Low-power radio design for the IoT
Exercise 4 (24.03.2022)

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Problem 1 Low-power Common Source RF Design

The circuit in the Figure below is a single MOSFET Common Source (CS) Amplifier.

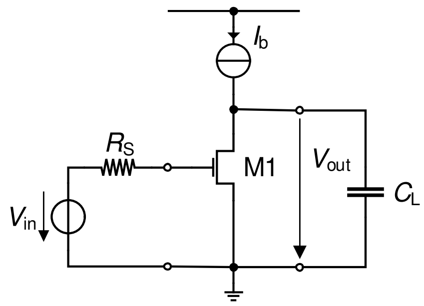


Figure 1: Common Source Amplifier.

- Find the optimum inversion coefficient of the common-source amplifier shown in **??** for maximizing the following figure-of-merit

$$FoM \triangleq \frac{\omega_u}{(F-1)\cdot I_b}. \tag{1}$$

- Calculate the bias current required to achieve a gain-bandwidth $f_u = 10\,GHz$ for a load capacitance $C_L = 10\,fF$ with $\lambda_c = 0.25$.
- Calculate the resulting noise factor F for a source impedance $R_S = 50\,\Omega$.
- How can you reduce this noise factor?

Problem 2 Common Source Design

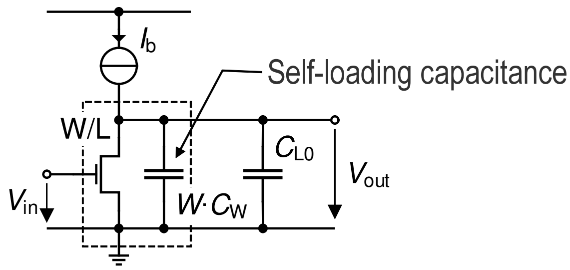


Figure 2: Common source amplifier including the self-loading capacitance.

The normalized bias current and aspect ratio for the CS amplifier

$$\begin{aligned} i_b &\triangleq \frac{I_D}{I_{spec\square}} \cdot \frac{1}{\Omega} = \frac{IC}{g_{ms} - \Theta}; \\ AR &\triangleq \frac{W}{L} \cdot \frac{1}{\Omega} = \frac{1}{g_{ms} - \Theta}, \end{aligned} \tag{2}$$

where $\Omega \triangleq \frac{\omega_u}{\omega_L}$ and Θ is equal to $\frac{C_w L}{C_{L0}} \cdot \frac{\omega_u}{\omega_L} = \frac{\omega_u}{\omega_{tspec}}$.

Design the CS amplifier, shown in Fig. 2, for the following specifications at room temperature:

$$f_u = 18\,\text{MHz} \quad C_{L0} = 60\,\text{fF} \quad V_{DD} = 1.8\,\text{V} \quad L = 40\,\text{nm} \quad C_w = 0.450\,\text{fF/nm}, \tag{3}$$

and by assuming the following values for the technology parameters

$$I_{spec\square} = 950\,\text{nA} \quad n = 1.5 \quad V_{T0} = 455\,\text{mV} \quad \lambda_c = 0.4875 \quad L_{sat} = 19.5\,\text{nm}. \tag{4}$$

- Find the IC_{opt}, the value of the inversion coefficient for which the bias current is minimum. Assume no velocity saturation.
- Based on the IC_{opt}, find the values of the bias current, I_q, and the transistor aspect ratio, W/L, to achieve the specified gain-bandwidth, ω_u.

