Low-power radio design for the IoT
Exercise 7 (07.04.2022)

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Problem 1 Noise Parameters of the Common-Gate Amplifier

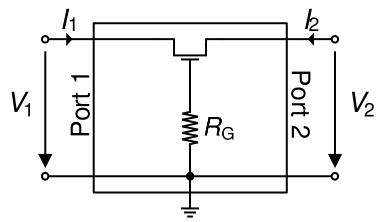

Figure 1: Two-port network corresponding to a common-gate amplifier (bias not shown).

Fig. 1 shows the two-port network corresponding to a common-gate stage including the gate resistance, but without the details of the bias network.

1.1 Small-signal schematic

Draw the small-signal schematic (only account for the gate-to-source capacitance C_{GS}, no other capacitances).

1.2 Two-port Noise Parameters

Calculate the noise parameters of the noisy two-port including the transistor channel thermal noise and the thermal noise of R_G but ignoring the induced gate noise.

- First calculate the parameters R_v, G_i, G_c and B_c.
- Then derive the noise parameters G_{opt}, B_{opt} and F_{min}.

1.3 Noise Factor

Calculate the effective noise factor,

- from the formula using the noise parameters R_v, G_i, G_c and B_c and then
- directly by calculating the total input referred noise current i_{neq} (or voltage v_{neq}).

Solutions to Exercise 7 (07.04.2022)

Problem 1 Noise Factor of Common-Gate Amplifier

1.1 Noise Parameters

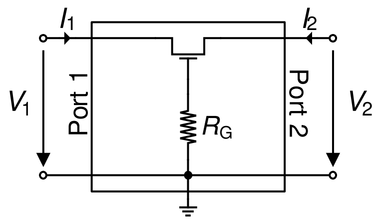

Figure 1: Two-port network corresponding to a common-gate amplifier (bias not shown).

Fig. 1 shows the two-port network corresponding to a common-gate stage including the gate resistance, but without the details of the bias network.

1.2 Small-signal equivalent schematic (in saturation)

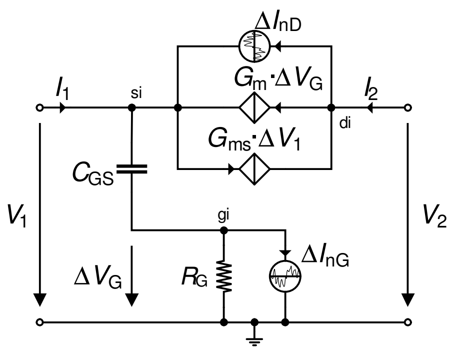

Figure 2: Simplified small-signal schematic of the common-gate amplifier of Fig. 1 assuming the transistor is biased in saturation.

The simplified small-signal schematic corresponding to the two-port network shown in Fig. 1 is presented in Fig. 2. In order to keep the derivation simple, it only includes the gate-to-source capacitance C_{GS}, neglecting all other capacitances and access resistances except the gate resistance R_G. On the other hand it includes the channel noise source ΔI_{nD} and the induced gate noise (IGN) source ΔI_{nG}. From Fig. 2, we clearly see that the IGN source has no effect if the gate resistance is zero. If $R_G > 0$, the IGN has two effects: a first effect trough the voltage ΔV_G generated across R_G producing a current at the drain thanks to the gate transconductance G_m. A second effect is through the gate-to-source capacitance C_{GS}, which produces a current at the input.

1.3 Calculation of two-port noise parameters R_v, G_i, G_c and B_c

From the small-signal schematic of Fig. 2, we can calculate the two-port noise parameters. We first have to calculate the noise sources V_n and I_n from their definitions given by

$$V_n = B \cdot I_2|_{V_1=V_2=0} = -\frac{1}{Y_{21}}\, I_2|_{V_1=V_2=0}\,, \tag{1a}$$

$$I_n = D \cdot I_2|_{I_1=V_2=0} = -\frac{Y_{11}}{Y_{21}}\, I_2|_{I_1=V_2=0}\,, \tag{1b}$$

which require the Y-parameters Y_{11} and Y_{21}, which are calculated as

$$Y_{11} = G_{ms} \cdot \frac{1 + sC_{GS}/G_{ms}(1 + R_G(G_{ms} - G_m))}{1 + sR_GC_{GS}} = G_{ms} \cdot \frac{1 + sC_{GS}/G_{ms}(1 + R_GG_m(n-1))}{1 + sR_GC_{GS}}, \tag{2a}$$

