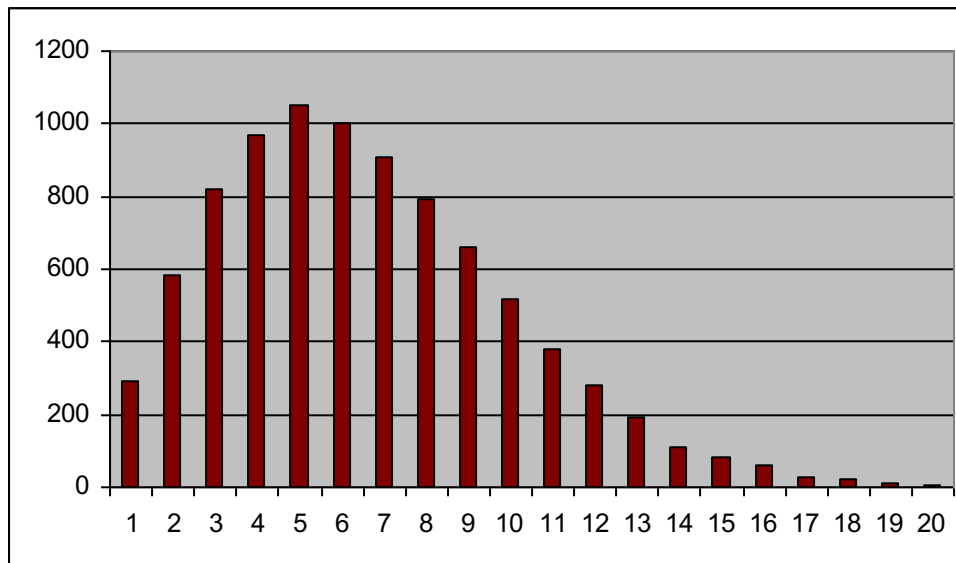


Electricity production and dimensioning of a wind turbine with given wind distribution at the site

- Wind turbine power at a wind of 10 m/s: 150 kW
(i.e. above $v_{nom} = 10$ m/s, the turbine power is constant at 150 kW)
- Wind velocities distribution [y hours at x m/s]:



x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
y	290	585	820	970	1050	1000	910	790	660	520	380	280	190	110	80	60	30	20	10	5

- Operating range of the wind turbine: $5 \text{ m/s} \leq v \leq 25 \text{ m/s}$
(i.e. below 5 m/s wind speed, the turbine does not operate)
- Mechanical efficiency of the wind turbine: 70 %
(or in other words, $C_p = 0.59 \cdot 0.7 = 0.41$)
- Air density: 1.22 kg/m^3

1) What is the diameter of the turbine?

Power (Betz formula) : $P_{wt} = (16/27) \cdot \epsilon_{wt} \cdot \rho_{air} \cdot (\pi \cdot D^2/4) \cdot v^3/2$

$$\text{Wind turbine diameter: } D_{wt} = \{P_{wt,n}/((16/27) \cdot \varepsilon_{wt} \cdot \rho_{air} \cdot (\pi/4) \cdot v_n^3/2)\}^{1/2}$$

$$= \{150 \text{ [kW]} \cdot 1000 \text{ [W/kW]} / (0.70 \cdot (16/27) \cdot 1.2 \text{ [kg/m}^3\text{]} \cdot (\pi/4) \cdot 10^3 \text{ [m}^3\text{/s}^3\text{]}/2)\}^{1/2} =$$

$$\mathbf{27.5 \text{ [m]}}$$

- 2) Evaluate the electricity produced during a year (cf. excel solution sheet)

Produced electricity (such as $5 \text{ [m/s]} \leq v_i \leq 20 \text{ [m/s]}$) :

$$\Sigma P_{wti} \cdot t_i = \mathbf{484'500 \text{ [kWh]}}$$

- 3) Evaluate the *mean* equivalent power of this turbine

Operating time of the wind turbine over one year:

$$t_{an} = \Sigma t_i \text{ (such as } 5 \text{ [m/s]} \leq v_i \leq 20 \text{ [m/s])}$$

$$= 6'095 \text{ [h]}$$

Produced annual electricity / actual operating hours ($5 \text{ [m/s]} \leq v_i \leq 20 \text{ [m/s]}$)

$$= 484'500 \text{ kWh} / 6095 \text{ h} = 79.5 \text{ kWh}$$

- 4) What is the equivalent annual load factor at nominal power?

Produced annual electricity / nominal power = $484500 \text{ kWh} / 150 \text{ kWh} = 3230 \text{ h} = 36.9\%$ of the year (8760 h).

This value is here rather high (normal is 20-25%), since it was assumed that the turbine operates at all times when the wind blows between 5 and 25 m/s. Also, the distribution disconsiders hours at $v=0 \text{ m/s}$ (periods of total absence of wind). These 2 factors overestimate the annual load factor in this case.

- 5) Calculate the wind mean velocity and mean cubic velocity (operating range). Compare the results.

Operating time of the wind turbine over one year: $t_{an} = 6'095 \text{ [h]}$

Mean velocity: $v_m = \{\Sigma v_i \cdot t_i \text{ (} 5 \text{ [m/s]} \leq v_i \leq 20 \text{ [m/s])}\} / t_{an} = \mathbf{8.20 \text{ [m/s]}}$

$$\rightarrow v_m^3 = 550 \text{ m}^3\text{/s}^3$$

Mean cubic velocity: $v_m^3 = \{\Sigma v_i^3 \cdot t_i \text{ (} 5 \text{ [m/s]} \leq v_i \leq 20 \text{ [m/s])}\} / t_{an} = \mathbf{765 \text{ [m}^3\text{/s}^3\text{]}}$

Conclusion: $v_m^3 \neq v_m^3$

- 6) Estimate the Weibull c- and k-parameters from the above wind velocities distribution

Weibull c-parameter estimate

$$c \approx \frac{2\bar{v}}{\sqrt{\pi}}$$

with \bar{v} the mean wind speed over the whole distribution (1-20 m/s):
 v_{avg} (1 – 20 m/s) = 6.6 m/s

$$c = 2 * 6.6 / \text{sqrt}(\text{Pi}) = 7.44$$

In a pure Weibull distribution the value of c indicates that 63% of wind speed values in the distribution should be below c . We can verify that the number of hours that the wind speed is below 7.44 m/s amounts to 5625 h (hours at 1 to 7 m/s in the distribution table). This corresponds to $5625 / 8760 = 64\%$, so this fits quite well.

Weibull k-parameter estimate

$$k \approx \frac{3}{2} \frac{c^3}{\langle v^3 \rangle} \sqrt{\pi}$$

For the mean cubic velocity, it has here again to be taken over the whole range 1-20 m/s, since this is the relevant range that covers the whole distribution and therefore also the shape (k = shape factor) of this distribution.

The mean cubic velocity over 1-20 m/s is calculated to $542 \text{ m}^3/\text{s}^3$, and thus

$$k = 1.5 * \text{sqrt}(\text{Pi}) * (7.44)^3 / 542 = 2.0$$