Problem 1: Low-power common source RF design

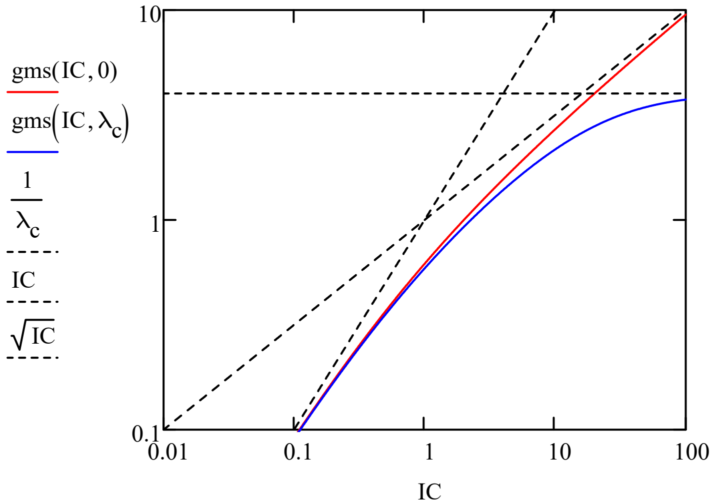
$$fF \equiv 10^{-15} \cdot F$$

$$K \equiv \frac{J}{1.38 \cdot 10^{-23}} \qquad k_B \equiv 1.38 \cdot 10^{-23} \cdot \frac{J}{K} \qquad q \equiv 1.6 \cdot 10^{-19} \cdot C \qquad T := 300 \cdot K$$

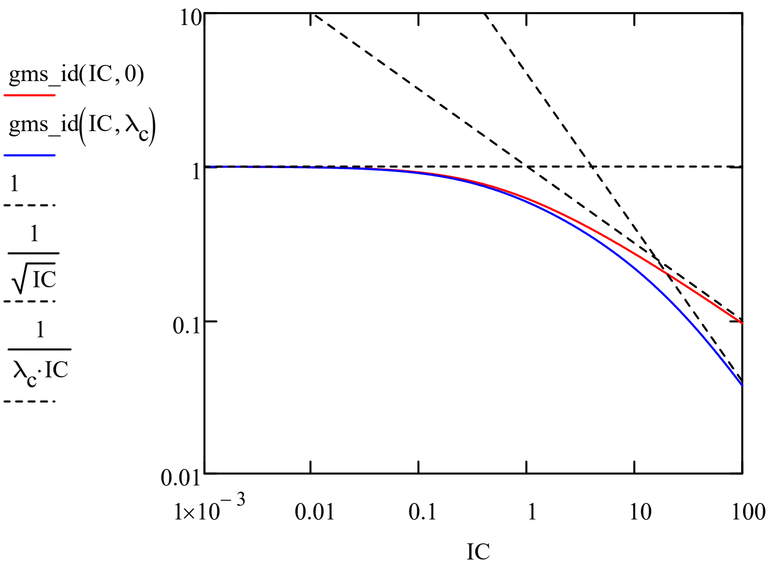
$$U_T := \frac{k_B \cdot T}{q} \qquad U_T = 25.875 \cdot mV$$

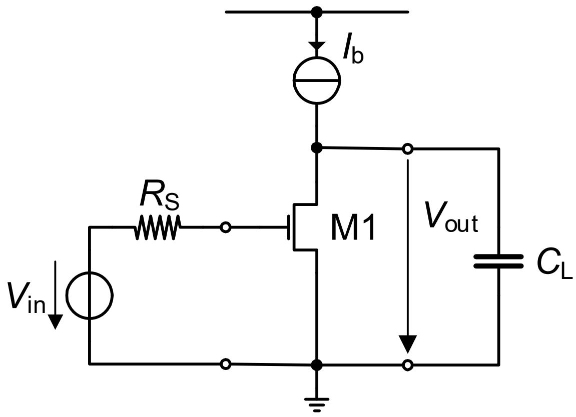
$$\lambda_c := 0.25$$

$$gms(IC, \lambda_c) := \frac{\sqrt{(\lambda_c \cdot IC + 1)^2 + 4 \cdot IC} - 1}{\lambda_c \cdot (\lambda_c \cdot IC + 1) + 2}$$



$$gms\_id(IC, \lambda_c) := \frac{gms(IC, \lambda_c)}{IC}$$





$$\mathrm{FoM} = \frac{\omega_u}{(F-1)\cdot I_b} = \frac{R_S}{\gamma_{nD}}\cdot\frac{G_m\cdot\omega_t}{I_b}$$

$n := 1.3$

$\lambda_c := 0.25$

$IC_{opt} := \dfrac{1}{\lambda_c^{\frac{4}{3}}}$ $\qquad IC_{opt} = 6.35$

$C_L := 20\cdot fF$

$f_u := 10\cdot GHz$ $\qquad \omega_u := 2\cdot\pi\cdot f_u$

$G_m := \omega_u\cdot C_L$ $\qquad G_m = 1.257\,\dfrac{mA}{V}$

$I_b := \dfrac{G_m\cdot n\cdot U_T}{gms\_id\left(IC_{opt},\lambda_c\right)}$ $\qquad I_b = 152.283\,\mu A$

