Low-power radio design for the IoT
Exercise 3 (10.03.2022)

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Problem 1 Nonlinearity and Time Variance

A list of input-output equations for several systems are given below. The instantaneous input is $x(t)$ and the instantaneous output is $y(t)$.

a) $2y(t) = 5x(t) + 7$

b) $2y(t) = 5x^2(t) + x(t)$

c) $2t\,y(t) = 5x(t) + 7$

d) $\frac{dy(t)}{dt} + 2y(t) = 3x(t)$

e) $\frac{d^2y(t)}{dt^2} + 3y(t)\frac{dy(t)}{dt} = 5x(t)$

- Classify each system as time invariant or time variant, linear or nonlinear and with memory or memoryless.

Problem 2 Third-Order Input Intercept Point

Consider the scenario shown in Fig. 1, where $\omega_3 - \omega_2 = \omega_2 - \omega_1$ and the band-pass filter (BPF) provides an attenuation of 17 dB at ω_2 and 37 dB at ω_3.

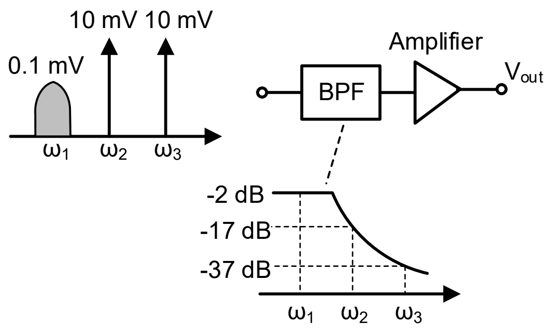

Figure 1: Cascade of BPF and amplifier.

- Compute the A_{IIP3} of the amplifier such that the intermodulation product falling at ω_1 is 20 dB below the desired signal.
- Suppose an amplifier with a voltage gain of 10 dB and $A_{IIP3} = 3.979$ dBm precedes the band-pass filter. Calculate the A_{IIP3} of the overall chain (neglect the second order nonlinearities and the nonlinearity of the BPF).

Problem 3 Cascade of Nonlinear Stages

The circuit in Fig. 2 is a cascade of two identical common-source stages loaded by a resistor, R_D.

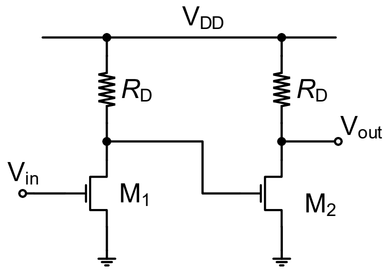

Figure 2: Cascade of common source stages.

- Assume that each transistor operates in saturation and SI and determine the A_{IIP3} of the cascade.

Solutions to Exercise 3 (10.03.2022)

Problem 1 Nonlinearity and Time Variance

- Classify each system as time invariant or time variant, linear or nonlinear and memoryless.

a) $2y(t) = 5x(t) + 7$: time invariant, nonlinear and memoryless

b) $2y(t) = 5x^2(t) + x(t)$: time invariant, nonlinear and memoryless

c) $2t\,y(t) = 5x(t) + 7$: time variant, nonlinear and memoryless

d) $\frac{dy(t)}{dt} + 2y(t) = 3x(t)$:time invariant, linear and with memory

e) $\frac{d^2y(t)}{dt^2} + 3y(t)\frac{dy(t)}{dt} = 5x(t)$: time invariant, non linear and with memory

Problem 2 Third-Order Input Intercept Point

- Compute the A_{IIP3} of the amplifier such that the intermodulation product falling at ω_1 is 20 dB below the desired signal.

At the output of the BPF we have

$$
\begin{aligned}
-2\,\text{dB} &= 10^{-0.1} = 0.794 \rightarrow A_{sig} = 0.1\,\text{mV} \cdot 0.7943 = 79.43\,\mu\text{V} \\
-17\,\text{dB} &= 10^{-0.85} = 0.1413 \rightarrow A_2 = 10\,\text{mV} \cdot 0.1413 = 1.413\,\text{mV} \\
-37\,\text{dB} &= 10^{-1.85} = 0.0141 \rightarrow A_3 = 10\,\text{mV} \cdot 0.0141 = 0.141\,\text{mV}
\end{aligned} \tag{1}
$$

At the output of the amplifier, the problem imposes to have the following condition

$$
\begin{aligned}
20\log|\alpha_1 \cdot A_{sig}| - 20\,\text{dB} &= 20\log\left|\frac{3}{4}\alpha_3 \cdot A_2^2 A_3\right| \\
\rightarrow |\alpha_1 \cdot 79.43\,\mu\text{V}| &= |\frac{30}{4}\alpha_3 \cdot (1.413\,\text{mV})^2 \cdot 0.141\,\text{mV}|
\end{aligned} \tag{2}
$$

From the latter, we can compute the A_{IIP3} as

$$
A_{IIP3} = \sqrt{\frac{4}{3}\frac{|\alpha_1|}{|\alpha_3|}} = \sqrt{\frac{4}{3} \cdot \frac{30}{4} \cdot \frac{(1.413\,\text{mV})^2 \cdot 0.141\,\text{mV}}{79.43\,\mu\text{V}}} = 5.95\,\text{mV} = 10\log\left(\frac{\frac{(5.95\,\text{mV})^2}{2 \cdot 50\,\Omega}}{1\,\text{mV}}\right) = -34.5\,\text{dBm}, \tag{3}
$$

- Suppose an amplifier with a voltage gain of 10 dB and $A_{IIP3} = 500\,\text{mV}$ precedes the band-pass filter. Calculate the A_{IIP3} of the overall chain (neglect the second order nonlinearities and the nonlinearity of the BPF).

