Astrophysics III, Dr. Yves Revaz EPFL

4th year physics Exercises week 4
12.10.2021 Autumn semester 2021

Astrophysics III: Stellar and galactic dynamics

Exercises

Problem 1:

Derive the density corresponding to the Plummer-Schuster potential:

$$\Phi(r) = -\frac{GM}{\sqrt{b^2 + r^2}}$$

Problem 2:

Derive analytically the circular velocity corresponding to the following potentials:

a) Point mass:

$$\Phi(r) = -\frac{GM}{r}$$

b) Homogeneous sphere of radius a:

$$\Phi(r) = \begin{cases} -\frac{GM}{a}\left(\frac{3}{2} - \frac{1}{2}\frac{r^2}{a^2}\right) & \text{if } r < a \\ -\frac{GM}{r} & \text{if } r \geq a \end{cases}$$

c) Plummer-Schuster potential:

$$\Phi(r) = -\frac{GM}{\sqrt{b^2 + r^2}}$$

d) Miyamoto-Nagai potential:

$$\Phi(R, z) = -\frac{GM}{\sqrt{R^2 + (a + \sqrt{b^2 + z^2})^2}}$$

Compute for $z = 0$.

Problem 3:

Using the scale length h_R and scale height h_z :

$$h_R = a + b$$
$$h_z = b$$

in the Miyamoto-Nagai potential, verify that the rotation curve is independent of the scale height h_z.

Problem 4:

Using a N-body model, calculate the circular velocity for the model in problem 1 and a realistic model of galaxy using the script vc_plummer.py and vc_galaxy.py. Modify the parameters at the beginning of the script to see their influence, then compare them to the theoretical results.

With vc_galaxy.py, you can compare the circular velocity of all the components together and their sum.

Problem 5:

Implement the other potentials in their respective files and reproduce the analysis done in Problem 4.

In the case of the Miyamoto potential, modify the number of particles and observe the numerical noise.