KFn=8.1E-24

VAR
Bias
VDD=1.8

VAR
Noise
SInD=4*kT*n/2*Gm
Gm=Ib/nUT
InD=sqrt(SInD)

VAR
Specifications
f0=1G
A=0.1
QL=10
C2=5E-12
IC=0.1
L1=0.18E-6

HARMONIC BALANCE
HarmonicBalance
HB1
Freq[1]=f0
Order[1]=15
Oversample[1]=8
NLNoiseMode=yes
NLNoiseStart=10 kHz
NLNoiseStop=10.0 MHz
NLNoiseDec=10
NoiseOutputPort=2
PhaseNoise=yes
NoiseNode[1]="D"
NoiseNode[2]="G2"

DC
DC
DC1

VAR
ParametersCalculation
w0=2*pi*f0
C1=C2
C3=0.1*C12
C12=C1*C2/(C1+C2)
Ceq=C12+C3
L2=1/(w0**2*Ceq*(1+1/QL**2))
L=(Gmcrit**2*C3+w0**2*(C1+C2)*(C1*C2+C1*C3+C2*C3))/(w0*((Gmcrit*C3)**2+w0**2*(C1*C2+C1*C3+C2*C3)**2))/w0
r=w0*L/QL
Gmcrit1=w0/QL*(C1+C2)*(1+C3/C12)
Gmcrit=w0*QL*C2*alpha1/(2*alpha3)*(1-sqrt(1-(2*alpha3/(alpha1*QL))**2*(alpha1+1)*(1+alpha1+alpha1/alpha3)))
Icrit=Gmcrit*nUT
x=A/nUT
a1=0.5
a2=0.2
chi=(1+a1*x+a2*x**2)/(a1*x+a2*x**2)

Ib=Icrit*x/2*chi
Ispec=Ib/IC
S=Ispec/Ispecn
W1=S*L1
alpha1=C1/C2
alpha3=C3/C2

TRANSIENT
Tran
Tran1
StopTime=500.0 nsec
MaxTimeStep=0.01 nsec

Pierce Oscillator – ADS HB Simulations (WI)

Technology and physical parameters

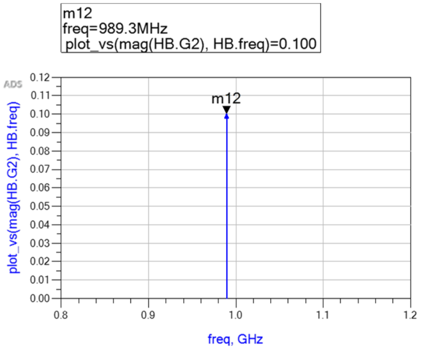
UT	n	nUT	VT0n	Ispecn
25.87 m	1.271	32.89 m	455.0 m	715.0 n

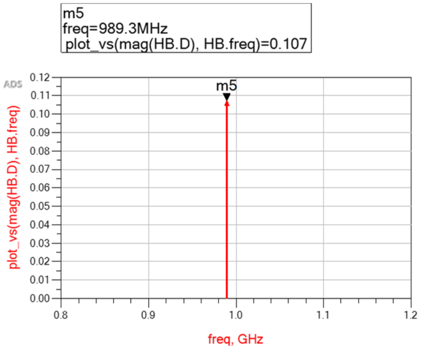
Specifications

f0	QL	C2	x	IC	L1
1.000 G	10.00	5.000 p	3.041	100.0 m	180.0 n

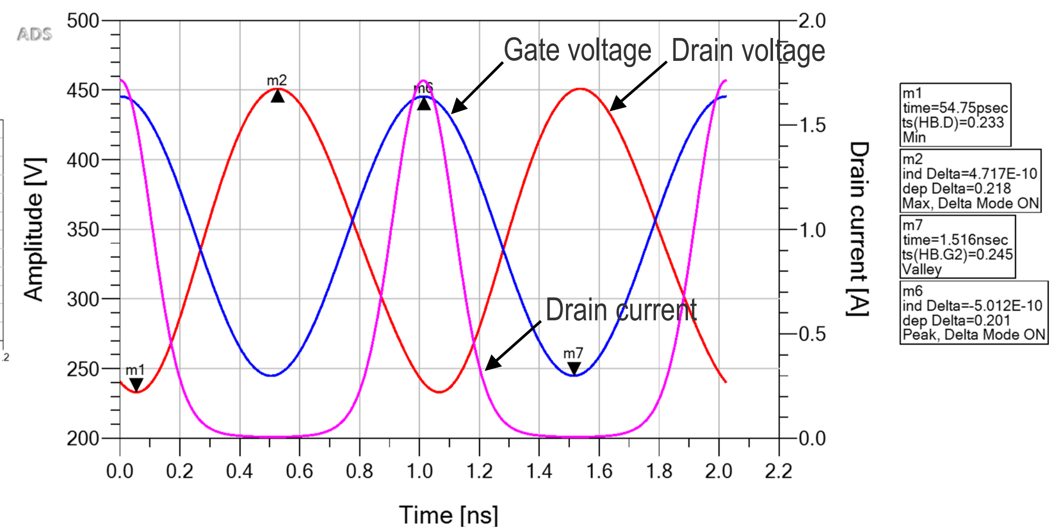
Calculated parameters

A	C1	C3	L	r	Gmcrit	Icrit	Ib	Ispec	W1
100.0 m	5.000 p	250.0 f	9.220 n	5.793	6.919 m	227.6 u	448.6 u	4.486 m	1.129 m





Waveforms



Slight shift in resonant frequency due to fairly low inductor Q ($Q_L = 10$).

Amplitude of the fundamental component at the gate is exactly equal to 100 mv and a bit larger at the drain (107 mV)

Amplitude of quasi-sinusoid is almost exactly 100mV (100.5mV) at the gate and slightly larger at the drain (116 mV)

Pierce Oscillator – ADS Transient Simulations (WI)

Technology and physical parameters

UT	n	nUT	VT0n	Ispecn
25.87 m	1.271	32.89 m	455.0 m	715.0 n

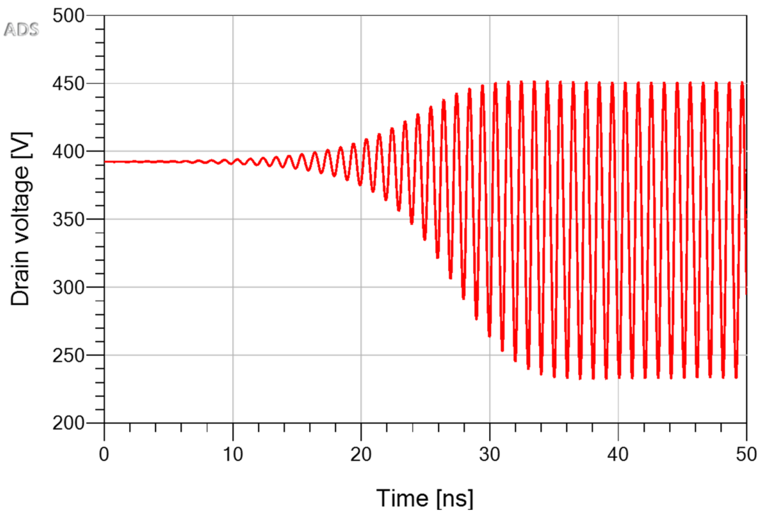
Specifications

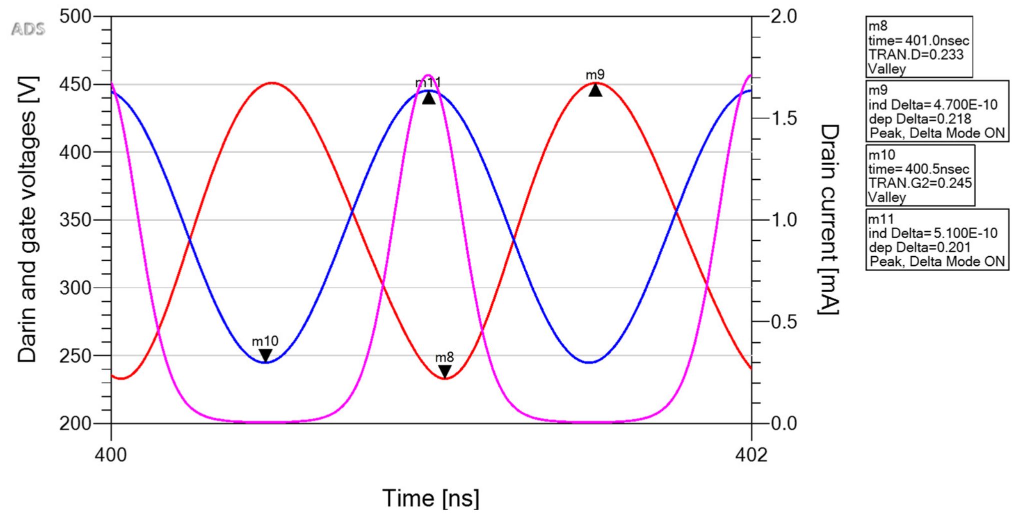
f0	QL	C2	x	IC	L1
1.000 G	10.00	5.000 p	3.041	100.0 m	180.0 n

Calculated parameters

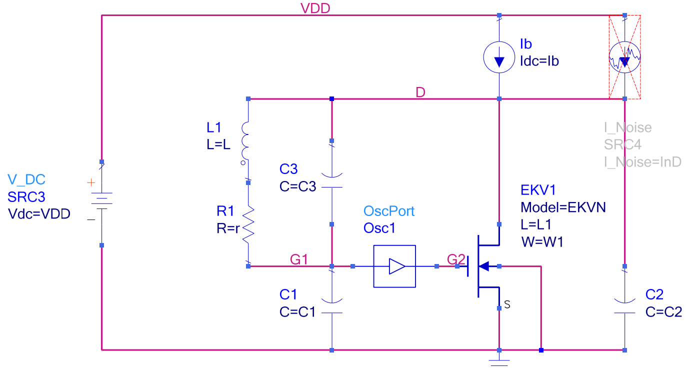
A	C1	C3	L	r	Gmcrit	Icrit	Ib	Ispec	W1
100.0 m	5.000 p	250.0 f	9.220 n	5.793	6.919 m	227.6 u	448.6 u	4.486 m	1.129 m

Transient simulation





Pierce Oscillator – ADS SSB Phase Noise Simulation (WI)



Technology and physical parameters

UT	n	nUT	VT0n	Ispecn
25.87 m	1.271	32.89 m	455.0 m	715.0 n

Specifications

f0	QL	C2	x	IC	L1
1.000 G	10.00	5.000 p	3.041	100.0 m	180.0 n

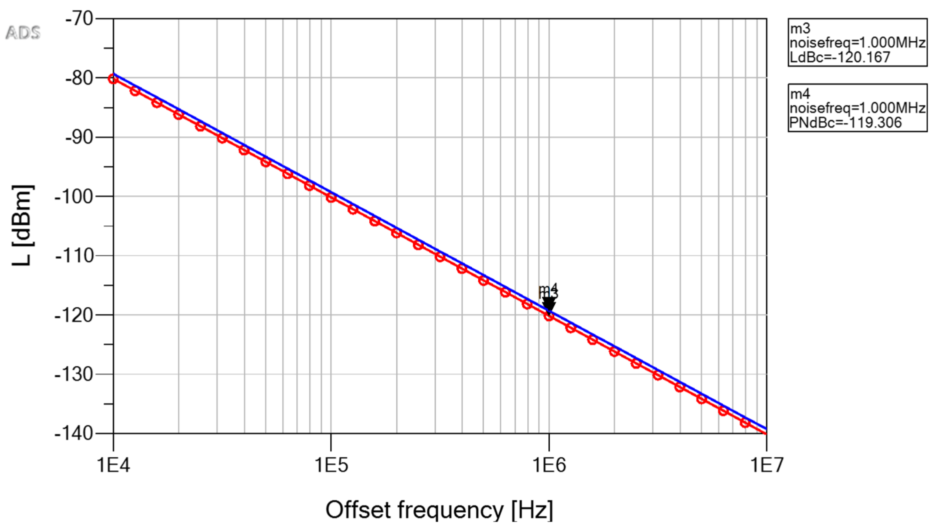
Calculated parameters

A	C1	C3	L	r	Gmcrit	Icrit	Ib	Ispec	W1
100.0 m	5.000 p	250.0 f	9.220 n	5.793	6.919 m	227.6 u	448.6 u	4.486 m	1.129 m

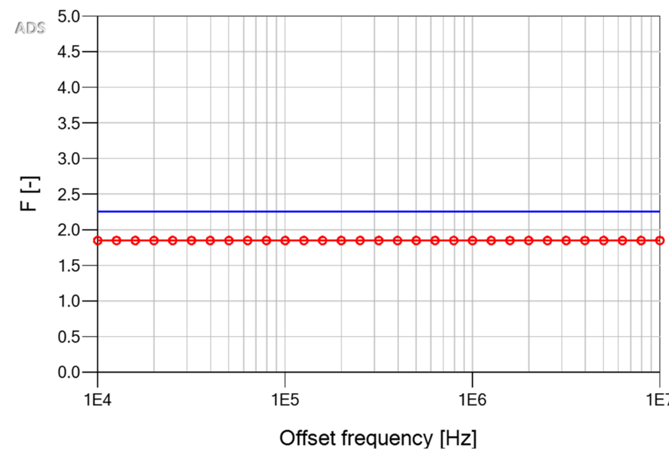
Only accounting for thermal coming from main transistor (current source is noiseless and flicker noise of transistor has been turned off by setting KF=0 see schematic)

Am	gamma	F
0.107	1.253	2.253

SSB Phase Noise (thermal noise only)



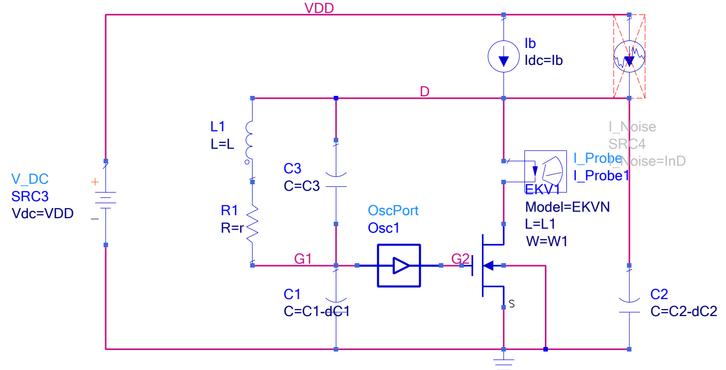
Extracted Noise Factor (thermal noise only)

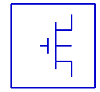


Phase noise calculated with simulated amplitude (since goes with the square)

Very good match between model and simulations despite the linear analysis

Pierce Oscillator – ADS HB Simulations (SI)





ekv_va_Model
EKVN
TYPE=1
L=10E-6
W=10E-6
TNOM=27
M=1.0
NS=1.0
COX=8.46e-3
XJ=1.6e-7
VTO=VT0n
TCV=6.03e-4
GAMMA=0.540
PHI=990.0e-3
KP=420.0e-6
BEX=-1.569
THETA=0.0
E0=5.917e+7
UCRIT=3.75e+6
UCEX=1.76
LAMBDA=0.340
DL=-7.6e-8
DW=3.9e-8
WETA=0.0
LETA=220.0e-3
Q0=0.000420
LK=3.80e-7
IBA=0.0
IBB=270.0e+6
IBBT=0.0
IBN=1.0
RSH=600.0
HDIF=2e-07
AVTO=1E-6
AKP=1E-6
AGAMMA=1E-6
AF=0.8265
KF=0
AllParams=

Simulation using the full EKV 2.6 model for a 180nm CMOS generic process.