$$Y_{21} = -G_{ms} \cdot \frac{1 + sR_GC_{GS}(1 - G_m/G_{ms})}{1 + sR_GC_{GS}} = -G_{ms} \cdot \frac{1 + sR_GC_{GS}(1 - 1/n)}{1 + sR_GC_{GS}}. \tag{2b}$$

which can be simplified assuming that $\omega C_{GS}/G_{ms} \ll 1$ and $\omega R_GC_{GS} \ll 1$, leading to

$$Y_{11} \cong G_{ms}, \tag{3a}$$

$$Y_{21} \cong -G_{ms}. \tag{3b}$$

The output current I_2 when the input and output are short-circuited is given by

$$I_2|_{V_1=V_2=0} = \Delta I_{nD} - \frac{G_mR_G}{1 + sR_GC_{GS}} \cdot \Delta I_{nG} \cong \Delta I_{nD} - G_mR_G \cdot \Delta I_{nG}, \tag{4}$$

from which we obtain V_n

$$V_n = -\frac{1}{Y_{21}}\, I_2|_{V_1=V_2=0} = \frac{(1 + sR_GC_{GS}) \cdot \Delta I_{nD} - G_mR_G \cdot \Delta I_{nG}}{G_{ms}(1 + sR_GC_{GS}(1 - 1/n))} \cong \frac{\Delta I_{nD}}{G_{ms}} - \frac{R_G}{n} \cdot \Delta I_{nG}. \tag{5}$$

The output current I_2 when the input is open and the output is short-circuited is

$$I_2|_{I_1=V_2=0} = \frac{sC_{GS}/G_{ms} \cdot \Delta I_{nD} + sR_GC_{GS}(1 - 1/n) \cdot \Delta I_{nG}}{1 + sC_{GS}/G_{ms}(1 - G_mR_G + G_{ms}R_G)} \tag{6a}$$

$$\cong sC_{GS}/G_{ms} \cdot \Delta I_{nD} + sR_GC_{GS}(1 - 1/n) \cdot \Delta I_{nG}, \tag{6b}$$

from which we get I_n

$$I_n = -\frac{Y_{11}}{Y_{21}}\, I_2|_{I_1=V_2=0} = \frac{sC_{GS}/G_{ms} \cdot \Delta I_{nD} + sR_GC_{GS}(1 - 1/n) \cdot \Delta I_{nG}}{1 + sR_GC_{GS}(1 - 1/n)} \tag{7a}$$

$$\cong sC_{GS}/G_{ms} \cdot \Delta I_{nD} + sR_GC_{GS}(1 - 1/n) \cdot \Delta I_{nG}. \tag{7b}$$

The mean square value of V_n assuming $\omega C_{GS}/G_{ms} \ll 1$ and $\omega R_GC_{GS} \ll 1$ is then given by

$$\overline{|V_n|^2} \cong \frac{\overline{|\Delta I_{nD}|^2}}{G_{ms}^2} + \frac{R_G^2}{n^2} \cdot \overline{|\Delta I_{nG}|^2} - \frac{R_G}{nG_{ms}} \cdot \left(\overline{\Delta I_{nG} \cdot \Delta I_{nD}^*} + \overline{\Delta I_{nG}^* \cdot \Delta I_{nD}}\right), \tag{8}$$

whereas the mean square value of I_n is given by

$$\begin{aligned}\overline{|I_n|^2} \cong{}& \left(\frac{\omega\, C_{GS}}{G_{ms}}\right)^2 \cdot \overline{|\Delta I_{nD}|^2} + (\omega\, R_GC_{GS}(1 - 1/n))^2 \cdot \overline{|\Delta I_{nG}|^2} \\ &+ \left(\frac{\omega\, C_{GS}}{G_{ms}}\right)^2 G_mR_G(1 - 1/n) \cdot \left(\overline{\Delta I_{nG} \cdot \Delta I_{nD}^*} + \overline{\Delta I_{nG}^* \cdot \Delta I_{nD}}\right).\end{aligned} \tag{9}$$

The mean-square values $\overline{|\Delta I_{nD}|^2}$ and $\overline{|\Delta I_{nG}|^2}$ can be expressed in terms of the PSD $S_{\Delta I_{nD}^2}$ and $S_{\Delta I_{nG}^2}$ as

$$\overline{|\Delta I_{nD}|^2} = S_{\Delta I_{nD}^2} \cdot B = 4kTB \cdot G_{nD}, \tag{10a}$$

$$\overline{|\Delta I_{nG}|^2} = S_{\Delta I_{nG}^2} \cdot B = 4kTB \cdot G_{nG}(\omega). \tag{10b}$$

