

# Astrophysics III: Stellar and galactic dynamics

## Exercises

**Problem 1:**

Derive the density and circular velocity corresponding to the NFW potential

$$\Phi(r) = v_s^2 \left[ 1 - \frac{\ln(1 + r/r_s)}{r/r_s} \right]$$

**Problem 2:**

Derive the potential of an infinite wire of constant linear density  $\lambda_0$ .

**Problem 3:**

The isochrone potential is given by

$$\Phi(r) = -\frac{GM}{b + \sqrt{b^2 + r^2}}$$

What is the density profile that gives this potential? What is the circular velocity?

**Problem 4:**

Using Gauss' theorem, derive the surface density for the Kuzmin disk potential at  $z=0$

$$\Phi_K(R, z) = -\frac{GM}{\sqrt{R^2 + (a + |z|)^2}}$$

**Problem 5:**

The surface density of a Mestel disk is defined as:

$$\Sigma(R) = \begin{cases} \frac{v_0^2}{2\pi GR} & (R < R_{\max}) \\ 0 & (R \geq R_{\max}) \end{cases} \quad (1)$$

Show that in the limit  $R_{\max} \rightarrow \infty$ , the circular velocity is the constant  $v_0$ .