$\gamma_{nD} := n\cdot\dfrac{2}{3}$

$R_S := 50\cdot\Omega$

$F := 1 + \dfrac{\gamma_{nD}}{G_m\cdot R_S}$ $\qquad F = 14.793$

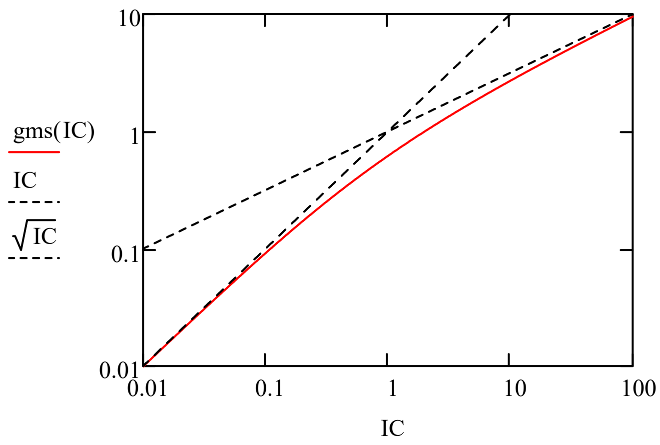
Problem 2: Common source design for given GBW

$fF \equiv 10^{-15} \cdot F$

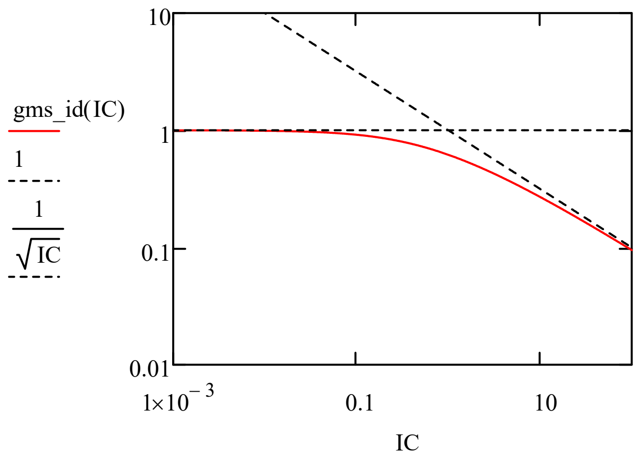
$K \equiv \dfrac{J}{1.38 \cdot 10^{-23}}$ $\quad k_B \equiv 1.38 \cdot 10^{-23} \cdot \dfrac{J}{K}$ $\quad q \equiv 1.6 \cdot 10^{-19} \cdot C$ $\quad T := 300 \cdot K$

$U_T := \dfrac{k_B \cdot T}{q}$ $\quad U_T = 25.875 \cdot mV$

$gms(IC) := \dfrac{\sqrt{4 \cdot IC + 1} - 1}{2}$



$gms\_id(IC) := \dfrac{gms(IC)}{IC}$



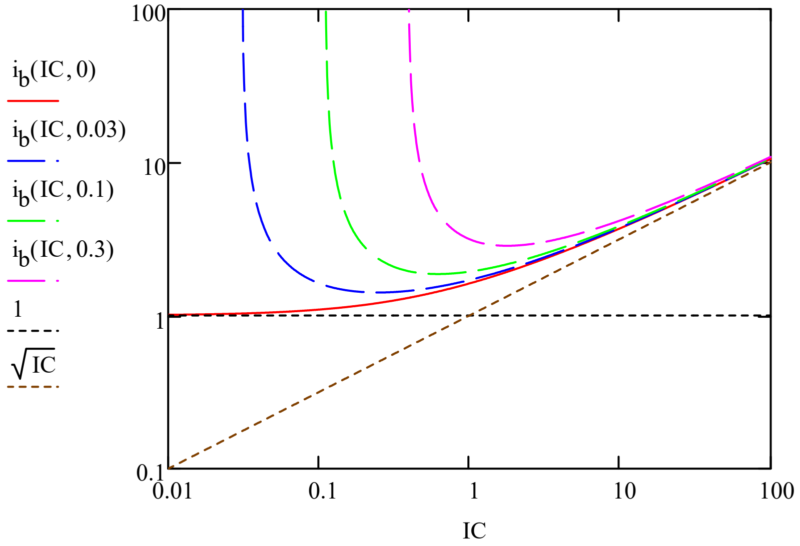
$i_b = \dfrac{I_b}{I_{specsq}} \cdot \dfrac{1}{\Omega}$ $\quad \Omega = \dfrac{\omega_u}{\omega_L}$ $\quad \omega_u = \dfrac{G_m}{C_L}$ $\quad \omega_L = \dfrac{I_{specsq}}{n \cdot U_T \cdot C_{L0}}$

