Solutions to Homework 4
CS-526 Learning Theory

Exercise 3 of Chapter 6
We prove that VC-dimension of $\mathcal{H}_{n-parity}$ is n. First, observe that $|\mathcal{H}_{n-parity}| = 2^n$ and it follows that VCdim$(H_{n-parity}) \leq \log(|H_{n-parity}|) = n$. Also VCdim$(H_{n-parity}) \geq n$ since it shatters the standard basis $\{e_i\}_{i=1}^n$, where e_i is a length-n vector that has 1 at position i and 0 everywhere else. To see that, observe that $h_J(e_j) = 1$ iff $i \in J$ and hence for any vector of labels $(y_1, \ldots, y_n)$ taking $J = \{i | y_i = 1\}$ will suffice.

Exercise 3 of Chapter 7

- For $h \in \mathcal{H}_n$, take $w(h) = \frac{2^{-n}}{|\mathcal{H}_n|}$. We get

$$\sum_{h \in \mathcal{H}} w(h) = \sum_{n} \sum_{h \in \mathcal{H}_n} \frac{2^{-n}}{|\mathcal{H}_n|} = \sum_{n \in \mathbb{N}} 2^{-n} = 1$$

- The size of $\mathcal{H}_n$ is countable, so we can enumerate the hypotheses with natural numbers as $h_{n,1}, h_{n,2}, \ldots$ and set $w(h_{n,k}) = 2^{-n}2^{-k}$. We get

$$\sum_{h \in \mathcal{H}} w(h) = \sum_{n} \sum_{h_{n,k} \in \mathcal{H}_n} 2^{-n}2^{-k} \leq \sum_{n \in \mathbb{N}} \sum_{k \in \mathbb{N}} 2^{-n}2^{-k} = 1$$