Low-power radio design for the IoT
Exercise 10 (05.05.2022)

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Problem 1 Dual Gate Mixer

Consider the dual-gate mixer shown in Fig. 1. Assume that the dc voltage applied at the gate of M2 is such that transistor M2 is biased in strong inversion and saturation and M1 is biased in the linear region when V_{LO} is high and that its on-resistance is R_{on1}. Also, assume that the LO signal has abrupt edges and a 50% duty cycle. You can also neglect the output conductances of transistors M1 and M2 ($G_{ds1} = G_{ds2} = 0$) and the source and drain access resistances.

- Compute the voltage conversion gain of the circuit. Assume M2 remains in saturation and denote its transconductance by G_{m2}.
- If R_{on1} is very small, determine the IP_2 of the circuit. Assume M2 has an overdrive of $V_{G0} - V_{T0}$ in the absence of signals (when it is on).

Problem 2 Active mixer with load mismatch

Consider the active mixer shown in Fig. 2, where the LO signal has abrupt edges and a 50% duty cycle. As above you can neglect the output conductances ($G_{ds1} = G_{ds2} = 0$) and the source and drain access resistance. The load resistors exhibit mismatch, but the circuit is otherwise symmetric. Assume M1 carries a bias current I_b.

- Determine the output offset voltage.
- Determine the IP_2 of the circuit in terms of the overdrive and bias current of M1.

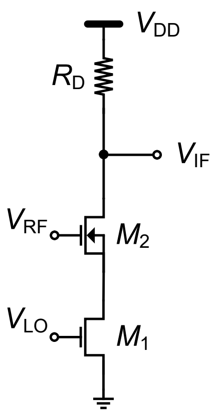

Figure 1: Dual-gate mixer

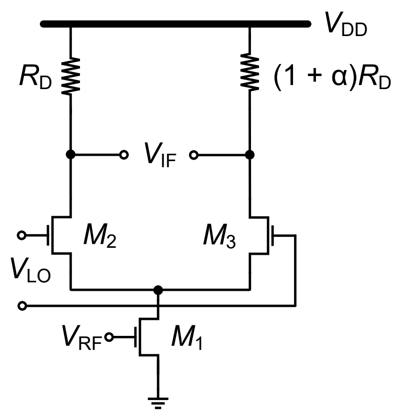

Figure 2: Active mixer with load mismatch

Solutions to Exercise 10 (05.05.2022)

Problem 1 Dual-Gate mixer

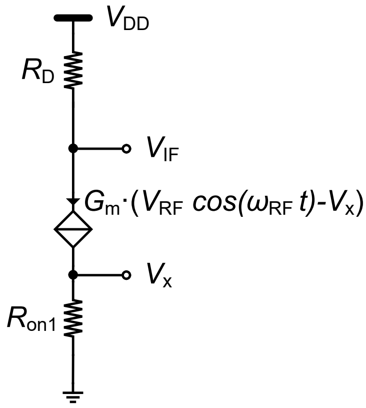

Figure 1: Small signal equivalent of the dual-gate mixer

Consider the dual-gate mixer shown in Fig. 1. Assume that the dc voltage applied at the gate of M2 is such that transistor M2 is biased in strong inversion and saturation and M1 is biased in the linear region when V_{LO} is high and that its on-resistance is R_{on1}. Also, assume that the LO signal has abrupt edges and a 50% duty cycle. You can also neglect the output conductances of transistors M1 and M2 ($G_{ds1} = G_{ds2} = 0$) and the source and drain access resistances.

- Compute the voltage conversion gain of the circuit. Assume M2 remains in saturation and denote its transconductance by G_{m2}.

The output voltage of the mixer can be written as

$$V_{IF}(t) = I_{RF}(t) \cdot R_D \cdot m_{LO}(t) \tag{1}$$

where m_{LO} is the square wave toggling between 1 and 0 applied to the gate of M1. This square wave can be approximated by its fundamental component and give rise to the following approximation

$$V_{IF}(t) \cong I_{RF}(t) \cdot R_D \cdot \frac{2}{\pi} \cos(\omega_{LO} t) \tag{2}$$

The I_{RF} current can be derived by solving the following small-signal equivalent circuit shown in Fig. 1 from which we get

$$I_{RF}(t) = \frac{G_{m2}}{1 + G_{m2} R_{on1}} V_{RF} \cos(\omega_{RF} t) \tag{3}$$

The output V_{IF} can be then re-written as

$$V_{IF}(t) = \frac{G_{m2}}{1 + G_{m2} R_{on1}} V_{RF} \cos(\omega_{RF} t) \cdot R_D \cdot \frac{2}{\pi} \cos(\omega_{LO} t) \tag{4}$$

which for the $\cos((\omega_{RF} - \omega_{IF})t)$ components results into

$$V_{IF}(t) = \frac{G_{m2}}{1 + G_{m2} R_{on1}} V_{RF} \cdot R_D \cdot \frac{1}{\pi} \cos((\omega_{RF} - \omega_{LO})t) \tag{5}$$

Finally, the conversion gain can be expressed as

$$\frac{V_{IF_p}}{V_{RF_p}} = \frac{1}{\pi} \cdot \frac{G_{m2}}{1 + G_{m2} R_{on1}} \cdot R_D \tag{6}$$

- If R_{on1} is very small, determine the IP_2 of the circuit. Assume M2 has an overdrive of $V_{G0} - V_{T0}$ in the absence of signals (when it is on).