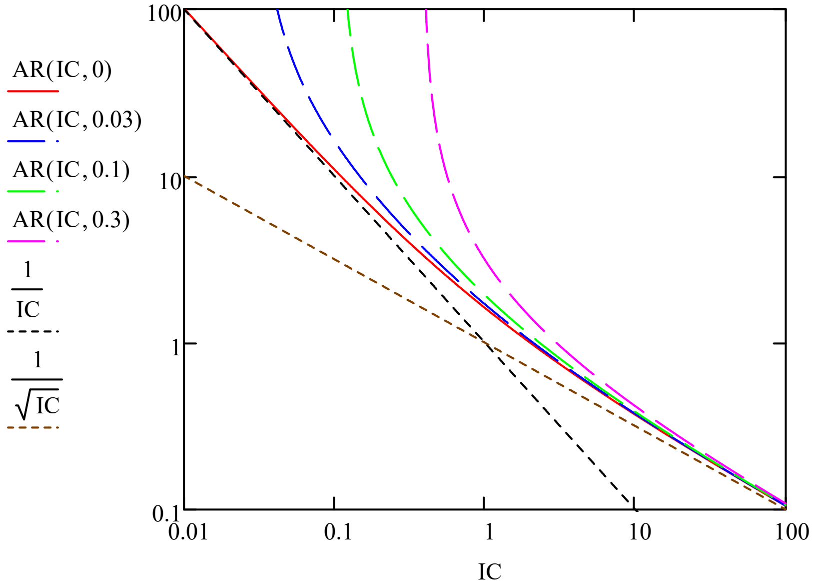
$\Theta = \dfrac{\omega_u}{\omega_w}$ $\quad \omega_w = \dfrac{I_{specsq}}{n \cdot U_T \cdot C_W \cdot L}$ $\quad AR = \dfrac{W}{L} \cdot \dfrac{1}{\Omega}$

$$i_b(IC,\Theta) := \frac{IC}{gms(IC) - \Theta}$$

$$i_b(1,0.1) = 1.93$$

$$AR(IC,\Theta) := \frac{1}{gms(IC) - \Theta}$$





$$IC_{opt}(\Theta) := 2\cdot\Theta\cdot(1+\Theta) + (1+2\cdot\Theta)\cdot\sqrt{\Theta\cdot(1+\Theta)}$$

$$IC_{opt}(\Theta) := \left[\sqrt{\Theta\cdot(1+\Theta)} + \Theta + \frac{1}{2}\right]^2 - \frac{1}{4}$$

$IC_{opt}(0.1) = 0.618$

$i_{bopt}(\Theta) := i_b(IC_{opt}(\Theta), \Theta)$

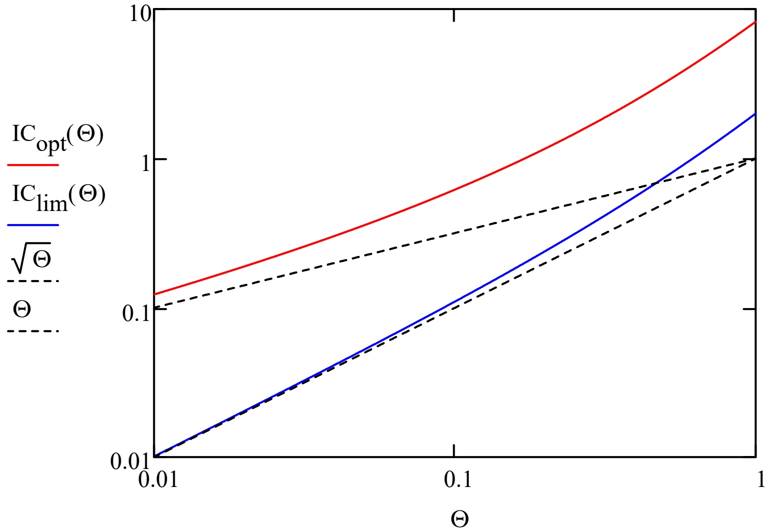
$i_{bopt}(\Theta) := 1 + 2\cdot\Theta + 2\cdot\sqrt{\Theta\cdot(1+\Theta)}$