For narrow-band systems, the definition of the correlation coefficient can be extended to the mean-square values as

$$\rho_{kl} \triangleq \frac{s_{\Delta i_{nk} \cdot \Delta i^*_{nl}}}{\sqrt{s_{\Delta i^2_{nk}} \cdot s_{\Delta i^2_{nl}}}} = \frac{\overline{\Delta I_{nk} \cdot \Delta I^*_{nl}}}{\sqrt{\overline{|\Delta I_{nk}|^2} \cdot \overline{|\Delta I_{nl}|^2}}}. \tag{11}$$

The mean-square values $\overline{\Delta I_{nG} \cdot \Delta I^*_{nD}}$ and $\overline{\Delta I^*_{nG} \cdot \Delta I_{nD}}$ can then be expressed in terms of mean-square values $\overline{|\Delta I_{nD}|^2}$ and $\overline{|\Delta I_{nG}|^2}$ and the correlation coefficient ρ_{GD} according to (11):

$$\overline{\Delta I_{nG} \cdot \Delta I^*_{nD}} = \rho_{GD} \cdot \sqrt{\overline{|\Delta I_{nG}|^2} \cdot \overline{|\Delta I_{nD}|^2}} = +jc_g \cdot 4kTB \cdot \sqrt{G_{nG} \cdot G_{nD}}, \tag{12a}$$

$$\overline{\Delta I^*_{nG} \cdot \Delta I_{nD}} = \rho^*_{GD} \cdot \sqrt{\overline{|\Delta I_{nG}|^2} \cdot \overline{|\Delta I_{nD}|^2}} = -jc_g \cdot 4kTB \cdot \sqrt{G_{nG} \cdot G_{nD}}. \tag{12b}$$

The noise parameters R_v and G_i are then given by

$$R_v = \frac{\overline{|V_n|^2}}{4kTB} = \frac{G_{nD}}{G^2_{ms}} + \left(\frac{R_G}{n}\right)^2 \cdot G_{nG}, \tag{13a}$$

$$G_i = \frac{\overline{|I_n|^2}}{4kTB} = \left(\frac{\omega\, C_{GS}}{G_{ms}}\right)^2 \cdot G_{nD} + (\omega\, R_G C_{GS}(1 - 1/n))^2 \cdot G_{nG}. \tag{13b}$$

For calculating Y_c according to

$$Y_c = \frac{\overline{I_n \cdot V^*_n}}{\overline{|V_n|^2}}, \tag{14}$$

we need $\overline{I_n \cdot V^*_n}$ which is given by

$$\overline{I_n \cdot V^*_n} = 4kTB \cdot \frac{\omega\, C_{GS}}{G_{ms}} \cdot \left\{-c_g \cdot R_G \cdot \sqrt{G_{nG} \cdot G_{nD}} + j\left[\frac{G_{nD}}{G_{ms}} - \left(1 - \frac{1}{n}\right) G_m R^2_G G_{nG}\right]\right\}, \tag{15}$$

$$\overline{I_n \cdot V^*_n} = 4kTB \cdot \frac{j\,\omega\, C_{GS}}{G_{ms}} \cdot \left[\frac{G_{nD}}{G_{ms}} - \left(1 - \frac{1}{n}\right) G_m R^2_G G_{nG} + j\; c_g \cdot R_G \cdot \sqrt{G_{nG} \cdot G_{nD}}\right], \tag{16}$$

from which we get

$$Y_c = \frac{j\omega C_{GS}\left(G_{nD} + R_G(-G^2_m G_{nG}(n-1)R_G + jc_g G_{ms}\sqrt{G_{nD}G_{nG}})\right)}{G^2_m G_{nG} R^2_G + G_{nD}} \tag{17}$$

Introducing the expressions of G_{nD} and G_{nG} finally results in

$$R_v = \frac{\gamma_{nD}}{G_m}, \tag{18a}$$

$$G_i = (\omega\, C_{GS})^2 \cdot \frac{\gamma_{nD}}{G_m} \cdot \left[1 + \frac{\beta_{nG}}{\gamma_{nD}} - 2c_g \cdot \sqrt{\frac{\beta_{nG}}{\gamma_{nD}}}\right], \tag{18b}$$

$$G_c = 0, \tag{18c}$$

$$B_c = \omega\, C_{GS} \cdot \left[1 - c_g \cdot \sqrt{\frac{\beta_{nG}}{\gamma_{nD}}}\right]. \tag{18d}$$

