Astrophysics III: Stellar and galactic dynamics

Exercises

Problem 1:

Derive the Jeans equations in spherical coordinates, starting from the collisionless Boltzmann equation. Assume a spherically symmetric system.

Hints:

- for the zeroth moment, you can express the zeroth moment of the Boltzmann equation

$$\frac{\partial \nu}{\partial t} + \sum_i \frac{\partial}{\partial x_i}(\nu \bar{v}_i) = 0, \quad \nu \equiv \int \mathrm{d}\mathbf{v}^3 f$$

  first in vectorial notation, then in spherical coordinates.

- for the first moment, use the collisionless Boltzmann equation in spherical coordinates, and compute the first moment by multiplying with the radial velocity v_r and integrate over all velocity space.

- Finally, find the solution assuming the system is stationary, isotropic, and express the equation using velocity dispersion σ.

- Here are a few properties to keep in mind :

  1) $f \to 0$ when $|v_i| \to \infty$ 2) $m \int f\, d^3\mathbf{v} = \rho$ 3) $m \int v_i\, f\, d^3\mathbf{v} = \rho \overline{v_i}$

  4) $m \int v_i\, v_j f\, d^3\mathbf{v} = \rho \overline{v_i v_j}$ 5) $\overline{v_i}\, \overline{v_j} + {\sigma_{ij}}^2 = \overline{v_i v_j}$

Problem 2:

Using the Jeans equations in spherical coordinates, determine the velocity dispersion analytically (assuming it is isotropic) for a Plummer sphere in equilibrium.

Problem 3:

Apply the results from the previous problems in the case of an N-body Plummer sphere. Verify the equilbrium nature of the system and integrate for several dynamical timescales to check.

First, create a snapshot using plummer.py and then use gtree to evolve your snapshot (use the help option).

Verify that your integral of motions are conserved with plotIntegrals.py.