Transistor biased in SI with $V_G - V_{T0} = 300mV$

Amplitude set to 300 mV

VAR
Technology
T0=273
UT=0.025875
kT=qelectron*UT
T=kT/boltzmann
Tcelsius=T-T0
n=1.271
nUT=n*UT
VT0n=0.455
Ispecn=0.715E-6
KFn=8.1E-24

VAR
Bias
VDD=1.8

VAR
Noise
SInD=4*kT*2/3*n*Gm
InD=sqrt(SInD)
Gm=2*Ib/(VG_VT0)

HARMONIC BALANCE
HarmonicBalance
HB1
Freq[1]=f0
Order[1]=15
Oversample[1]=8
NLNoiseMode=
NLNoiseStart=10 kHz
NLNoiseStop=10.0 MHz
NLNoiseDec=10
NoiseOutputPort=2
PhaseNoise=no
NoiseNode[1]="D"
NoiseNode[2]="G2"

VAR
Specifications
f0=1G
A=0.3
QL=10
C2=5E-12
VG_VT0=0.3
L1=1E-6

DC
DC
DC1

HB NOISE CONTROLLER
NoiseCon
NC1
NoiseNode[1]=G2
NoiseNode[2]=G1
NoiseNode[3]=D

VAR
ParametersCalculation
w0=2*Pi*f0
vgt=VG_VT0/nUT
x=A/nUT
C1=C2
C3=0.1*C12
C12=C1*C2/(C1+C2)
Ceq=C12+C3
L=(Gmcrit**2*C3+w0**2*(C1+C2)*(C1*C2+C1*C3+C2*C3))/(w0*((Gmcrit*C3)**2+w0**2*(C1*C2+C1*C3+C2*C3)**2))/w0
r=w0*L/QL
Gmcrit=w0*QL*C2*alpha1/(2*alpha3)*(1-sqrt(1-(2*alpha3/(alpha1*QL))**2*(alpha1+1)*(1+alpha1+alpha1/alpha3)))
vgtcrit=sqrt(vgt**2-x**2/2)
icrit=(vgtcrit/2)**2
IC=icrit+x**2/8
Ispec=2*nUT*Gmcrit/vgtcrit
Ib=Ispec*IC
Ibcrit=Ispec*icrit
S=Ispec/Ispecn
W1=S*L1
alpha1=C1/C2
alpha3=C3/C2
Ib1=1E-3
dC1=0.25E-12
dC2=0.25E-12

TRANSIENT
Tran
Tran1
StopTime=1000 nsec
MaxTimeStep=0.01 nsec

Pierce Oscillator – ADS HB Simulations (SI)

Technology and physical parameters

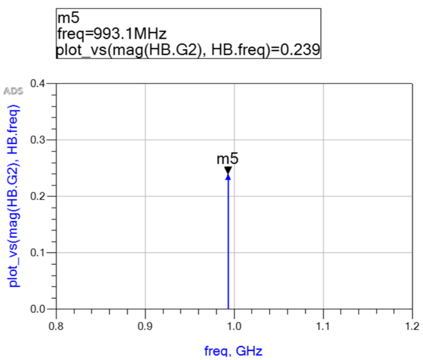
UT	n	nUT	VT0n	Ispecn
25.87 m	1.271	32.89 m	455.0 m	715.0 n

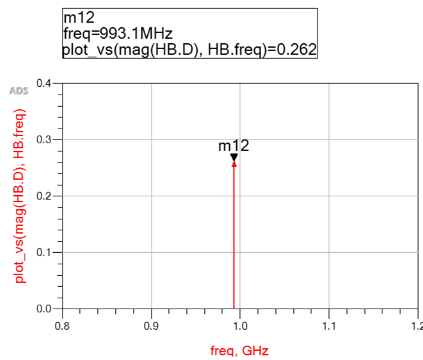
Specifications

f0	QL	C2	x	IC	L1
1.000 G	10.00	5.000 p	9.122	20.80	1.000 u

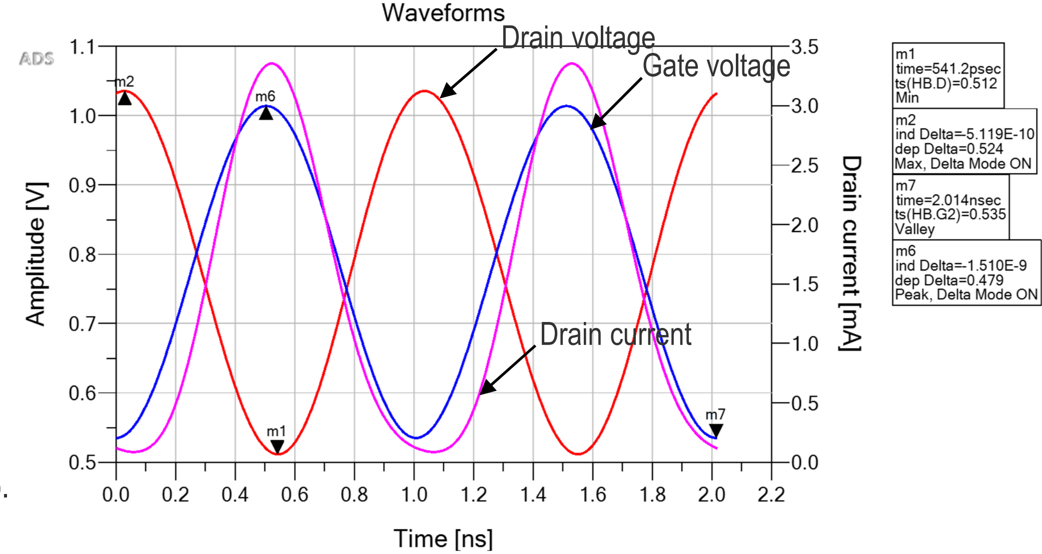
Calculated parameters

A	C1	C3	L	r	Gmcrit	Ibcrit	Ib	Ispec	W1
300.0 m	5.000 p	250.0 f	9.220 n	5.793	6.919 m	733.9 u	1.468 m	70.55 u	98.68 u





Waveforms



Slight shift in resonant frequency due to fairly low inductor Q ($Q_L = 10$). Amplitude of the fundamental component at the gate is lower than expected (239 mV). This probably due to the fact that we have additional effects (such mobility reduction and velocity saturation) which are not accounted for in the simple quadratic model.

Simulations much more sensitive than in WI. Does not always converge. Amplitude is slightly lower than 300mV (240mV) at the gate and at the drain (260mV)

Pierce Oscillator – ADS Transient Simulations (SI)

Technology and physical parameters

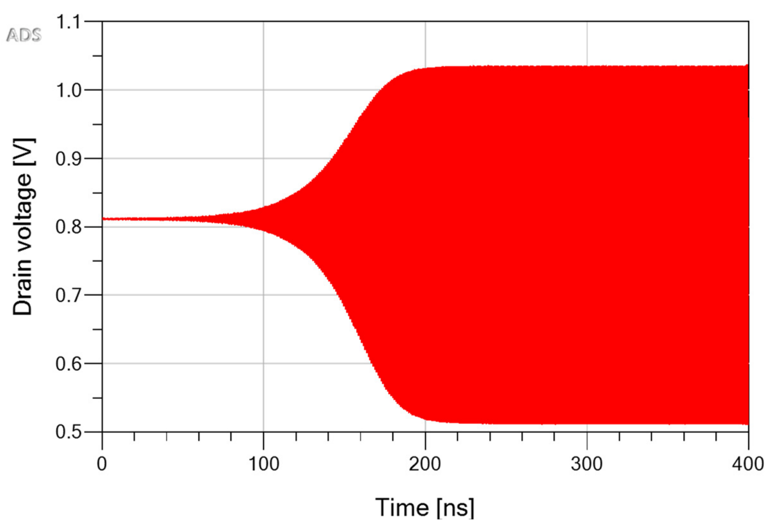
UT	n	nUT	VT0n	Ispecn
25.87 m	1.271	32.89 m	455.0 m	715.0 n

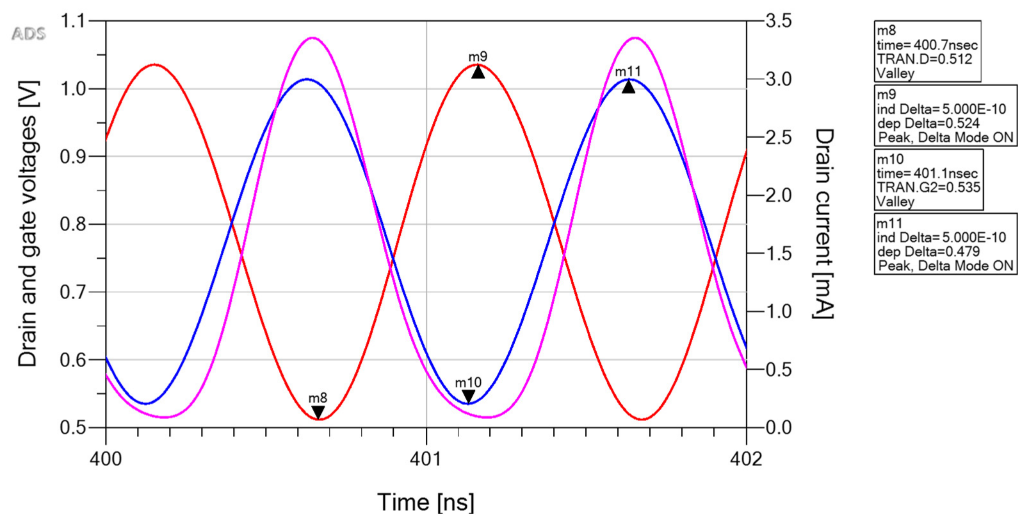
Specifications

f0	QL	C2	x	IC	L1
1.000 G	10.00	5.000 p	9.122	20.80	1.000 u

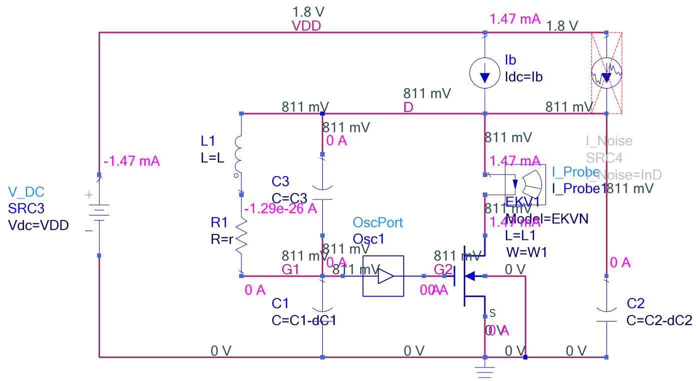
Calculated parameters

A	C1	C3	L	r	Gmcrit	Ibcrit	Ib	Ispec	W1
300.0 m	5.000 p	250.0 f	9.220 n	5.793	6.919 m	733.9 u	1.468 m	70.55 u	98.68 u





Pierce Oscillator – ADS SSB Phase Noise Simulation (SI)



Technology and physical parameters

UT	n	nUT	VT0n	Ispecn
25.87 m	1.271	32.89 m	455.0 m	715.0 n

Specifications

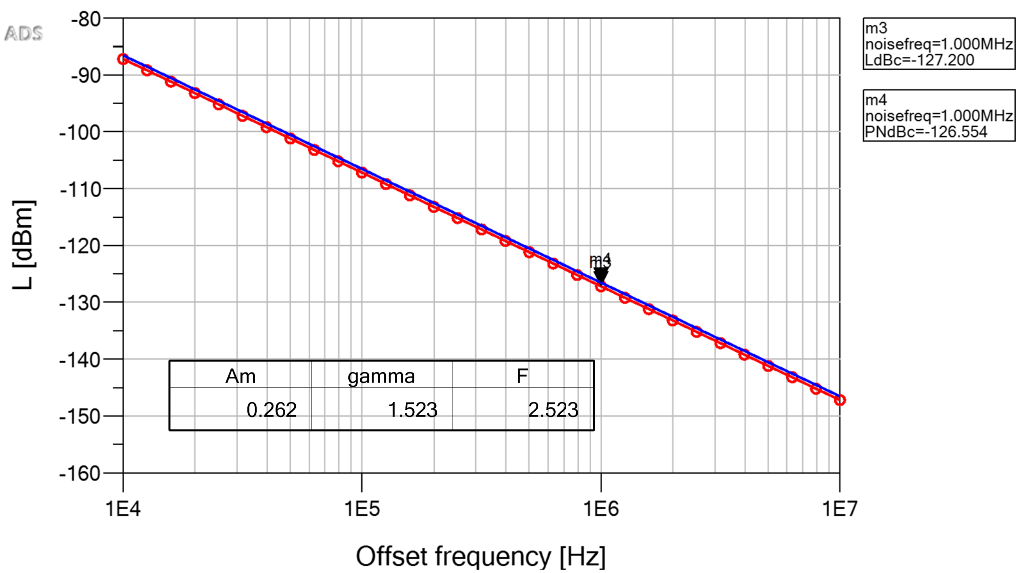
f0	QL	C2	x	IC	L1
1.000 G	10.00	5.000 p	9.122	20.80	1.000 u