The uncorrelated and correlated parts of G_i are then obtained using

$$G_{iu} = G_i - |Y_c|^2 \cdot R_v = (1 - |c|^2) \cdot G_i, \tag{19a}$$

$$G_{ic} = |Y_c|^2 \cdot R_v = |c|^2 \cdot G_i, \tag{19b}$$

$$G_i = G_{iu} + G_{ic}, \tag{19c}$$

and resulting in

$$G_{iu} = (\omega\, C_{GS})^2 \cdot \frac{\beta_{nG}}{G_m} \cdot (1 - c^2_g), \tag{20a}$$

$$G_{ic} = (\omega\, C_{GS})^2 \cdot \frac{\gamma_{nD}}{G_m} \cdot \left[1 + c^2_g \cdot \frac{\beta_{nG}}{\gamma_{nD}} - 2c_g \cdot \sqrt{\frac{\beta_{nG}}{\gamma_{nD}}}\right]. \tag{20b}$$

1.4 Calculation of G_{opt}, B_{opt} and F_{min}

From the R_v, G_i, Y_c parameters we can derive the four noise parameters R_v, G_{opt}, B_{opt} and F_{min} as a function of the circuit parameters using

$$G_{opt} = \sqrt{\frac{G_{iu}}{R_v} + G_c^2} = \sqrt{\frac{G_i}{R_v} - B_c^2}, \tag{21a}$$

$$B_{opt} = -B_c, \tag{21b}$$

$$F_{min} = 1 + 2R_v(G_{opt} + G_c) = 1 + 2R_v\left(\sqrt{\frac{G_{iu}}{R_v} + G_c^2} + G_c\right), \tag{21c}$$

resulting in

$$G_{opt} = \omega\, C_{GS} \cdot \sqrt{\frac{\beta_{nG}}{\gamma_{nD}} \cdot (1 - c_g^2)}, \tag{22a}$$

$$B_{opt} = -\omega\, C_{GS} \cdot \left[1 - c_g \cdot \sqrt{\frac{\beta_{nG}}{\gamma_{nD}}}\right], \tag{22b}$$

$$\begin{aligned} F_{min} &= 1 + 2\omega\, C_{GS} \cdot \frac{\gamma_{nD}}{G_m} \cdot \sqrt{\frac{\beta_{nG}}{\gamma_{nD}} \cdot (1 - c_g^2)} \\ &\cong 1 + 2\gamma_{nD} \cdot \frac{\omega}{\omega_t} \cdot \sqrt{\frac{\beta_{nG}}{\gamma_{nD}} \cdot (1 - c_g^2)}, \end{aligned} \tag{22c}$$

where the G_m/C_{GS} ratio has been approximated by the transit frequency $\omega_t \cong G_m/C_{GS}$. Equations (22) reveal that, due to the induced gate noise, the noise matching condition is slightly different than the gain matching condition which would require $B_s = -\omega\, C_{GS}$. Also, the minimum noise factor is strongly depending on the induced gate noise through parameters β_{nG} and c_g. If induced gate noise was ignored (by setting $\beta_{nG} = 0$), the minimum noise factor would be equal to unity! This surprising result can be explained as follows: if the induced gate noise is ignored there is only the drain noise left and the optimum source conductance is null whereas the optimum source susceptance is $-\omega\, C_{GS}$. This noise matching situation corresponds to having an inductor with a susceptance value being $-\omega\, C_{GS}$ and no internal conductance. The input circuit is then an inductance in series with the transistor gate-to-source capacitance. This source inductance will then resonate with the input transistor capacitance at the operating frequency providing an infinite voltage gain at the input. The input referred noise is then nulled, resulting in a unity noise factor.

Equation (22c) indicates that the minimum noise factor increases with frequency for a given bias. For a given operating frequency it can be decreased by increasing the transistor transit frequency. This can be achieved by increasing the transistor bias or by reducing the transistor length (or both). Technology scaling leads to an improved noise factor at a given frequency and bias.

It is also interesting to notice that the correlation between the drain noise and the induced gate noise reduces the noise factor compared to the situation where they would be fully uncorrelated.

The noise parameters (22) can be further simplified by replacing γ_{nD} and β_{nG} by their expression valid in strong inversion and saturation leading to

$$G_{opt} \cong 0.45\, \omega\, C_{GS}, \tag{23a}$$

$$B_{opt} \cong -0.8\, \omega\, C_{GS}, \tag{23b}$$

$$F_{min} \cong 1 + 0.77 \cdot \frac{\omega}{\omega_t}, \tag{23c}$$

where $n = 1.3$ has been used. Equations (23) give a 1^{st}-order relation for the noise parameters of a common-source amplifier.

1.5 Direct calculation of the noise factor