

Astrophysics III: Stellar and galactic dynamics

Exercises

Problem 1:

For the following potentials, derive the analytic expression for the acceleration \vec{a} at an arbitrary point \vec{x} in Cartesian coordinates. Also, derive the orbital circular period for a radius r .

a) Point mass:

$$\Phi(r) = -\frac{GM}{r}$$

b) Plummer-Schuster:

$$\Phi(r) = -\frac{GM}{\sqrt{e^2 + r^2}}$$

c) Miyamoto-Nagai:

$$\Phi(R, z) = -\frac{GM}{\sqrt{R^2 + (a + \sqrt{b^2 + z^2})^2}}$$

d) Harmonic potential:

$$\Phi(x, y, z) = \frac{1}{2}\omega_x^2 x^2 + \frac{1}{2}\omega_y^2 y^2 + \frac{1}{2}\omega_z^2 z^2$$

Problem 2:

Insert the results from Problem 1 into an orbital integrator provided (in `pm.py`, `plummer.py`, `miyamoto.py` and `harmonique.py`), which uses a Runge-Kutta solver. You should start with the point mass (`pm.py`). (Specifically the forces, circular velocities and periods in the relevant python routines.)

You can verify your results by using the script `orbit.py`. The following command print some information about the script.

```
./orbit.py --help
```

Problem 3:

Using the integrator for the orbits, attempt to find the appropriate initial conditions for each of the potentials proposed in problem 1 in order to generate:

- circular orbits
- quasi-periodic orbits
- resonant orbits

For each case, examine the conserved integrals (energy, angular momentum, the projected angular momentum on z).