Astrophysics III, Dr. Yves Revaz | EPFL

4th year physics | Exercises week 8
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Astrophysics III : Stellar and galactic dynamics

Exercises

Problem 1 :

Derive the epicycle frequencies for the following potentials :

a) Point mass :

$$\Phi(r) = -\frac{GM}{r}$$

b) Plummer-Schuster :

$$\Phi(r) = -\frac{GM}{\sqrt{e^2 + r^2}}$$

c) Miyamoto-Nagai :

$$\Phi(R, z) = -\frac{GM}{\sqrt{R^2 + (a + \sqrt{b^2 + z^2})^2}}$$

Problem 2 :

Implement the previous computation in plummer.py, miyamoto.py and pm.py. Examine the precision of the predictions above by playing around with them on the numerical integration of the above potentials.

With orbit.py, you will obtain 3 figures :

— Figure 1 : the blue lines show the trajectories integrated over time in fixed coordinate planes. The green dashed line on the top-left panel shows the theoretical prediction.

— Figure 2 : the blue line shows the integrated trajectories versus time for the three cartesian coordinates in the rotating frame. The red dashed line shows these for a circular orbit.

— Figure 3 : the blue line shows the trajectories integrated over time in rotating coordinate planes. The red dashed line shows again these trajectories for a circular orbit.

These scripts also print in your terminal the ratio between the frequencies. You can compare these to the theoretical values, that you can find using the plotfreq.py script, i.e :

```
./plotfreq.py --potentialfile pm.py
```

Compare the theoretical predictions to the results of the integration from orbit.py.

Problem 3 :

Show that the following relations hold :

$$A(R) \equiv \frac{1}{2}\left(\frac{v_c}{R} - \frac{\mathrm{d}v_c}{\mathrm{d}R}\right) = -\frac{1}{2}R\frac{\mathrm{d}\Omega}{\mathrm{d}R}$$

$$B(R) \equiv -\frac{1}{2}\left(\frac{v_c}{R} + \frac{\mathrm{d}v_c}{\mathrm{d}R}\right) = -\left(\Omega + \frac{1}{2}R\frac{\mathrm{d}\Omega}{\mathrm{d}R}\right)$$

$$\Omega = A - B$$

$$\kappa^2 \equiv \left(R\frac{\mathrm{d}(\Omega^2)}{\mathrm{d}R} + 4\Omega^2\right) = -4B(A-B) = -4B\Omega$$

Problem 4 :

Derive the relation between R and $\dot{R}$ in the $z = 0$ plane (approximation of the third integral), if we assume that the total angular momentum is conserved in an axisymmetric potential.