

MCAA lecture 1: quiz

A) Let $(X_n, n \in \mathbb{N})$ be a sequence of independent & identically distributed (i.i.d.) random variables such that $\mathbb{P}(X_n = +1) = \mathbb{P}(X_n = -1) = \frac{1}{2} \quad \forall n \in \mathbb{N}$.

Which of the following processes are Markov chains?

1. The process $(X_n, n \in \mathbb{N})$ itself.

2. The process $(Y_n, n \in \mathbb{N})$ defined as:

$$Y_0 = X_0, \quad Y_{n+1} = aY_n + X_{n+1} \quad n \in \mathbb{N}, \quad a \in \mathbb{R}$$

3. The process $(Z_n, n \in \mathbb{N})$ defined as:

$$Z_0 = 0, Z_{n+1} = X_n + X_{n+1}, n \in \mathbb{N}$$

4. The process $(U_n, n \in \mathbb{N})$ defined as:

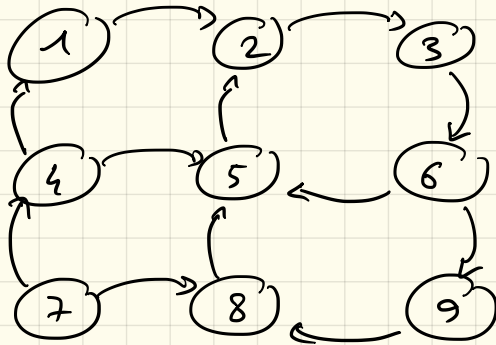
$$U_0 = X_0, U_1 = U_0 + X_1, U_{n+1} = U_n + U_{n-1} + X_{n+1}, n \geq 1$$

5. The process $(V_n, n \in \mathbb{N})$ defined as:

$$V_0 = X_0, V_{n+1} = f(V_n, X_{n+1}) \quad n \in \mathbb{N}$$

(where $f: \mathbb{Z}^2 \rightarrow \mathbb{Z}$ is some function)

B) Consider the following Markov chain:



[arrow from i to j
 \Leftrightarrow probability $p_{ij} > 0$]

1) What are the equivalence classes?

2) What is the periodicity of each class?