Exercise 1  *The Young double slit experiment (1803)*

1) The scheme of the experiment is as follows:

If $D >> d$, we use the approximation

$$|\psi(\vec{r}_P)|^2 \approx \frac{A^2}{D^2} \left| e^{\frac{2\pi i}{\lambda} |\vec{r}_B - \vec{r}_P|} + e^{\frac{2\pi i}{\lambda} |\vec{r}_C - \vec{r}_P|} \right|^2.$$  

By factoring out the factor whose modulus is 1, we then have

$$|\psi(\vec{r}_P)|^2 \approx \frac{A^2}{D^2} \left| 1 + e^{\frac{2\pi i}{\lambda} (|\vec{r}_C - \vec{r}_P| - |\vec{r}_B - \vec{r}_P|)} \right|^2.$$  

As shown in the above figure, the difference of lengths between the two beams $|\vec{r}_C - \vec{r}_P| - |\vec{r}_B - \vec{r}_P|$ is $d \sin \theta$. Therefore, we have

$$|\psi(\vec{r}_P)|^2 \approx \frac{A^2}{D^2} \left| 1 + e^{\frac{2\pi i d \sin \theta}{\lambda}} \right|^2 = A^2 \left[ \left( 1 + \cos \left( \frac{2\pi d \sin \theta}{\lambda} \right) \right)^2 + \sin^2 \left( \frac{2\pi d \sin \theta}{\lambda} \right) \right]$$

$$= A^2 \left[ 2 + 2 \cos \left( \frac{2\pi d \sin \theta}{\lambda} \right) \right]$$

$$= \frac{4A^2}{D^2} \cos^2 \left( \frac{\pi d}{\lambda} \sin \theta \right),$$

where the last line uses $\cos 2\alpha = 2 \cos^2 \alpha - 1$.  

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Note: We used an approximation which was deduced with a geometric argument, but the same results can also be obtained algebraically. For instance, let \( M = \frac{1}{2}(B + C) \) be the middle of the segment \([BC]\) and \( \phi \) the direct angle between \( \vec{MO} \) and \( \vec{MP} \).

\[
|\vec{r}_P - \vec{r}_B| = \|\vec{B}P\| = \|BM + \vec{MP}\| \\
= \sqrt{BM^2 + MP^2 - 2\vec{MP} \cdot \vec{MB}} \\
= MP \sqrt{1 - 2\frac{\vec{MP} \cdot \vec{MB}}{MP^2} + \left(\frac{MB}{MP}\right)^2} \\
\simeq MP \left(1 - \frac{\vec{MP} \cdot \vec{MB}}{MP^2} + \frac{1}{2} \left(\frac{MB}{MP}\right)^2 + o\left(\frac{d^2}{D^2}\right)\right)
\]

where we used \( \frac{\vec{AP} \cdot \vec{AB}}{AP^2} \sim \frac{d\phi}{D^2} \) and \( \frac{\vec{AP} \cdot \vec{AB}}{AP^2} \sim \left(\frac{d}{D}\right)^2 \). Similarly:

\[
CP \simeq MP \left(1 - \frac{\vec{MP} \cdot \vec{MC}}{MP^2} + \frac{1}{2} \left(\frac{MC}{MP}\right)^2 + o\left(\frac{d^2}{D^2}\right)\right)
\]

Therefore, using \( MB = MC \) we have:

\[
CP - BP \simeq \frac{\vec{MP}}{MP} \cdot (\vec{MB} - \vec{MC}) = \frac{\vec{MP}}{MP} \cdot \vec{CB} = d \sin(\phi)
\]

and notice that for large \( D \) compared to \( d \) and \( \rho \), the geometric angle \( \theta \) can be assimilated with \( \phi \) with \( \phi \simeq \theta \).

2) The intensity attains its minima at 0 when \( \sin \theta = \left(m + \frac{1}{2}\right) \frac{\lambda}{d} \). The intensity attains its maxima when the cosine function equals \( \pm 1 \), whereby \( \sin \theta = m \frac{\lambda}{d} \) for some integer \( m \).

3) For \( D >> d \), we use the approximation \( \tan \theta \approx \theta \approx \sin \theta \) so that the intensity is given by

\[
|\psi(\vec{r}_P)|^2 \approx \frac{4A^2}{D^2} \cos^2 \left(\frac{\pi d\rho}{D\lambda}\right).
\]

As the location of maxima satisfies \( \frac{d\rho_m}{D\lambda} = m \in \mathbb{N} \), the distance between two successive minima is

\[
\rho_{m+1} - \rho_m = \lambda \frac{D}{d}.
\]

With \( d = 0.25\text{mm}, D = 10\text{m} \) and \( \lambda = 652\text{nm} \), the \( \rho_{m+1} - \rho_m \) is 26.1mm.
Exercise 2  Modern Young’s experiment

1) For a molecule of $C_{60}$, $p = mv$, where $m = \frac{M_{mol}}{N_A} \times 60$. The De Broglie wavelength is
\[ \lambda = \frac{h}{p} = \frac{hN_A}{60M_{mol}v}. \]
For an average velocity of 220m/s, the wavelength is $2.518 \times 10^{-12}$m.

2) Take the results known for waves. We should observe interference fringes with a distance
\[ \rho_{m+1} - \rho_m = \lambda \frac{d}{D} = 31.48 \mu m. \]

3) The wavelength is $5.30 \times 10^{-35}$m, which is not a measurable distance.

Exercise 3  Photoelectric effect

According to Einstein’s formula, the kinetic energy of the ejected electrons is
\[ \frac{1}{2}mv^2 = h\nu - W_0 \]
where $W_0 = h\nu_0 = \frac{hc}{\lambda_0}$ is the minimal energy for extraction. The equation can be rewritten as
\[ \frac{hc}{\lambda} = \frac{1}{2}mv^2 + \frac{hc}{\lambda_0}. \]
Therefore, the necessary wavelength is
\[ \lambda = \left( \frac{1}{2}mv^2 + \frac{hc}{\lambda_0} \right)^{-1} hc. \]

Numerics can be calculated using $\frac{1}{2}mv^2 = 1.5eV$ and one finds $\lambda = 180nm$. 