Calculated parameters

A	C1	C3	L	r	Gmcrit	Ibcrit	Ib	Ispec	W1
300.0 m	5.000 p	250.0 f	9.220 n	5.793	6.919 m	733.9 u	1.468 m	70.55 u	98.68 u

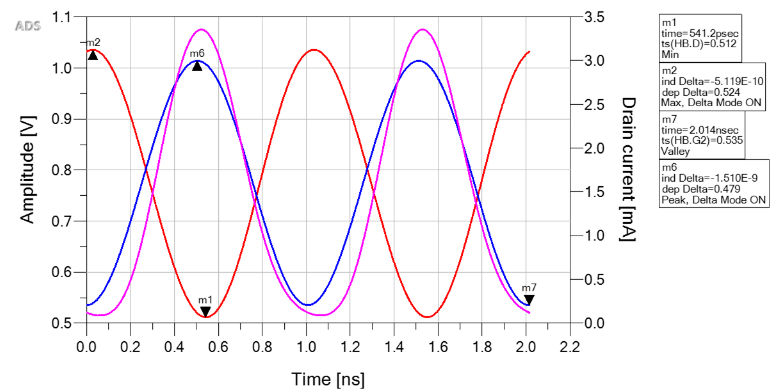
Only accounting for thermal coming from main transistor (current source is noiseless)

SSB Phase Noise (thermal noise only)

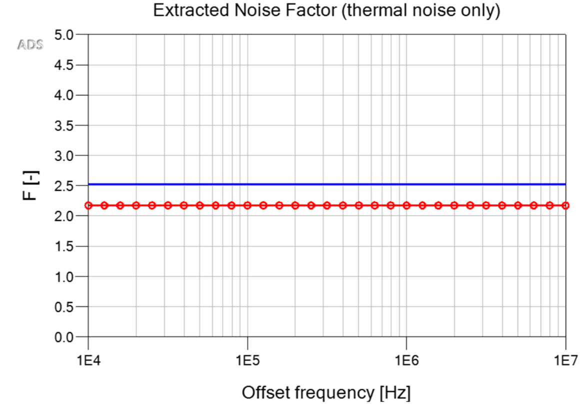


Am	gamma	F
0.262	1.523	2.523

Waveforms



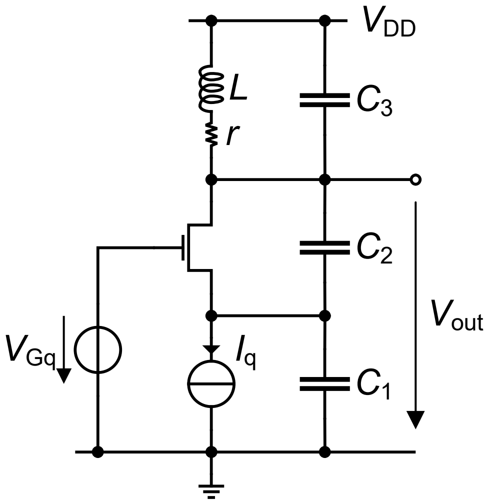
Extracted Noise Factor (thermal noise only)

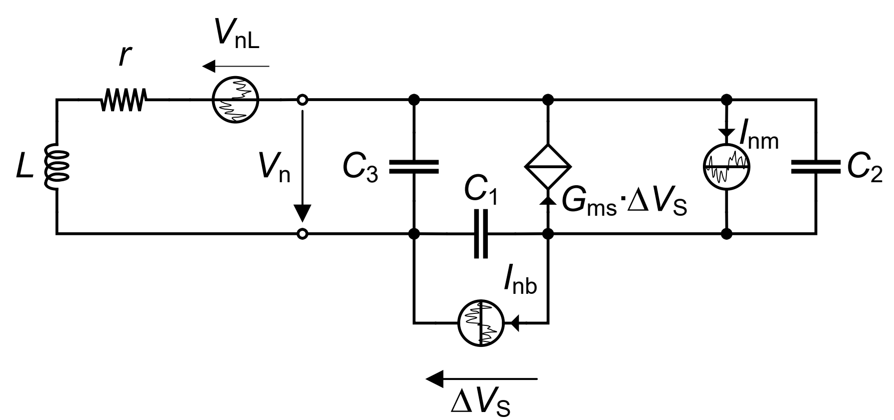


Very good match between model and simulations despite the linear analysis

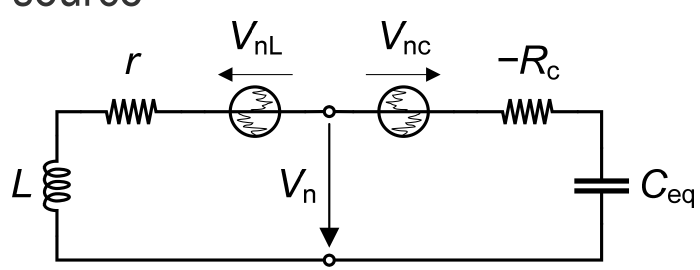
Linear Noise Analysis of Colpitts Oscillator

- The equivalent small-signal circuit including the noise sources from the inductor, MOS transistor and bias current source is given below





- The active circuit, including its noise sources, can be replaced by its Thévenin source

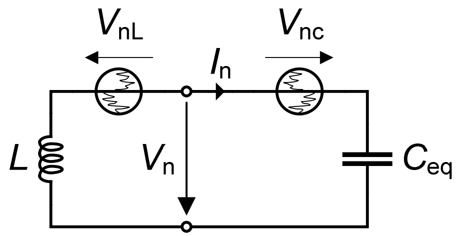


$$R_c \cong \frac{G_{ms} C_1 C_2}{\omega_0^2 (C_1 C_2 + C_1 C_3 + C_2 C_3)^2} = \frac{G_{ms}}{(\omega_0 C_{eq})^2} \cdot \frac{C_1 C_2}{(C_1 + C_2)^2}$$

$$C_{eq} = C_3 + C_{12} = C_3 + \frac{C_1 C_2}{C_1 + C_2} = \frac{C_1 C_2 + C_1 C_3 + C_2 C_3}{C_1 + C_2}$$

Linear Noise Analysis of Colpitts Oscillator

- At the resonance frequency the inductor loss r is compensated by the negative resistance $-R_c$ provided by the circuit. The latter then simplifies to



- The noise transfer function from sources V_{nL} and V_{nc} to V_n are given by

$$H_{nL}(\omega) = \frac{V_n}{V_{nL}} = \frac{1}{1-(\omega/\omega_0)^2} \quad \text{and} \quad H_{nc}(\omega) = \frac{V_n}{V_{nc}} = -\frac{(\omega/\omega_0)^2}{1-(\omega/\omega_0)^2} \quad \text{with} \quad \omega_0 = \frac{1}{\sqrt{LC_{eq}}}$$

- At an offset frequency $\Delta\omega \ll \omega_0$ from the carrier we have

$$\omega = \omega_0 + \Delta\omega \;\rightarrow\; \frac{\omega}{\omega_0} = 1 + \frac{\Delta\omega}{\omega_0} \;\rightarrow\; \left(\frac{\omega}{\omega_0}\right)^2 = \left(1 + \frac{\Delta\omega}{\omega_0}\right)^2 \cong 1 + 2\frac{\Delta\omega}{\omega_0}$$

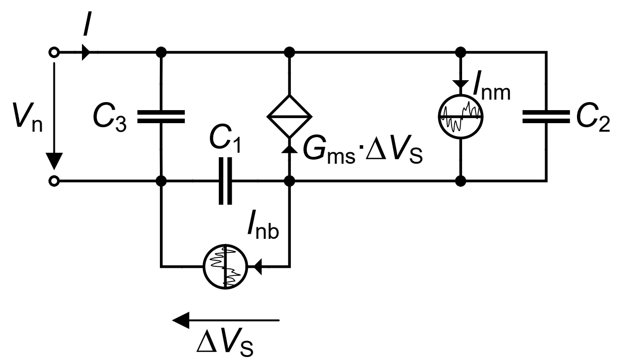
$$H_{nL}(\Delta\omega) \cong -\frac{\omega_0}{2\Delta\omega} \quad \text{and} \quad H_{nc}(\Delta\omega) \cong \frac{\omega_0}{2\Delta\omega}$$

Linear Noise Analysis of Colpitts Oscillator

- The noise voltage PSD is given by

$$S_{V_n} = |H_{nL}|^2 S_{V_{nL}} + |H_{nc}|^2 S_{V_{nc}} \cong \left(\frac{\omega_0}{2\Delta\omega}\right)^2 \left(S_{V_{nL}} + S_{V_{nc}}\right) \quad \text{with} \quad S_{V_{nL}} = 4kT \cdot r$$

where $S_{V_{nc}}$ has to be evaluated from the following circuit



$$V_{nc} = Z_{nm} \cdot I_{nm} + Z_{nb} \cdot I_{nb}$$

$$S_{V_{nc}} = |Z_{nm}|^2 \cdot S_{I_{nm}} + |Z_{nb}|^2 \cdot S_{I_{nb}}$$

$$S_{I_{nm}} = 4kT \cdot \delta_{nm} G_{msm} \quad S_{I_{nb}} = 4kT \cdot \gamma_{nb} G_{mb}$$

$$Z_{nm} = \frac{C_1}{G_{ms}C_3 + s(C_1C_2 + C_1C_3 + C_2C_3)} \cong \frac{C_1}{s(C_1C_2 + C_1C_3 + C_2C_3)} = \frac{C_1}{C_1 + C_2} \cdot \frac{1}{sC_{eq}}$$

$$Z_{nb} = \frac{G_{ms} + sC_2}{sG_{ms}C_3 + s^2(C_1C_2 + C_1C_3 + C_2C_3)} \cong \frac{C_2}{s(C_1C_2 + C_1C_3 + C_2C_3)} = \frac{C_2}{C_1 + C_2} \cdot \frac{1}{sC_{eq}}$$

$$S_{V_{nc}} = 4kT \cdot \left(\frac{C_1}{C_1 + C_2} \cdot \frac{1}{\omega_0 C_{eq}}\right)^2 \cdot \left(\delta_{nm} G_{msm} + \left(\frac{C_2}{C_1}\right)^2 \cdot \gamma_{nb} G_{mb}\right)$$

Linear Noise Analysis of Colpitts Oscillator

- Finally the noise voltage PSD is given by

$$S_{V_n} \cong kT \cdot r \cdot (1+\gamma) \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2$$

where γ is the noise excess factor representing the noise contribution of the circuit and given by

$$\gamma = \frac{1}{r} \cdot \left(\frac{C_1}{C_1+C_2} \cdot \frac{1}{\omega_0 C_{eq}}\right)^2 \cdot \left(\delta_{nm} G_{msm} + \left(\frac{C_2}{C_1}\right)^2 \cdot \gamma_{nb} G_{mb}\right)$$

$$= \frac{Q}{\omega_0 C_{eq}} \cdot \left(\frac{C_1}{C_1+C_2}\right)^2 \cdot \left(\delta_{nm} G_{msm} + \left(\frac{C_2}{C_1}\right)^2 \cdot \gamma_{nb} G_{mb}\right)$$

- Since the critical transconductance is given by

$$G_{mscrit} = r \cdot \omega_0^2 \cdot \frac{(C_1C_2 + C_1C_3 + C_2C_3)^2}{C_1C_2} = r \cdot \omega_0^2 \cdot \frac{C_1}{C_2} \cdot \left(\frac{C_1+C_2}{C_1}\right)^2 \cdot C_{eq}^2$$

- The γ noise excess factor can also be written as

$$\gamma = \frac{C_1}{C_2} \cdot \frac{\delta_{nm} G_{msm}}{G_{mscrit}} + \frac{C_2}{C_1} \cdot \frac{\gamma_{nb} G_{mb}}{G_{mscrit}}$$

Linear Noise Analysis of Colpitts Oscillator

- Since minimum G_{mscrit} is obtained for $C_1 = C_2$, the γ noise excess factor reduces to

$$\gamma = \frac{\delta_{nm} G_{msm} + \gamma_{nb} G_{mb}}{G_{mscrit}} \quad \text{for} \quad C_1 = C_2$$

- Since usually $G_{msm}/G_{mscrit} > 3$ for ensuring start-up and reaching the desired amplitude, the noise can be significantly degraded by the active part of the oscillator