The total voltage applied to the gate of M2 to evaluate the IP_2 is equal to

$$V_{RF} = V_m \cos(\omega_1 t) + V_m \cos(\omega_2 t) + V_{GS0} \tag{7}$$

This determines a second-order intermodulation component that can be written as

$$I_{IM2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_m^2 \cos((\omega_1 - \omega_2)t). \tag{8}$$

To derive the IP_2, V_{IIP2}, we equate the two expressions

$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{IIP2}^2 R_D = \frac{1}{\pi}\cdot\frac{G_{m2}}{1+G_{m2}R_{on1}}\cdot R_D V_{IIP2} \tag{9}$$

and we obtain, for $G_{m2}\cdot R_{on1} \ll 1$, the following expression

$$V_{IIP2} \cong \frac{2}{\pi}(V_{GS0} - V_{T0}). \tag{10}$$

Problem 2 Active mixer with load mismatch

Consider the active mixer shown in Fig. 2, where the LO signal has abrupt edges and a 50% duty cycle. As above you can neglect the output conductances ($G_{ds1} = G_{ds2} = 0$) and the source and drain access resistance. The load resistors exhibit mismatch, but the circuit is otherwise symmetric. Assume M1 carries a bias current I_b.

- Determine the output offset voltage.

In the circuit in Fig. 2, transistor M1 produces a small-signal drain current equal to $G_{m1}V_{RF}$. With abrupt LO switching, M2 multiplies I_b by a square wave toggling between 0 and 1, $m(t)$, and M3 by $m\big(t - \frac{T_{LO}}{2}\big)$ because the input pair is differentially driven. It follows that

$$I_2(t) = I_b \cdot m(t), \tag{11a}$$

$$I_3(t) = I_b \cdot m\left(t - \frac{T_{\mathrm{LO}}}{2}\right). \tag{11b}$$

It follows:

$$\begin{aligned} V_{out}(t) = I_3 R_D(1+\alpha) - I_2 R_D = \underbrace{I_b \alpha R_D m\left(t - \frac{T_{LO}}{2}\right)}_{\text{Offset term}} \quad & + \underbrace{I_b R_D \left(m\left(t-\frac{T_{LO}}{2}\right) - m(t)\right)}_{\text{Original term}} \approx \\ \approx I_b\alpha R_D \frac{2}{\pi}\cos(\omega_{LO}t) \quad & + I_b R_D\frac{4}{\pi}\cos(\omega_{LO}t); \end{aligned} \tag{12}$$

where the approximation takes into consideration the fundamental amplitude of the square wave. Therefore the output offset is $I_b\alpha R_D\frac{2}{\pi}$.

- Determine the IP_2 of the circuit in terms of the overdrive and bias current of M1.

In order to calculate the IP_2 we assume that the gate voltage of M1 is given by

$$V_{RF} = V_{ov}\cos(\omega_1 t) + V_{ov}\cos(\omega_2 t) + V_{GS0}, \tag{13}$$

where V_{GS0} is the bias gate-source voltage of M1 and V_{ov} is the overdrive voltage. With a square-law device, the second order intermodulation IM_2 product emerges in the current of M1 as

$$IM_2 = \frac{1}{2}\mu C_{ox}\frac{W}{L}V_m^2 \cos\left((\omega_1 - \omega_2)t\right) \tag{14}$$

Multiplying this quantity by $\alpha R_D \frac{2}{\pi}$ yields the direct feed-through to the output

$$V_{IM2,out} = \left(\frac{1}{2}\mu C_{ox}\frac{W}{L}V_m^2 \cos\left((\omega_1 - \omega_2)t\right)\right)\alpha R_D \frac{2}{\pi}. \tag{15}$$

To calculate the IP_2, the value of V_{ov} must be raised until the amplitude of $V_{IM2,out}$ becomes equal to the amplitude of the main downconverted components. This amplitude is simply given by $(2/\pi)G_{m1}R_D V_{ov}$. Thus

$$\begin{aligned}\frac{1}{2}\mu C_{ox}\frac{W}{L}V_m^2 \alpha R_D \frac{2}{\pi} &= \frac{2}{\pi}G_{m1}R_D V_{IIP2} \\ &= \frac{2}{\pi}\mu C_{ox}\frac{W}{L}V_{ov1}R_D V_{IIP2},\end{aligned} \tag{16}$$

where $G_{m1} = \mu C_{ox}(W/L)V_{ov1}$. Finally,

$$V_{IIP2} = \frac{\alpha V_{ov1}}{2}. \tag{17}$$