MCAA lecture 4: quiz

1) On $S=\{0,1,2\}$, which of the following distributions has the smallest/largest total variation distance from $\mu=(\frac{1}{2},0,\frac{1}{2})$?

a) $\nu=(1,0,0)$

b) $\nu=(\frac{1}{3},\frac{1}{3},\frac{1}{3})$

c) $\nu=(\frac{1}{2},\frac{1}{2},0)$

d) $\nu=(0,1,0)$

2) On $S=\{0,1,2\}$, let $\mu=(\frac{1}{2},0,\frac{1}{2})$ and $\nu=(1,0,0)$.

For which $A\subset S$ does it hold that $\|\mu-\nu\|_{TV}=\mu(A)-\nu(A)$?

a) $A=\{0,1\}$

b) $A=\{0,2\}$

c) $A=\{1,2\}$

d) $A=\{0,1,2\}$

3) On $S=\{0,1\}$, let $\mu=\nu=\left(\frac{1}{4},\frac{3}{4}\right)$. Let also (X,Y) be a couple of random variables with values in $S\times S$ such that $\mathbb{P}(X=Y=0)=p_{00}$, $\mathbb{P}(X=0,Y=1)=p_{01}$, $\mathbb{P}(X=1,Y=0)=p_{10}$ and $\mathbb{P}(X=Y=1)=p_{11}$. In which case(s) is (X,Y) a coupling of μ & ν?

a) $p_{00}=\frac{1}{4}$, $p_{11}=\frac{3}{4}$, $p_{01}=p_{10}=0$

b) $p_{00}=\frac{1}{8}$, $p_{11}=\frac{3}{8}$, $p_{01}=p_{10}=\frac{1}{4}$

c) $p_{00}=\frac{1}{8}$, $p_{11}=\frac{3}{8}$, $p_{01}=\frac{1}{8}$, $p_{10}=\frac{3}{8}$

d) $p_{00}=\frac{1}{8}$, $p_{11}=\frac{5}{8}$, $p_{01}=p_{10}=\frac{1}{8}$

e) $p_{00}=\frac{1}{16}$, $p_{11}=\frac{9}{16}$, $p_{01}=p_{10}=\frac{3}{16}$

f) $p_{00}=0$, $p_{11}=\frac{1}{2}$, $p_{01}=p_{10}=\frac{1}{4}$

<u>Subsidiary question:</u>

For which coupling(s) is it the case that $\|\mu-\nu\|_{TV}=\mathbb{P}(X\neq Y)$?