- The SSB phase noise is then given by

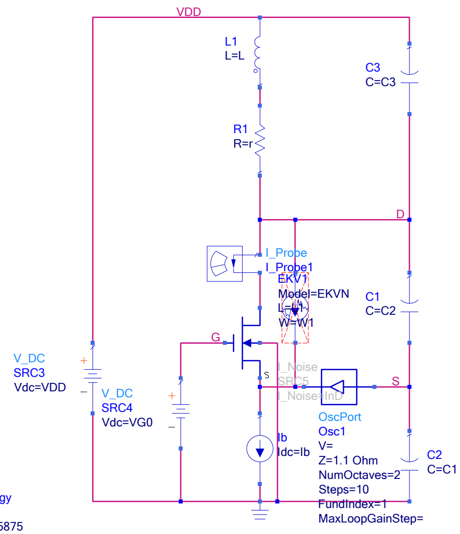
$$\mathcal{L}(\Delta\omega) = \frac{S_{V_n}/2}{A^2/2} = \frac{kT \cdot r \cdot (1+\gamma)}{A^2} \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2 = \frac{kT \cdot (1+\gamma)}{A^2 \cdot Q \cdot \omega_0 C_{eq}} \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2$$

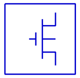
with Q given by

$$Q = \frac{\omega_0 L}{r} = \frac{1}{\omega_0 C_{eq} r}$$

- Note that this is still a linear analysis not accounting for the time variance of the circuit

Colpitts Oscillator – ADS HB Simulations





ekv_va_Model
EKVN
TYPE=1
L=10E-6
W=10E-6
TNOM=27
M=1.0
NS=1.0
COX=8.46e-3
XJ=1.6e-7
VTO=VT0n
TCV=6.03e-4
GAMMA=0.540
PHI=990.0e-3
KP=420.0e-6
BEX=-1.569
THETA=0.0
E0=5.917e+7
UCRIT=3.75e+6
UCEX=1.76
LAMBDA=0.340
DL=-7.6e-8
DW=3.9e-8
WETA=0.0
LETA=220.0e-3
Q0=0.000420
LK=3.80e-7
IBA=0.0
IBB=270.0e+6
IBBT=0.0
IBN=1.0
RSH=600.0
HDIF=2e-07
AVTO=1E-6
AKP=1E-6
AGAMMA=1E-6
AF=0.8265
KF=0
AllParams=

VAR
Technology
T0=273
UT=0.025875
kT=qelectron*UT
T=kT/boltzmann
Tcelsius=T-T0
n=1.271
nUT=n*UT
VT0n=0.455
Ispecn=0.715E-6
betan=420E-6
KFn=8.1E-24

VAR
Specifications
f0=1G
A=0.1
QL=10
C2=5E-12
IC=0.1
L1=0.18E-6
VG0=1

VAR
Bias
VDD=1.8

VAR
Noise
SInD=4*kT*2/3*Gms
InD=sqrt(SInD)
Gms=Ib/UT

VAR
ParametersCalculation
w0=2*pi*f0
C1=C2
C3=0.1*C12
C12=C1*C2/(C1+C2)
Ceq=C12+C3
L2=1/(w0**2*Ceq)
L=(Gmscrit**2*C3+w0**2*(C1+C2)*(C1*C2+C1*C3+C2*C3))/(w0*((Gmscrit*C3)**2+w0**2*(C1*C2+C1*C3+C2*C3)**2))/w0
r=w0*L/QL
Gmscrit1=w0/QL*(C1+C2)*(1+C3/C12)
Gmscrit=w0*QL*C2*alpha1/(2*alpha3)*(1-sqrt(1-(2*alpha3/(alpha1*QL))**2*(alpha1+1)*(1+alpha1+alpha1/alpha3)))
Icrit=Gmscrit*UT
x=A/UT
a1=0.5
a2=0.2
chi=(1+a1*x+a2*x**2)/(a1*x+a2*x**2)

Ib=Icrit*x/2*chi
Ispec=Ib/IC
S=Ispec/Ispecn
W1=S*L1
alpha1=C1/C2
alpha3=C3/C2

HARMONIC BALANCE
HarmonicBalance
HB1
Freq[1]=f0
Order[1]=15
Oversample[1]=8
NLNoiseMode=
NLNoiseStart=10 kHz
NLNoiseStop=10.0 MHz
NLNoiseDec=10
NoiseOutputPort=2
PhaseNoise=no
NoiseNode[1]="D"
NoiseNode[2]="S"

DC
DC
DC1

TRANSIENT
Tran
Tran1
StopTime=500.0 nsec
MaxTimeStep=0.01 nsec

HB NOISE CONTROLLER
NoiseCon
NC1
NoiseNode[1]=D
NoiseNode[2]=S

Parameter	Value
f_0	1 GHz
A	100 mV
Q_L	10
C_2	5 pF
a_1	1
a_3	1
IC	0.1
L	0.18 µm

Colpitts Oscillator – ADS HB Simulations

Technology and physical parameters

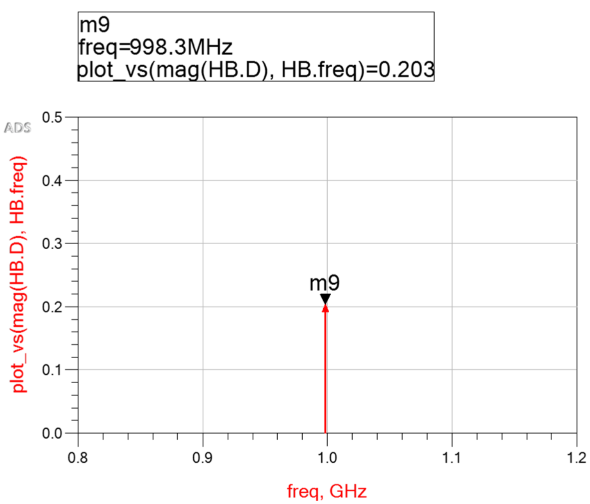
UT	n	nUT	VT0n	Ispecn
25.87 m	1.271	32.89 m	455.0 m	715.0 n

Specifications

f0	QL	C2	x	IC	L1
1.000 G	10.00	5.000 p	3.865	100.0 m	180.0 n

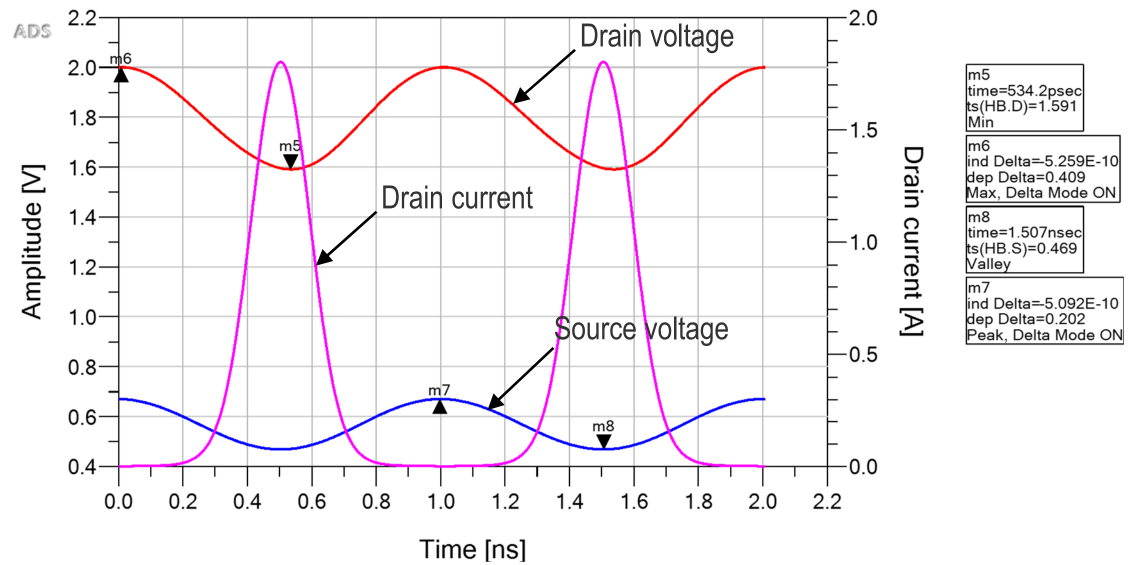
Calculated parameters

A	C1	C3	L	r	Icrit	Ib	Ispec	W1	Gmscrit	Icrit
100.0 m	5.000 p	250.0 f	9.220 n	5.793	179.0 u	416.3 u	4.163 m	1.048 m	6.919 m	179.0 u



Slight shift in resonant frequency due to parasitic capacitances coming from transistor and not accounted for

Waveforms



Amplitude is almost exactly 100mV (100.5mV) at the source and slightly larger at the drain (116 mV)

Colpitts Oscillator – ADS Transient Simulations

Technology and physical parameters

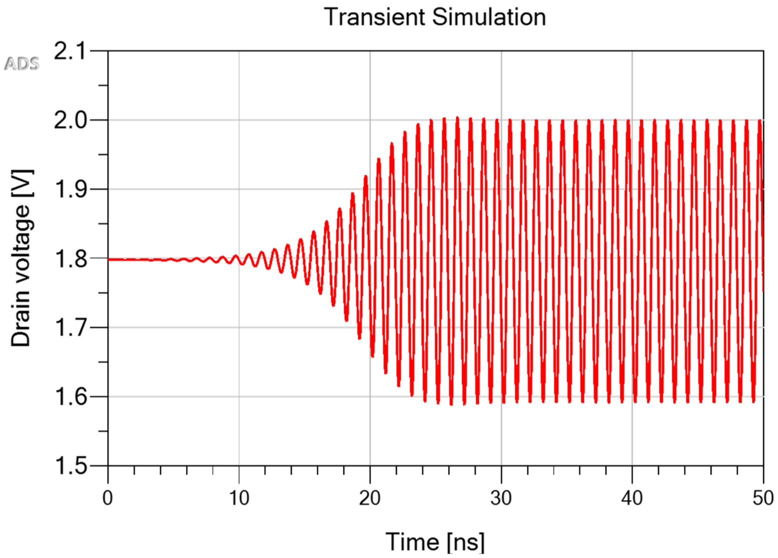
UT	n	nUT	VT0n	Ispecn
25.87 m	1.271	32.89 m	455.0 m	715.0 n

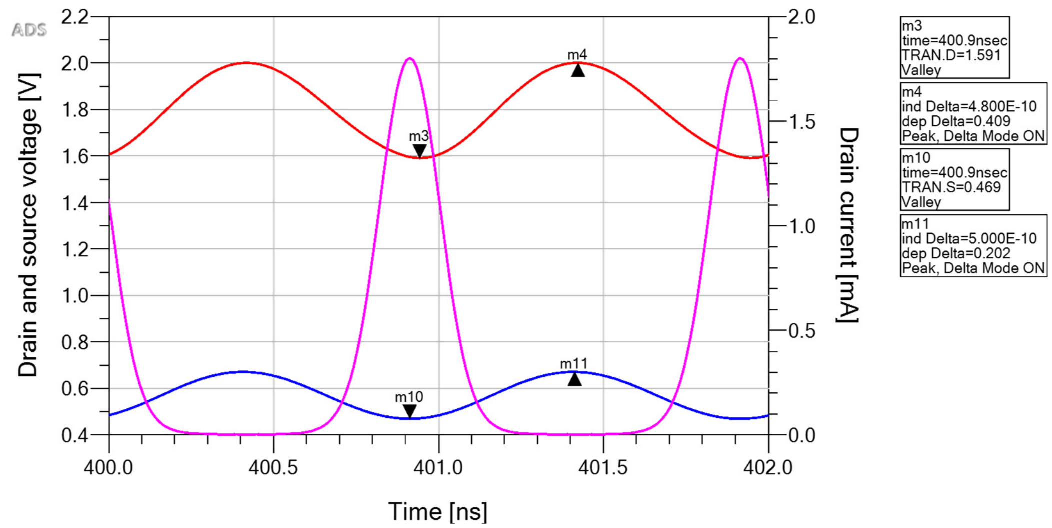
Specifications

f0	QL	C2	x	IC	L1
1.000 G	10.00	5.000 p	3.865	100.0 m	180.0 n

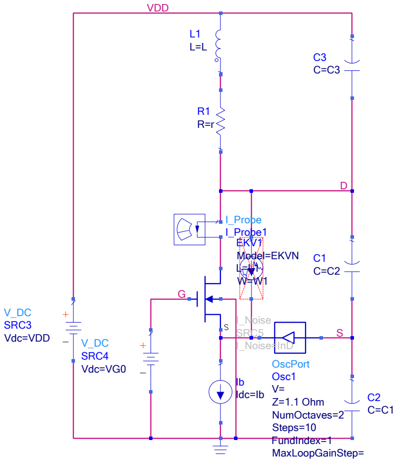
Calculated parameters

A	C1	C3	L	r	Icrit	Ib	Ispec	W1	Gmscrit	Icrit
100.0 m	5.000 p	250.0 f	9.220 n	5.793	179.0 u	416.3 u	4.163 m	1.048 m	6.919 m	179.0 u





Colpitts Oscillator – ADS SSB Phase Noise Simulation



Technology and physical parameters

UT	n	nUT	VT0n	Ispecn
25.87 m	1.271	32.89 m	455.0 m	715.0 n

Specifications

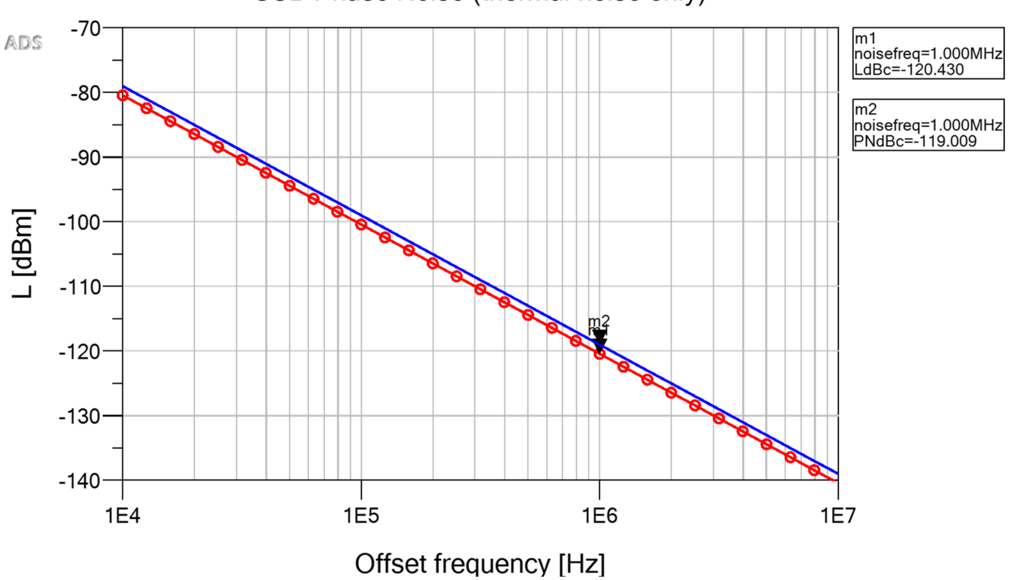
f0	QL	C2	x	IC	L1
1.000 G	10.00	5.000 p	3.865	100.0 m	180.0 n

