MCAA lecture 2: quiz

1) Let $(X_n, n \geq 0)$ be a Markov chain with state space S.
Which of the following statement(s) is correct?

a) $i \in S$ is transient iff $\mathbb{P}(X_n \neq i \ \forall n \geq 1 \mid X_0 = i) = 1$

b) $i \in S$ is transient iff $\mathbb{P}(\exists n \geq 1 \text{ s.t. } X_n = i \mid X_0 = i) = 0$

c) $i \in S$ is recurrent iff $\exists n \geq 1$ s.t. $\mathbb{P}(X_n = i \mid X_0 = i) = 1$

d) $i \in S$ is recurrent iff $\mathbb{P}(X_n \neq i \ \forall n \geq 1 \mid X_0 = i) = 0$

e) $i \in S$ is recurrent iff $\mathbb{P}(T_i = +\infty \mid X_0 = i) < 1$

(where $T_i = \inf\{n \geq 1 : X_n = i\}$)

2) Let X be a Markov chain with <u>finite</u> state space S. Which of the following changes can impact the recurrence/transience of some states?

a) Changing the weights of some arrows in the transition graph (while keeping them all strictly positive)

b) Changing the directions of some arrows in the transition graph

c) Adding self-loops in the transition graph

d) Removing some arrows in the transition graph

3) Let X be a randam variable with values in $\mathbb{N}^* = \{1, 2, 3, \dots\}$

We have seen that it is possible that $\mathbb{P}(X < +\infty) = 1$ and $\mathbb{E}(X) = +\infty$ simultaneausly. Some examples:

a) If $\mathbb{P}(X=n) = 2^{-n}$, then $\mathbb{P}(X \geq n) \simeq ?$

& $\mathbb{E}(X) < +\infty$ or $\mathbb{E}(X) = +\infty$?

b) If $\mathbb{P}(X=n) = \frac{c}{n^2}$, then $\mathbb{P}(X \geq n) \simeq ?$

& $\mathbb{E}(X) < +\infty$ or $\mathbb{E}(X) = +\infty$?

c) If $\mathbb{P}(X=n) = \frac{c}{n}$, then $\mathbb{P}(X \geq n) \simeq ?$

& $\mathbb{E}(X) < +\infty$ or $\mathbb{E}(X) = +\infty$?