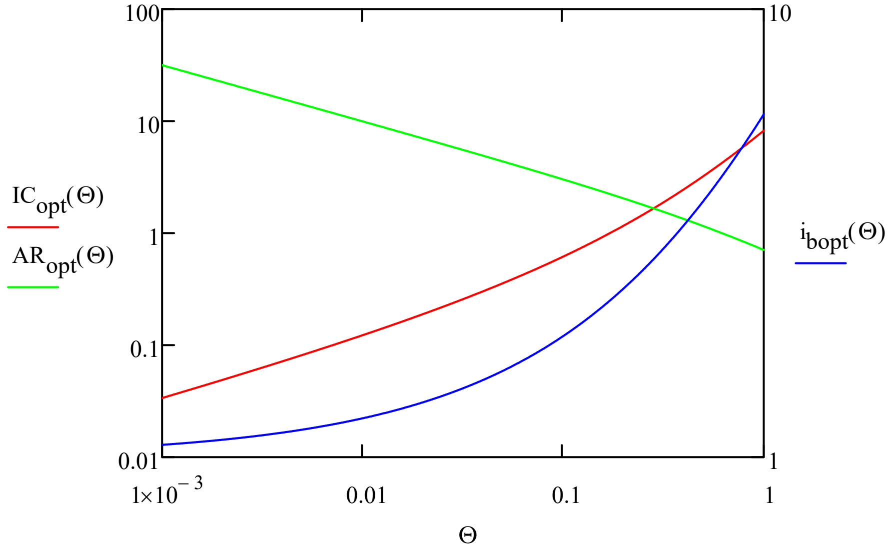
$AR_{opt}(\Theta) := AR(IC_{opt}(\Theta), \Theta)$