Calculated parameters

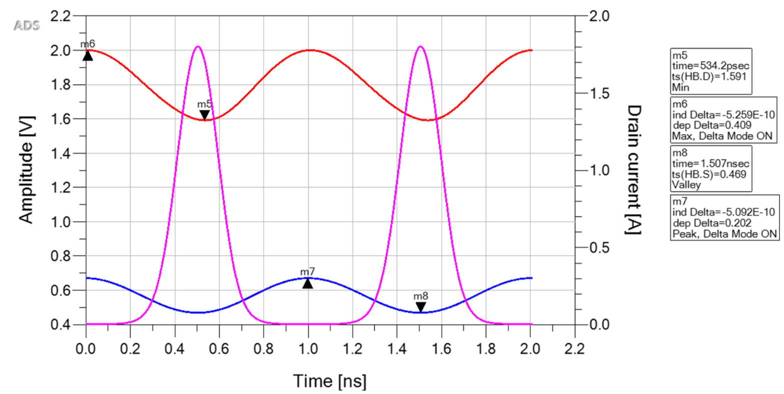
A	C1	C3	L	r	Icrit	Ib	Ispec	W1	Gmscrit	Icrit
100.0 m	5.000 p	250.0 f	9.220 n	5.793	179.0 u	416.3 u	4.163 m	1.048 m	6.919 m	179.0 u

Only accounting for thermal coming from main transistor (current source is noiseless)

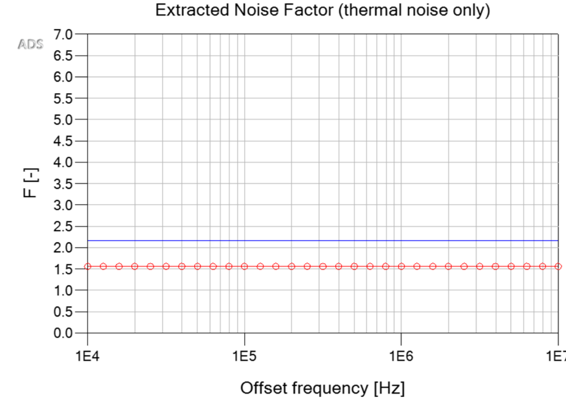
SSB Phase Noise (thermal noise only)



Waveforms



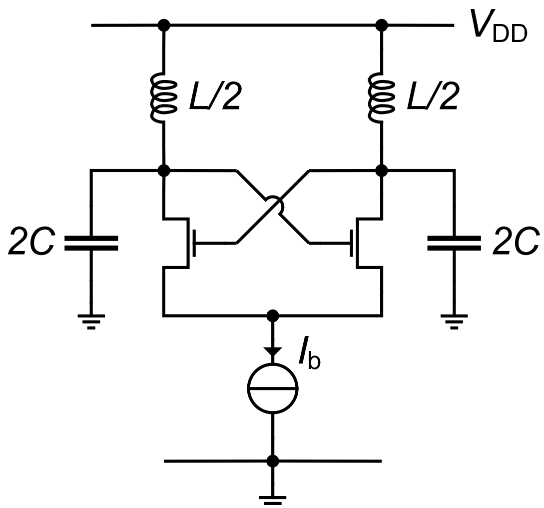
Extracted Noise Factor (thermal noise only)

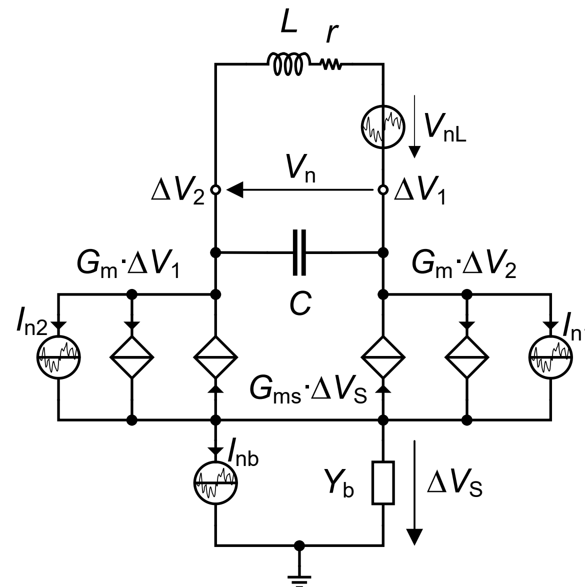


Very good match between model and simulations despite the linear analysis

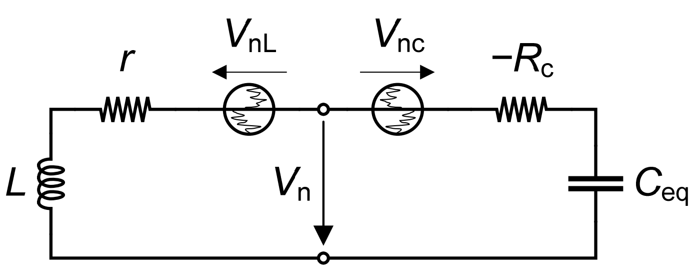
Linear Noise Analysis of the Cross-coupled Oscillator

- The same approach can be used for the cross-coupled pair oscillator
- The small-signal circuit including the noise sources is given below





- The cross-coupled pair , including its noise sources, can be replaced by its Thévenin source

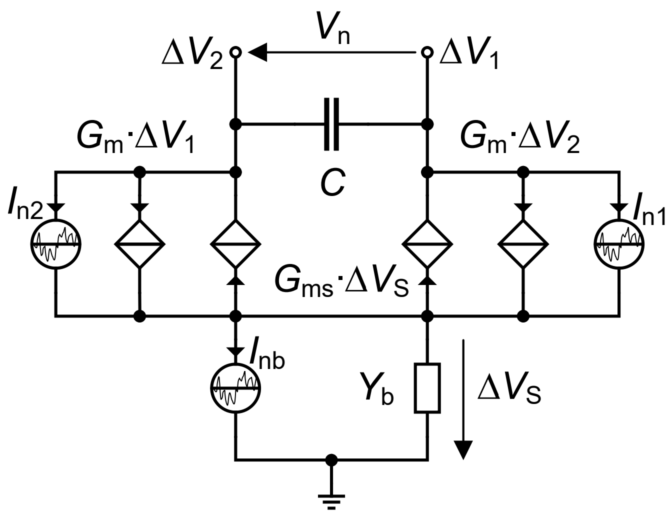


$$R_c \cong \frac{G_m}{2\omega_0^2 \cdot C^2}$$

$$C_{eq} = C$$

Linear Noise Analysis of Cross-coupled Oscillator

- The equivalent Thévenin noise source of the circuit V_{nc} is obtained from the circuit shown below
- Note that if perfect matching is assumed, under the small-signal approximation, the noise coming from the bias source I_{nb} does not contribute to the differential noise source V_n



$$V_{nc} = Z_{m1} \cdot I_{n1} + Z_{m2} \cdot I_{n2}$$

with

$$Z_{m1} = -Z_{m2} = Z_m = \frac{1}{G_m - s \cdot 2C} \cong -\frac{1}{s \cdot 2C}$$

- The noise voltage PSD due to the circuit is then given by

$$S_{V_{nc}} = 2 \cdot \left(\frac{1}{\omega_0 2C} \right)^2 \cdot 4kT \cdot \gamma_n G_m = \frac{2 \cdot kT \cdot \gamma_n G_m}{\omega_0^2 \cdot C^2}$$

Linear Noise Analysis of Cross-coupled Oscillator

- Similarly to the Colpitts oscillator, the noise voltage PSD is given by

$$S_{V_n} = |H_{nL}|^2 S_{V_{nL}} + |H_{nc}|^2 S_{V_{nc}} \cong \left(\frac{\omega_0}{2\Delta\omega}\right)^2 \left(S_{V_{nL}} + S_{V_{nc}}\right) \quad \text{with} \quad S_{V_{nL}} = 4kT \cdot r$$

- Which reduces to

$$S_{V_n} \cong kT \cdot r \cdot (1+\gamma) \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2$$

- The phase noise is then given by

$$\mathcal{L}(\Delta\omega) = \frac{S_{V_n}/2}{A^2/2} = \frac{kT \cdot r \cdot (1+\gamma)}{A^2} \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2 = \frac{kT \cdot (1+\gamma)}{A^2 \cdot Q_L \cdot \omega_0 C} \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2$$

where γ is the noise excess factor representing the noise contribution of the circuit and given by

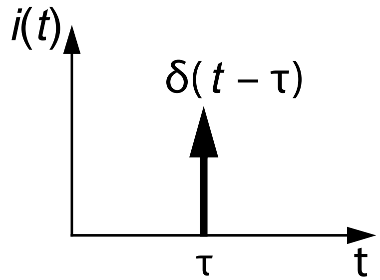
$$\gamma = \frac{\gamma_n G_m}{2r \cdot \omega_0^2 \cdot C^2} = \frac{\gamma_n G_m \cdot Q_L}{2 \cdot \omega_0 \cdot C} = \gamma_n \cdot \frac{G_m}{G_{mcrit}}$$

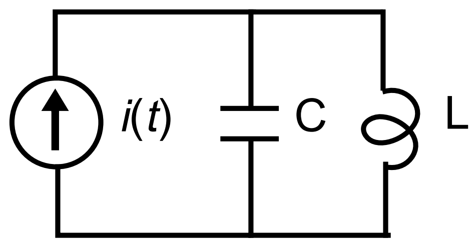
- Since $G_{mcrit} = 2r \cdot \omega_0^2 \cdot C^2 = \dfrac{2C \cdot \omega_0}{Q_L}$ and $Q_L = \dfrac{\omega_0 L}{r} = \dfrac{1}{\omega_0 r C}$

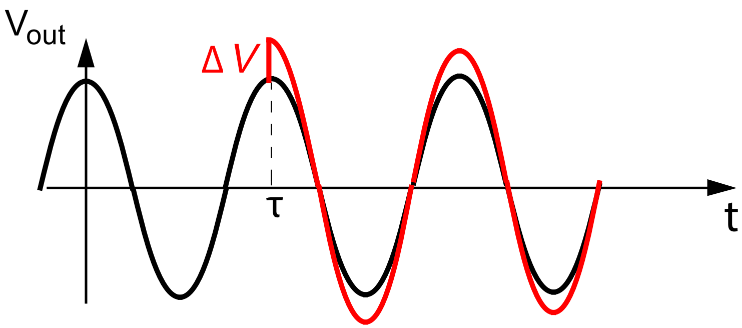
Outline

- Fundamentals
- Linear analysis
- Nonlinear analysis

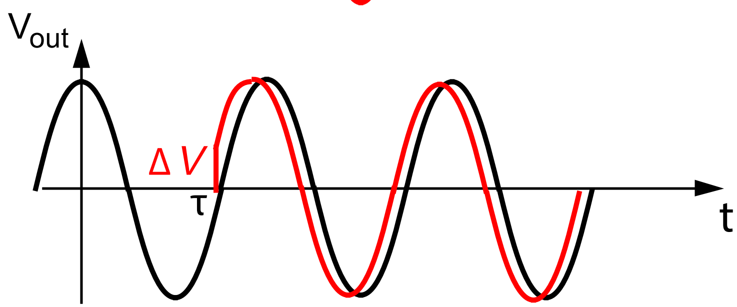
Oscillators are Time-Variant Systems







Impulse injected at the peak of amplitude.

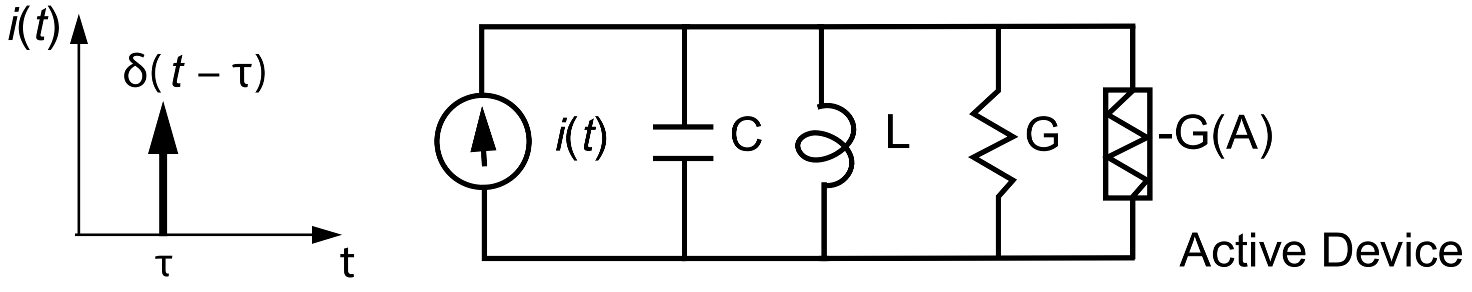


Impulse injected at zero crossing.