Only considering the first and third order terms, the output of the first amplifier, $y_1(t)$, can be written as

$$
y_1(t) = \alpha_1\,x(t) + \alpha_3\,x^3(t). \tag{4}
$$

In the same way, the output of the second amplifier, $y_2(t)$, can be written as

$$
y_2(t) = \beta_1\,y_1(t) + \beta_3\,y_1^3(t). \tag{5}
$$

If we replace Equation (4) into Equation (5), we then obtain

$$
y_2(t) = \beta_1\,(\alpha_1\,x(t) + \alpha_3\,x^3(t)) + \beta_3\,(\alpha_1\,x(t) + \alpha_3\,x^3(t))^3(t). \tag{6}
$$

If we limit the expression for $y_2(t)$ to the first and third order terms we have

$$
y_2(t) = \alpha_1\beta_1\,x(t) + (\alpha_3\beta_1 + \alpha_1^3\beta_3)\,x^3(t). \tag{7}
$$

As seen in the slides, this leads to the following formula for the cascaded stages

$$\frac{1}{IIP3_{tot}^2} \approx \frac{1}{IIP3_1^2} + \frac{\alpha_1^2}{IIP3_2^2} \tag{8}$$

Once evaluated the latter results into

$$\frac{1}{IIP3_{tot}^2} \approx \frac{1}{(500\,\text{mV})^2} + \frac{10}{(5.95\,\text{mV})^2} \rightarrow IIP3_{tot}^2 = 1.881\,\text{mV} \tag{9}$$

Problem 3 Cascade of Nonlinear Stages

- Assume that each transistor operates in saturation and SI and determine the A_{IIP3} of the cascade.

For the MOS transistor biased in saturation and SI we have

$$I_D = \frac{\beta}{2n} \cdot (V_G - V_{T0} - n \cdot V_S)^2, \tag{10}$$

where V_S is equal to zero, since the source of both transistors is connected to the bulk and to the ground. Then, by analyzing the circuit we obtain

$$V_{out} = V_{DD} - R_D \cdot \frac{\beta}{2n} \cdot (V_x - V_{T0})^2 \tag{11}$$

and

$$V_x = V_{DD} - R_D \cdot \frac{\beta}{2n} \cdot (V_{in} - V_{T0})^2 \tag{12}$$

where V_x is the voltage at the gate of M_2 and at the drain of M_1. By replacing Equation (12) into Equation (11), we obtain

$$V_{out} = V_{DD} - R_D \cdot \frac{\beta}{2n} \cdot \left[V_{DD} - R_D \cdot \frac{\beta}{2n} \cdot (V_{in} - V_{T0})^2 - V_{T0}\right]^2. \tag{13}$$

After some manipulations, this expression results into

$$V_{out} = V_{DD} - R_D \cdot \frac{\beta}{2n} \cdot \left[(V_{DD} - V_{T0})^2 + R_D^2 \cdot \left(\frac{\beta}{2n}\right)^2 \cdot (V_{in} - V_{T0})^4 - 2R_D \cdot \left(\frac{\beta}{2n}\right) \cdot (V_{DD} - V_{T0}) \cdot (V_{in} - V_{T0})^2\right]. \tag{14}$$

From Equation (14) is possible to extract the first order terms of Vin, leading to

$$\left[4(V_{DD} - V_{T0}) \cdot \left(\frac{\beta}{2n}\right) R_D V_{T0} - 4\left(\frac{\beta}{2n}\right)^2 R_D^2 V_{T0}^3\right] \cdot V_{in}. \tag{15}$$

On the other hand, the third order term of V_{in} is equal to

$$\left[-4\left(\frac{\beta}{2n}\right)^2 R_D^2 V_{T0}\right] \cdot V_{in}^3. \tag{16}$$

Finally, the A_{IIP3} can be expressed as

$$A_{IIP3} = \sqrt{\frac{4}{3}\frac{|\alpha_1|}{|\alpha_3|}} = \sqrt{\frac{4}{3} \cdot \frac{4(V_{DD} - V_{T0}) \cdot \left(\frac{\beta}{2n}\right) R_D V_{T0} - 4\left(\frac{\beta}{2n}\right)^2 R_D^2 V_{T0}^3}{4\left(\frac{\beta}{2n}\right)^2 R_D^2 V_{T0}}}. \tag{17}$$