$AR_{opt}(\Theta) := \frac{1}{\sqrt{\Theta\cdot(1+\Theta)}}$

$IC_{lim}(\Theta) := \Theta\cdot(1+\Theta)$

$IC_{lim}(0.1) = 0.11$

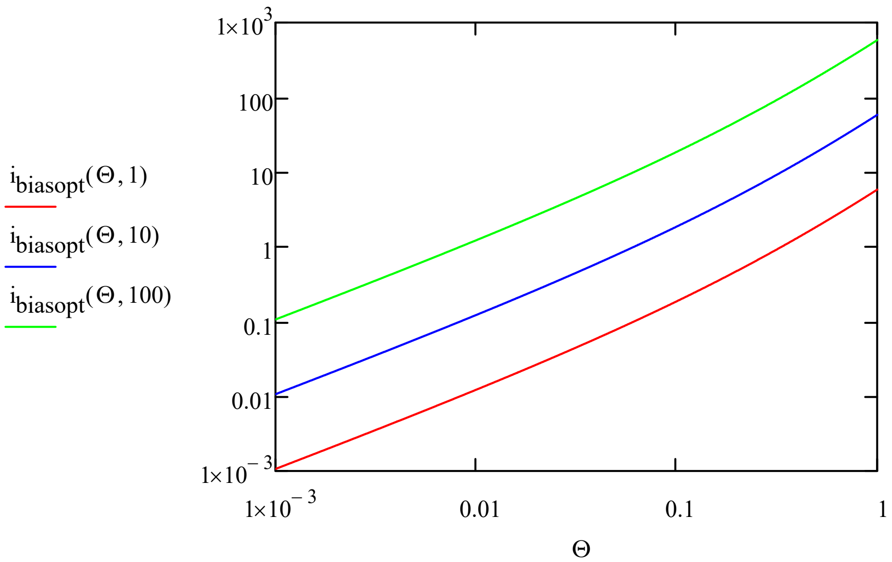




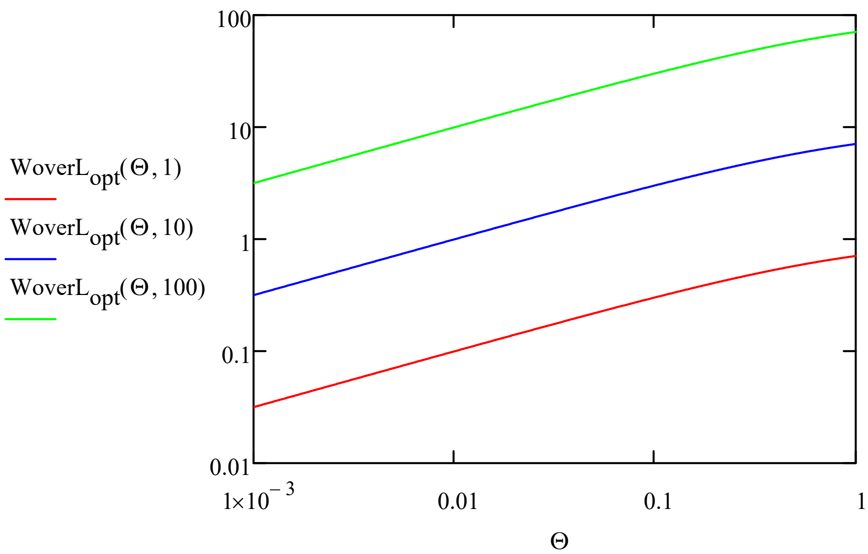
The effective optimum bias current is then given by

$$i_{bias} = \frac{I_b}{I_{specsq}} = \Omega \cdot i_b = \kappa \cdot \Theta \cdot i_b \qquad \kappa = \frac{\omega_W}{\omega_L} = \frac{C_{L0}}{C_W \cdot L}$$

$$i_{biasopt}(\Theta, \kappa) := \kappa \cdot \Theta \cdot i_{bopt}(\Theta)$$



$$WoverL_{opt}(\Theta, \kappa) := \kappa \cdot \Theta \cdot AR_{opt}(\Theta)$$



2) Design example in 40nm CMOS technology

$I_{specsq} := 950 \cdot nA$

$n := 1.5$ $\qquad n = 1.5$

$nU_T := n \cdot U_T$ $\qquad nU_T = 38.813 \cdot mV$

$C_W := 0.45 \cdot \frac{fF}{\mu m}$

$f_u := 10 \cdot GHz$ $\qquad \omega_u := 2 \cdot \pi \cdot f_u$

$L_{min} := 40 \cdot nm$

$L := L_{min}$

$C_{L0} := 60 \cdot fF$

$\omega_L := \frac{I_{specsq}}{nU_T \cdot C_{L0}}$ $\qquad \frac{\omega_L}{2 \cdot \pi} = 64.926 \cdot MHz$

$\Omega_n := \frac{\omega_u}{\omega_L}$ $\qquad \Omega_n = 154.021$

$\omega_w := \frac{I_{specsq}}{nU_T \cdot C_W \cdot L}$ $\qquad \frac{\omega_w}{2 \cdot \pi} = 216.421 \cdot GHz$

$\Theta := \frac{\omega_u}{\omega_w}$ $\qquad \Theta = 0.046$

$IC_{op} := IC_{opt}(\Theta)$ $\qquad IC_{op} = 0.337$

$i_{bop} := i_{bopt}(\Theta)$ $\qquad i_{bop} = 1.532$

$\kappa = \frac{\omega_w}{\omega_L} = \frac{C_{L0}}{C_W \cdot L}$

$C_W \cdot L = 0.018 \cdot fF$

$\kappa := \frac{C_{L0}}{C_W \cdot L}$ $\qquad \kappa = 3.333 \times 10^3$ $\qquad \frac{\omega_w}{\omega_L} = 3.333 \times 10^3$

$i_{bop} := i_{biasopt}(\Theta, \kappa)$ $\qquad i_{bop} = 235.982$

$I_{bias} := I_{specsq} \cdot i_{biasopt}(\Theta, \kappa)$ $\qquad I_{bias} = 224.183 \cdot \mu A$

$WoverL := WoverL_{opt}(\Theta, \kappa)$ $\qquad WoverL = 700.52$

$W_{opt} := WoverL \cdot L$ $\qquad W_{opt} = 28.021 \cdot \mu m$

$G_m := \frac{I_{specsq}}{nU_T} \cdot WoverL \cdot gms(IC_{op})$ $\qquad G_m = 4.562 \times 10^3 \cdot \frac{\mu A}{V}$

$C_{Ltot} := C_{L0} + W_{opt} \cdot C_W$ $\qquad C_{Ltot} = 72.609 \cdot fF$

$$\frac{G_m}{2 \cdot \pi \cdot C_{Ltot}} = 10 \cdot GHz$$