Even for an ideal LC oscillator, the phase response is *Time Variant.*

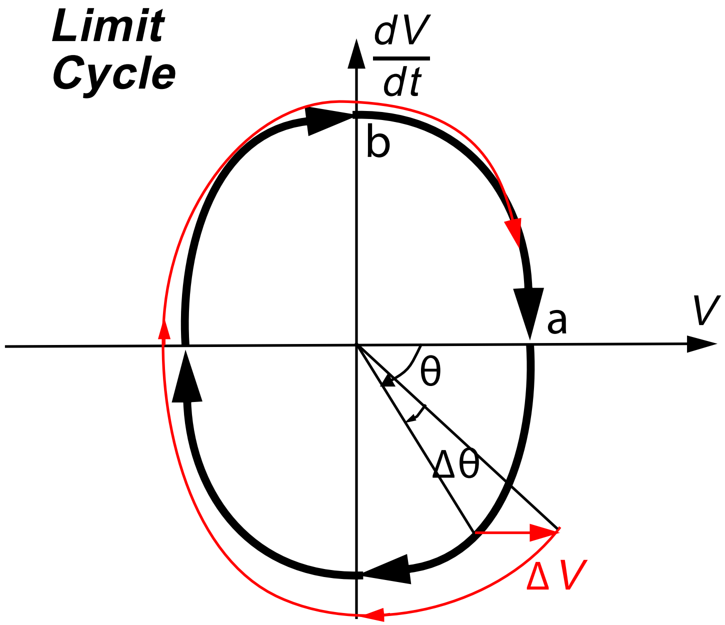
A. Hajimiri and T. Lee, JSSC, Feb. 1998; T. Lee and A. Hajimiri, JSSC, March 2000; T. Lee, Cambridge, 2nd-ed. 2004.

Amplitude Restoring Mechanism



Once Introduced, phase error persists indefinitely.

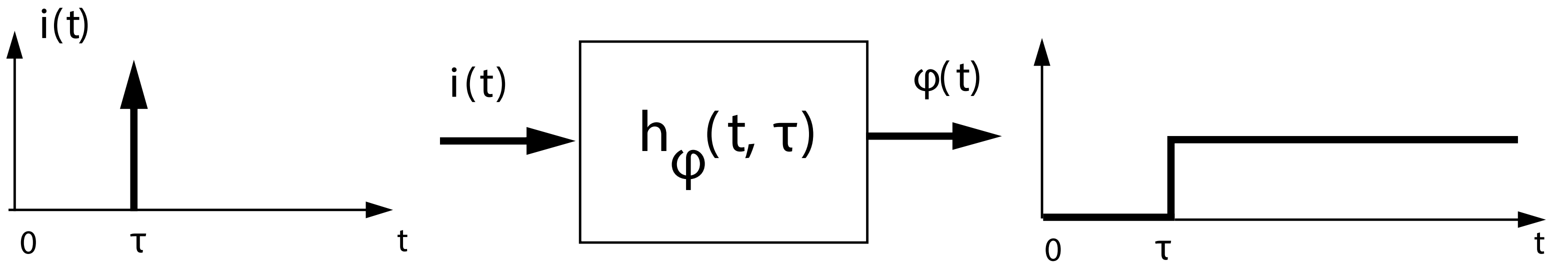
Non-linearity quenches amplitude changes over time.



📖 A. Hajimiri and T. Lee, JSSC, Feb. 1998; T. Lee and A. Hajimiri, JSSC, March 2000; T. Lee, Cambridge, 2nd-ed. 2004.

Phase Impulse Response

- The phase impulse response of an arbitrary oscillator is a time varying step



- The unit impulse response is

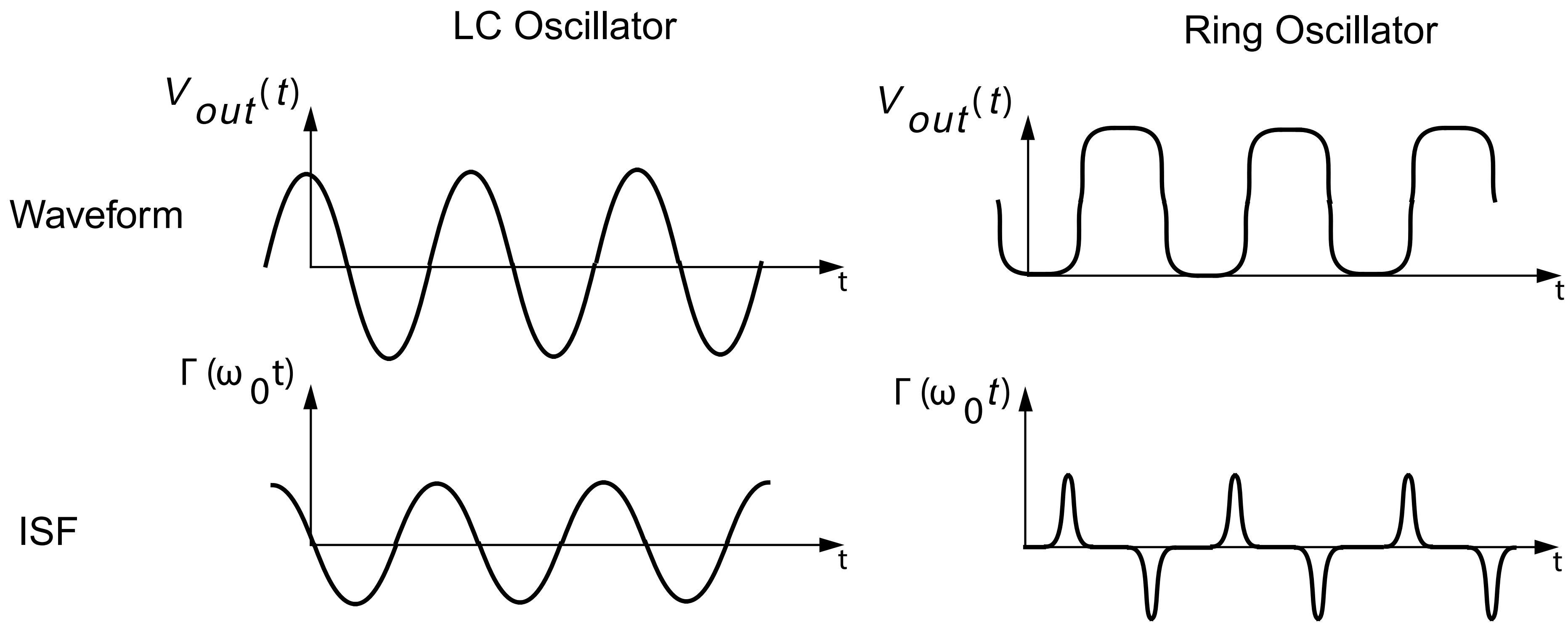
$$h_\phi(t,\tau) = \frac{\Gamma(\omega_0 \tau)}{q_{\max}} \cdot u(t-\tau)$$

- $\Gamma(x)$ is a dimensionless function periodic in 2π describing how much phase change results from applying an impulse at time $t = T \cdot x/(2\pi)$ and $u(t)$ is the unit step
- Dividing $\Gamma(x)$ by q_{max} makes the response independent of the amplitude
- q_{max} is the maximum charge on the tank capacitor C for an amplitude A

$$q_{\max} = C \cdot A$$

A. Hajimiri and T. Lee, JSSC, Feb. 1998; T. Lee and A. Hajimiri, JSSC, March 2000; T. Lee, Cambridge, 2nd-ed. 2004.

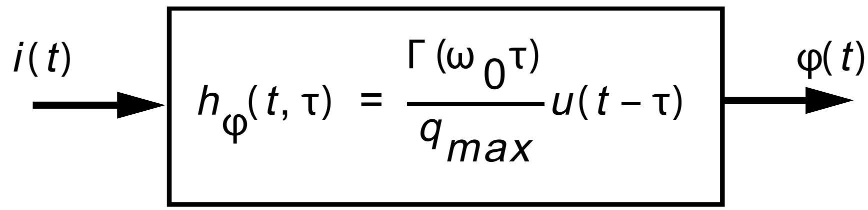
Impulse Sensitivity Function (ISF)



The ISF quantifies the sensitivity of every point in the waveform to perturbations.

📖 A. Hajimiri and T. Lee, JSSC, Feb. 1998; T. Lee and A. Hajimiri, JSSC, March 2000; T. Lee, Cambridge, 2nd-ed. 2004.

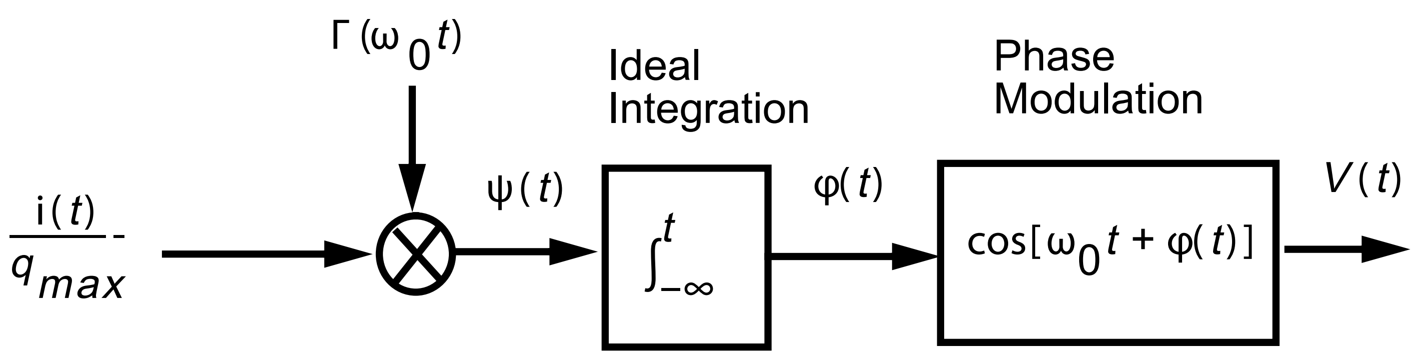
Phase Response to an Arbitrary Source



- The phase response is then given by

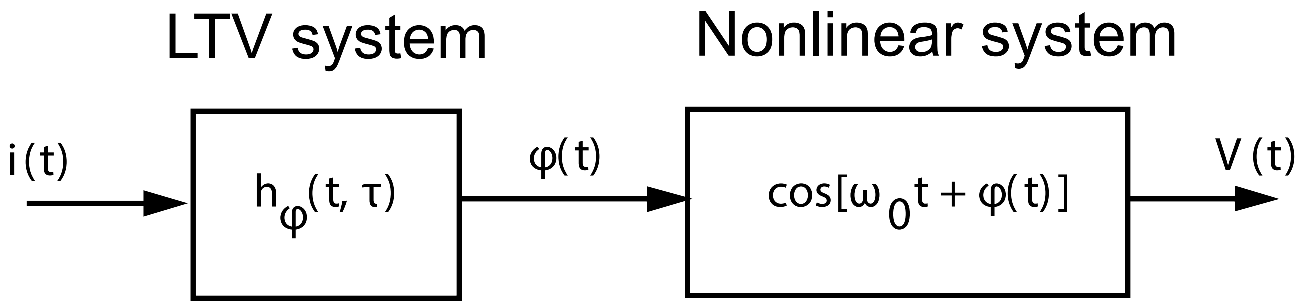
$$\varphi(t) = \int_{-\infty}^{+\infty} h_{\varphi}(t,\tau) \cdot i(\tau) \cdot d\tau = \frac{1}{q_{\max}} \cdot \int_{-\infty}^{t} \Gamma(\omega_0 \tau) \cdot i(\tau) \cdot d\tau$$

- This corresponds to the following equivalent block diagram



A. Hajimiri and T. Lee, JSSC, Feb. 1998; T. Lee and A. Hajimiri, JSSC, March 2000; T. Lee, Cambridge, 2nd-ed. 2004.

Phase Noise Due to White Noise



- Assuming that the source $i(t)$ is a white noise of PSD S_i, the phase noise is given by

$$\mathcal{L}(\Delta\omega) = \frac{\Gamma_{rms}^2}{q_{\max}^2 \cdot 2\Delta\omega^2} \cdot S_i$$

- Where Γ_{rms}^2 is the rms value of the ISF Γ

$$\Gamma_{rms}^2 = \frac{1}{2\pi} \cdot \int_0^{2\pi} |\Gamma(x)|^2 \cdot dx = \frac{1}{2} \cdot \sum_{n=0}^{+\infty} |c_n|^2$$

A. Hajimiri and T. Lee, JSSC, Feb. 1998; T. Lee and A. Hajimiri, JSSC, March 2000; T. Lee, Cambridge, 2nd-ed. 2004.

Nonlinear Expression under Linear Operation

- We can check that for linear operation we get back to the earlier expressions derived above

$$\mathcal{L}(\Delta\omega) = \frac{\Gamma_{rms}^2}{q_{\max}^2 \cdot 2\Delta\omega^2} \cdot S_i$$

- Replacing $S_i = 4kT(1+\gamma)G$ and $q_{\max} = C \cdot A$
- In case of linear operation Γ_{rms}^2 is simply ½, resulting in

$$\mathcal{L}(\Delta\omega) = \frac{kT(1+\gamma)G}{C^2 \cdot A^2 \cdot \Delta\omega^2}$$

- Remembering that $Q = \frac{\omega_0 C}{G} \rightarrow C = \frac{Q \cdot G}{\omega_0}$

$$\mathcal{L}(\Delta\omega) = \frac{kT(1+\gamma)\omega_0^2}{A^2 \cdot Q^2 \cdot G \cdot \Delta\omega^2}$$

which corresponds to one of the expression obtained from the linear analysis (slide 8)

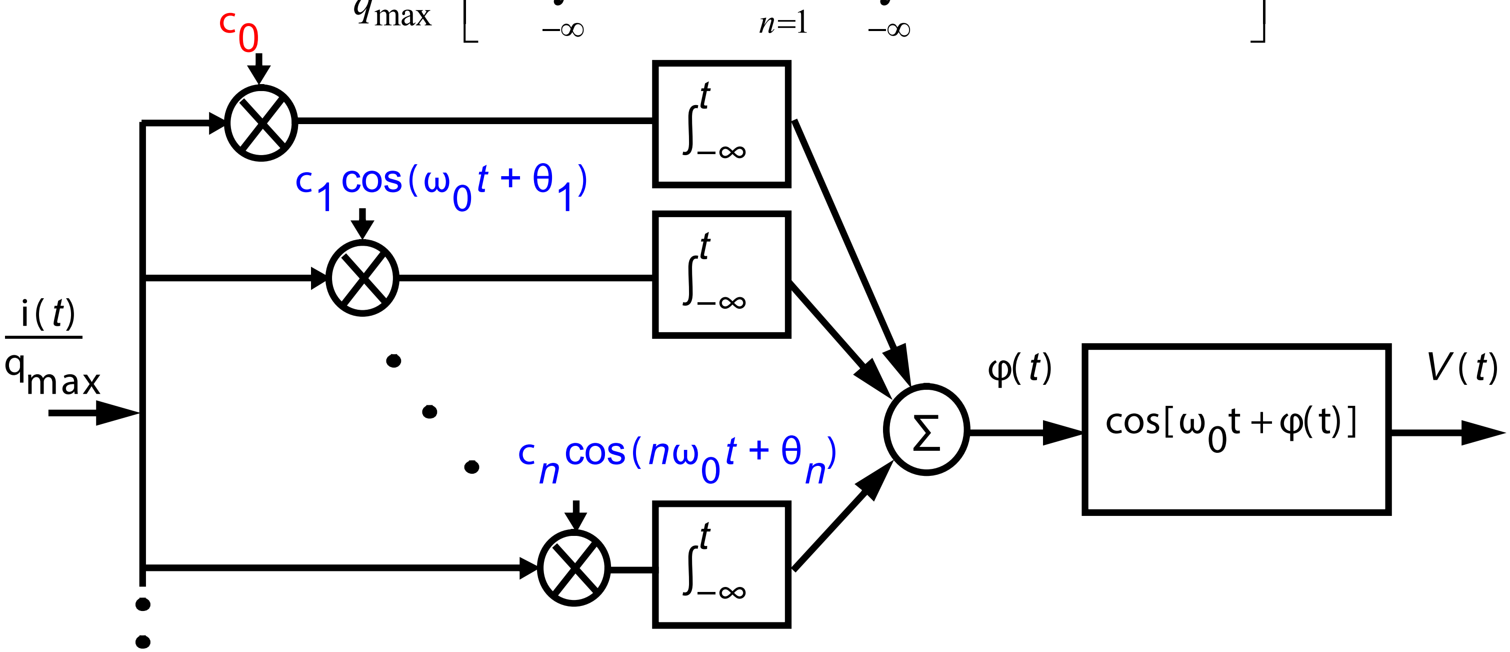
ISF Fourier Series Decomposition

- Since the ISF is periodic, it can be expanded into a Fourier series

$$\Gamma(\omega_0 t) = c_0 + \sum_{n=1}^{+\infty} c_n \cdot \cos\left(n\,\omega_0 t + \theta_n\right)$$

- The phase response can then be written as

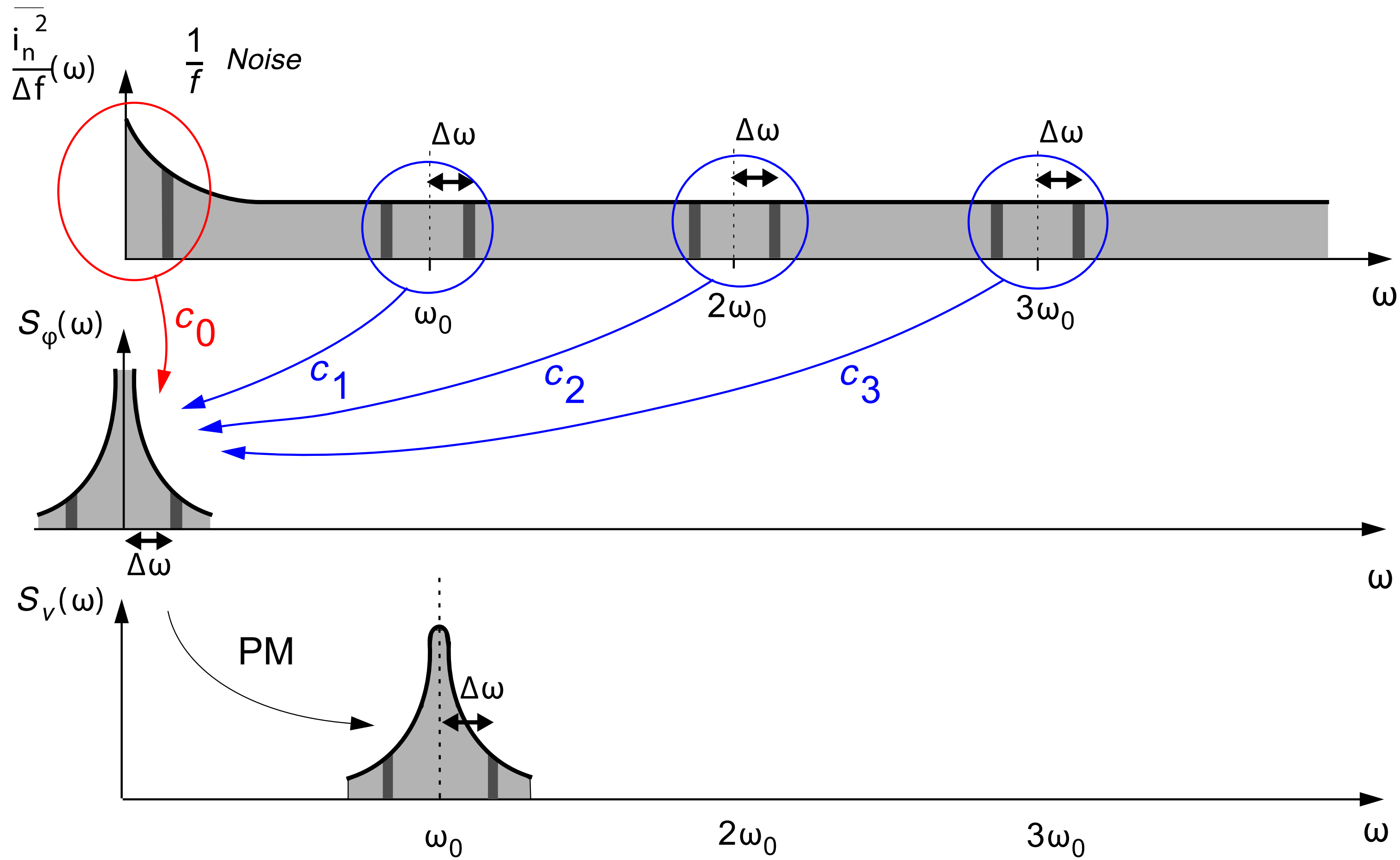
$$\phi(t) = \frac{1}{q_{max}} \cdot \left[c_0 \cdot \int_{-\infty}^{t} i(\tau) \cdot d\tau + \sum_{n=1}^{+\infty} c_n \cdot \int_{-\infty}^{t} i(\tau) \cdot \cos\left(n\,\omega_0 t\right) \cdot d\tau \right]$$



A. Hajimiri and T. Lee, JSSC, Feb. 1998; T. Lee and A. Hajimiri, JSSC, March 2000; T. Lee, Cambridge, 2nd-ed. 2004.

Noise Folding

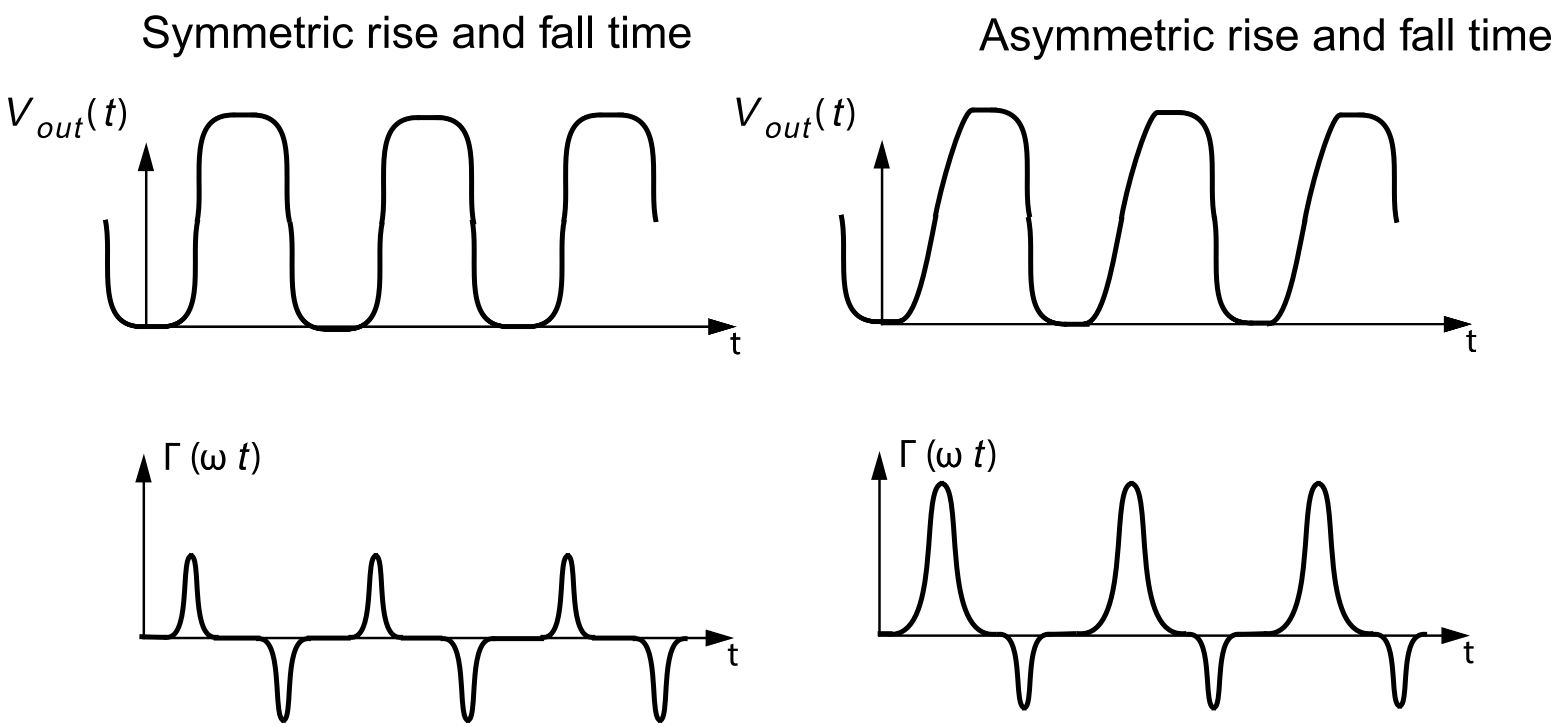
$$\phi(t) = \frac{1}{q_{max}} \cdot \left[c_0 \cdot \int_{-\infty}^{t} i(\tau) \cdot d\tau + \sum_{n=1}^{+\infty} c_n \cdot \int_{-\infty}^{t} i(\tau) \cdot \cos(n\omega_0 t) \cdot d\tau \right]$$



A. Hajimiri and T. Lee, JSSC, Feb. 1998; T. Lee and A. Hajimiri, JSSC, March 2000; T. Lee, Cambridge, 2nd-ed. 2004.

Effect of Symmetry

$$c_0 = \frac{1}{2\pi} \cdot \int_0^{2\pi} \Gamma(x) \cdot dx$$

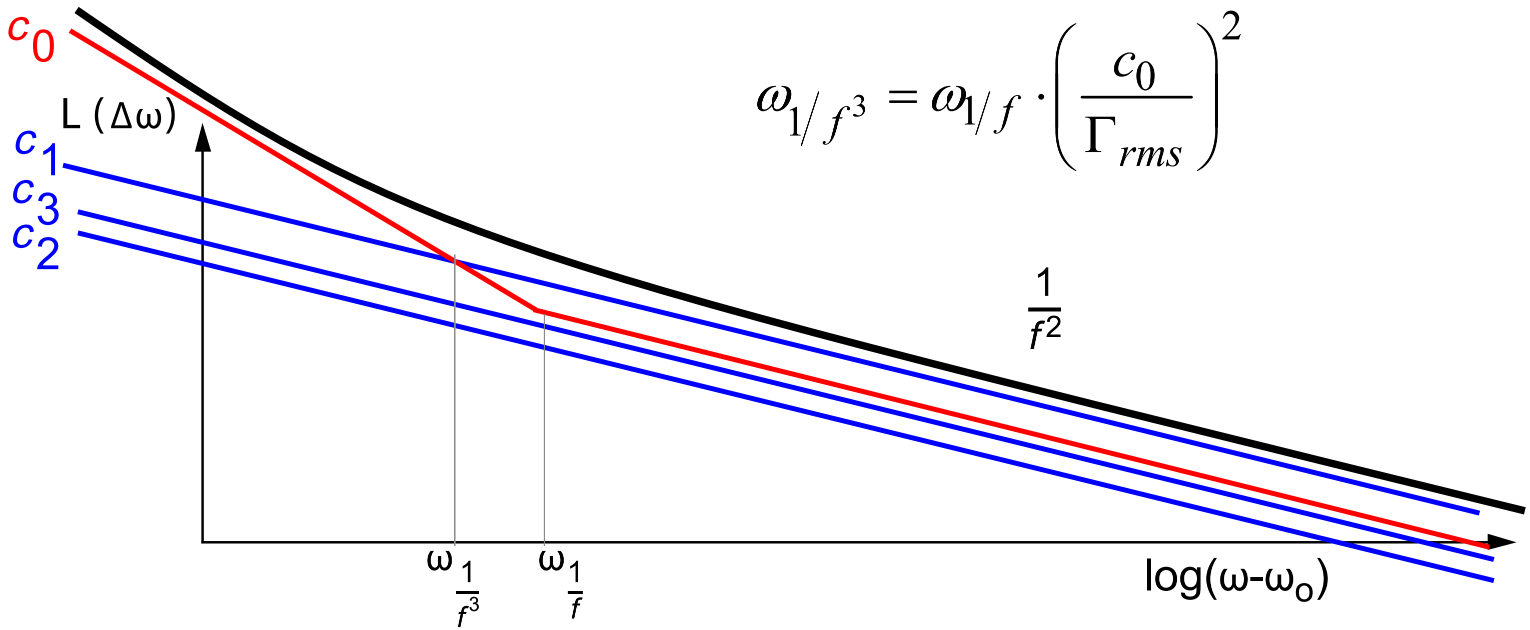


The dc value of the ISF is governed by rise and fall time symmetry, and controls the contribution of low frequency noise to the phase noise.

A. Hajimiri and T. Lee, JSSC, Feb. 1998; T. Lee and A. Hajimiri, JSSC, March 2000; T. Lee, Cambridge, 2nd-ed. 2004.

$1/f^3$ Corner of Phase Noise Spectrum

- Due to noise folding, the $1/f^3$ noise corner of the phase noise is not the same as the 1/f noise of the device noise source (it is usually smaller)

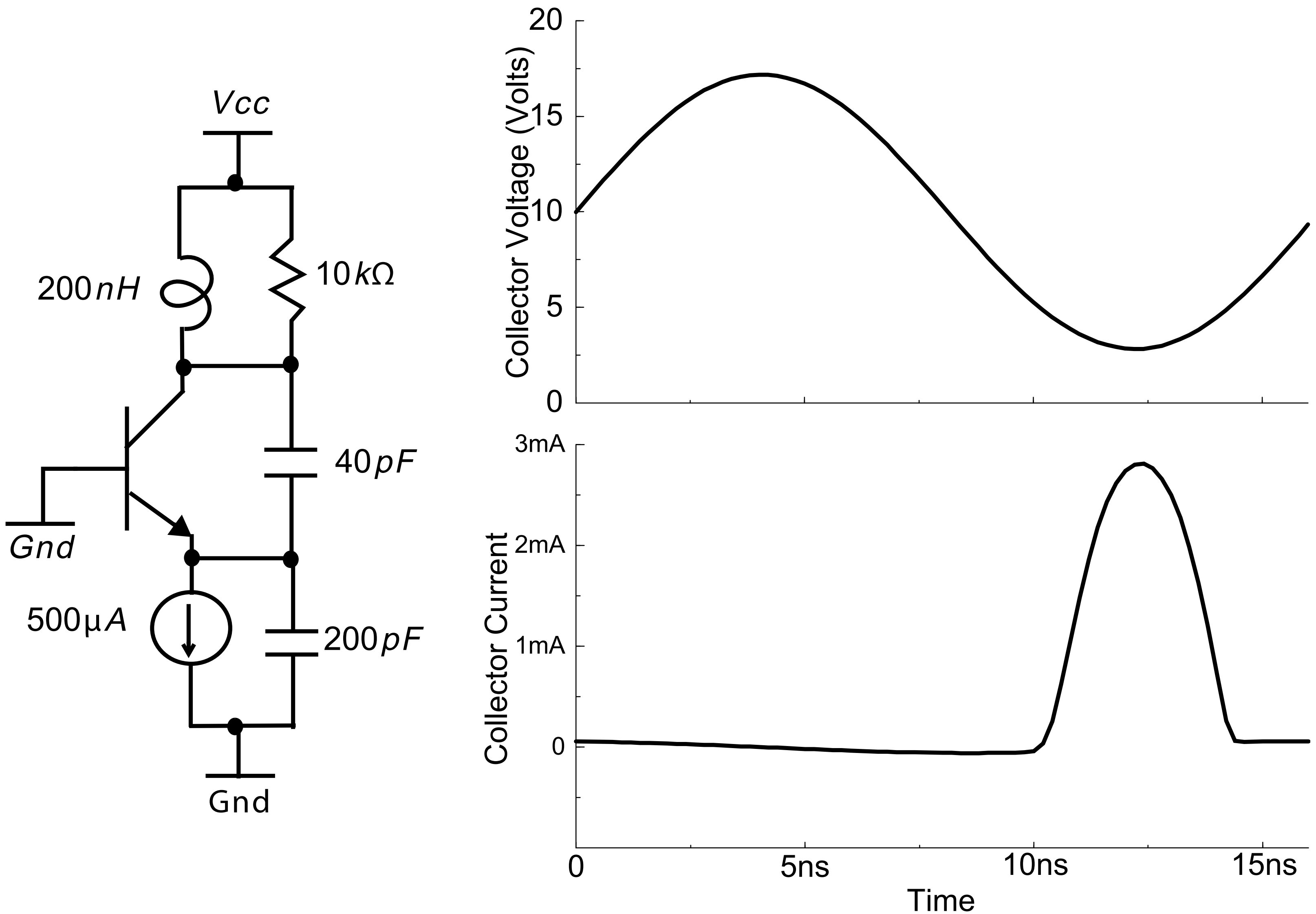


$$\omega_{1/f^3} = \omega_{1/f} \cdot \left(\frac{c_0}{\Gamma_{rms}}\right)^2$$

- By designing for a symmetric waveform, the performance degradation due to low frequency noise can be minimized (by minimizing coefficient c_0)

A. Hajimiri and T. Lee, JSSC, Feb. 1998; T. Lee and A. Hajimiri, JSSC, March 2000; T. Lee, Cambridge, 2nd-ed. 2004.

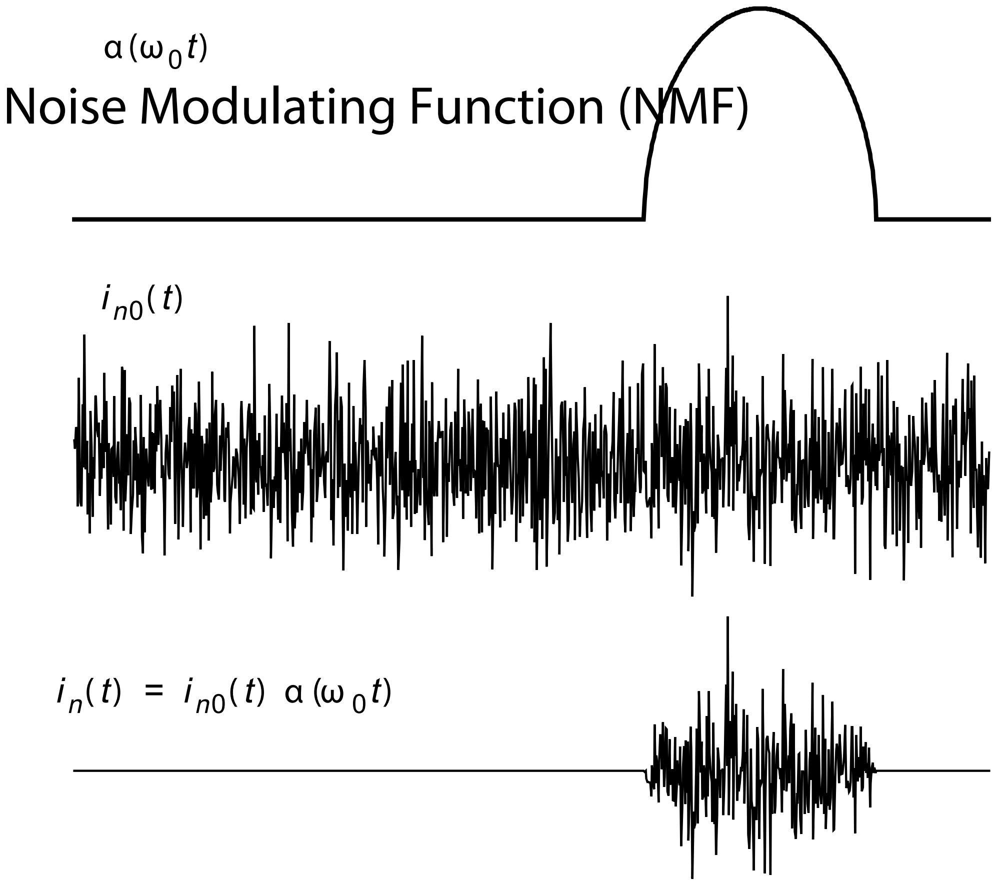
Time Varying Current in Colpitts Oscillator



A. Hajimiri and T. Lee, JSSC, Feb. 1998; T. Lee and A. Hajimiri, JSSC, March 2000; T. Lee, Cambridge, 2nd-ed. 2004.

Cyclostationary Properties, Time Domain

- Noise sources are not stationnary but cyclo-stationnary
- This can be modeled by a noise modulating function defining a new effective ISF



$$\phi(t) = \int_{-\infty}^{t} i_n(\tau) \cdot \frac{\Gamma(\omega_0 \tau)}{q_{\max}} \cdot d\tau =$$

$$= \int_{-\infty}^{t} i_{n0}(\tau) \cdot \frac{\alpha(\omega_0 \tau) \cdot \Gamma(\omega_0 \tau)}{q_{\max}} \cdot d\tau =$$

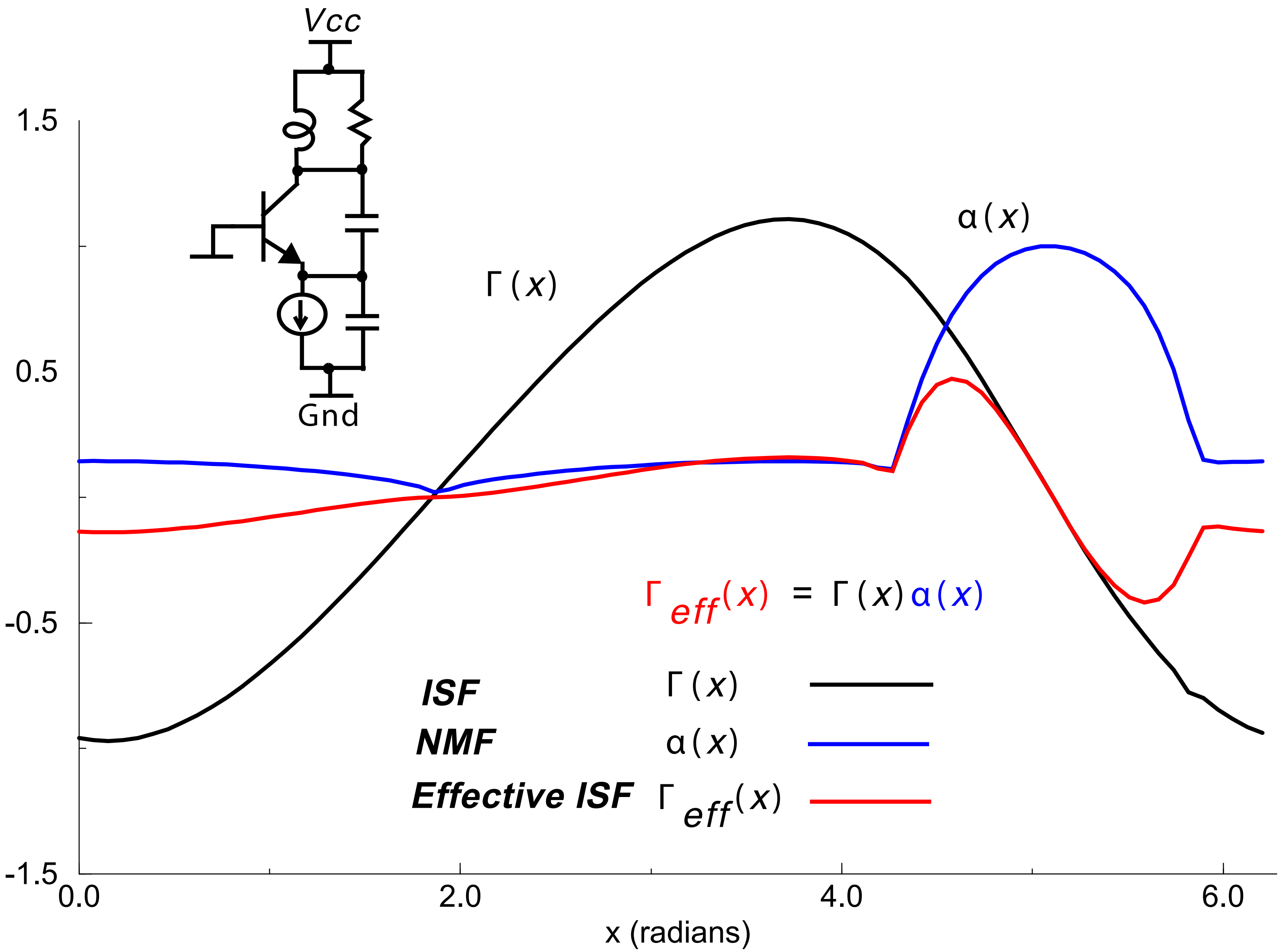
$$= \int_{-\infty}^{t} i_{n0}(\tau) \cdot \frac{\Gamma_{eff}(\omega_0 \tau)}{q_{\max}} \cdot d\tau$$

where i_{n0} is a stationary noise source and the effective ISF is defined as

$$\Gamma_{eff}(x) \triangleq \alpha(x) \cdot \Gamma(x)$$

A. Hajimiri and T. Lee, JSSC, Feb. 1998; T. Lee and A. Hajimiri, JSSC, March 2000; T. Lee, Cambridge, 2nd-ed. 2004.

Effective ISF of the Colpitts Oscillator



A. Hajimiri and T. Lee, JSSC, Feb. 1998; T. Lee and A. Hajimiri, JSSC, March 2000; T. Lee, Cambridge, 2nd-ed. 2004.