Low-power radio design for the IoT
Exercise 11 (12.05.2022)

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Problem 1 Pierce Oscillator

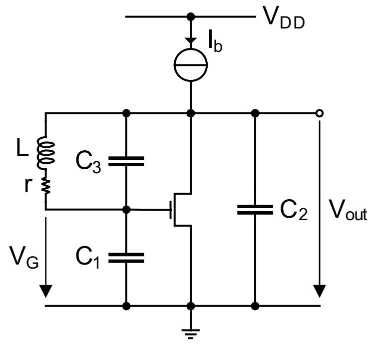

Figure 1: Pierce oscillator.

Design the Pierce oscillator shown in Fig. 1 for the following specifications:

$$f_0 = 2.4\,\text{GHz}, \quad C_2 = 1\,\text{pF}, \quad C_3 = 1\,\text{pF}, \quad Q_\text{L} = 10, \quad \mathcal{L}(\Delta\omega = 1\,\text{MHz}) = -112.37\,\text{dBc} \cdot \text{Hz}^{-1}$$

- Find the inductance value.
- Find the critical transconductance value.
- Find the critical current value assuming the transistor is biased in weak inversion (take $n = 1.3$).
- Find the output oscillation amplitude $\hat{V}_\text{out}$ for the given phase noise specification. Assume first that the transistor is biased in weak inversion and that the noise excess factor γ is equal to 1.2. Then repeat the calculus assuming that the transistor is biased in strong inversion and that the noise excess factor γ is equal to 0.89.
- Find the bias current I_b for the specified amplitude assuming the transistor is biased in weak inversion (take $n = 1.3$).
- Find the bias current I_b for the specified amplitude assuming the transistor is biased in strong inversion with $V_G - V_{T0} = 300\,\text{mV}$.

Problem 2 Complementary cross-coupled oscillator

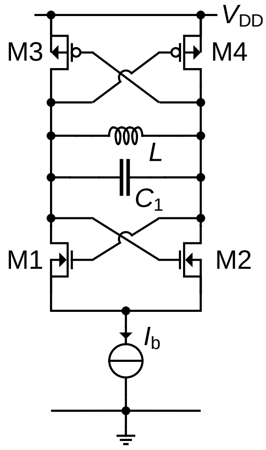

Figure 2: Complementary cross-coupled oscillator

2.1 Oscillator analysis

Fig. 2 shows a complementary cross-coupled oscillator. In the first part of the problem we will derive expressions for quantities that will be used in the second part. All transistors are biased in weak inversion and have transconductances equal to G_m. Quality factor of the inductor is Q_{L}.

- Draw the small signal equivalent circuit.
- Derive the expression for the impedance seen from the inductor Z_c, and find $R_c = -\Re Z_c$ and $X_c = -\Im Z_c$.
- Derive the expression for the oscillation frequency ω_0.
- Derive the expression for the G_{mcrit}.

2.2 Oscillator design

The derived expressions will now be used to design the oscillator with the following specifications:

$$f_0 = 2.4\,\mathrm{GHz}, \quad C = 0.5\,\mathrm{pF}, \quad Q_{\mathrm{L}} = 10, \quad V_{\mathrm{out}} = 325\,\mathrm{mV}, \quad U_{\mathrm{T}} = 25\,\mathrm{mV}, \quad n = 1.3$$

Again, assume that all transistors are biased in weak inversion.

- Find the inductance value for the given oscillation frequency.
- Find the value of G_{mcrit}.
- Calculate the bias current needed to achieve the desired amplitude of the output voltage V_{out}. You can assume here that the condition $V_{\mathrm{out}} \gg 2nU_{\mathrm{T}}$ is fulfilled.

Solutions to Exercise 11 (12.05.2022)

Problem 1 Pierce Oscillator

Design the Pierce oscillator shown in Fig. 1 for the following specifications:

$$f_0 = 2.4\,\text{GHz}, \quad C_2 = 1\,\text{pF}, \quad C_3 = 1\,\text{pF}, \quad Q_\text{L} = 10, \quad \mathcal{L}(\Delta\omega = 1\,\text{MHz}) = -112.37\,\text{dBc}\cdot\text{Hz}^{-1}$$

- Find the inductance value.

To find the inductance value we first need to know the value of C_1. We know that G_mcrit and hence the power consumption are minimum for $C_1 = C_2$, hence $C_1 = 1\,\text{pF}$. The inductance is approximately given by

$$L \cong \frac{1}{\omega_0^2 \cdot (C_3 + C_{12})} \tag{1}$$

where $\omega_0 = 2\pi f_0$ and $C_{12} = C_1 \cdot C_2/(C_1 + C_2) = 0.5\,\text{pF}$. This leads to $L = 2.932\,\text{nH}$.

- Find the critical transconductance value.

The critical transconductance is given by

$$G_\text{mcrit} \cong \frac{\omega_0}{Q_\text{L}} \cdot (C_1 + C_2) \cdot \left(1 + \frac{C_3}{C_{12}}\right) = 9.048\,\text{mS}. \tag{2}$$

- Find the critical current value assuming the transistor is biased in weak inversion (take $n = 1.3$).

The critical current assuming the transistor is biased in weak inversion is then given by

$$I_\text{crit} = G_\text{mcrit} \cdot n \cdot U_\text{T} = 304.345\,\mu\text{A}, \tag{3}$$

where $n = 1.3$ is the slope factor and $U_\text{T} \triangleq kT/q = 25\,\text{mV}$ is the thermodynamic voltage.

- Find the output oscillation amplitude $\hat{V}_\text{out}$ for the given phase noise specification. Assume first that the transistor is biased in weak inversion and that the noise excess factor γ is equal to 1.2. Then repeat the calculus assuming that the transistor is biased in strong inversion and that the noise excess factor γ is equal to 0.89.

The phase noise at 1 MHz offset frequency is given by:

$$\mathcal{L}_{\Delta\omega=1\,\text{MHz}} = \frac{S_{V_n}}{\hat{V}_\text{out}^2} = \frac{kT \cdot r \cdot (1+\gamma)}{\hat{V}_\text{out}^2} \left(\frac{C_1}{C_1 + C_2}\right)^2 \cdot \left(\frac{\omega_0}{\Delta\omega}\right)^2. \tag{4}$$

Therefore, the output amplitude can be derived:

$$\hat{V}_\text{out} = \sqrt{\frac{kT \cdot (1+\gamma)}{\mathcal{L}_{\Delta\omega=1\,\text{MHz}} \cdot Q_\text{L} \cdot \omega_0 \cdot (C_3 + C_{12})}} \left(\frac{C_1}{C_1 + C_2}\right) \cdot \left(\frac{\omega_0}{\Delta\omega}\right) \tag{5}$$

In WI, $\hat{V}_\text{out} = 100.058\ mV$, while in SI, $\hat{V}_\text{out} = 92.741\ mV$.

- Find the bias current I_b, assuming the transistor is biased in weak inversion (take $n = 1.3$).

We first need to calculate the normalized amplitude $x \triangleq \hat{V}_\text{out}/(n \cdot U_\text{T}) = 2.975$. The bias current is then given by

$$I_\text{b} = I_\text{crit} \cdot \frac{x \cdot I_\text{B0}(x)}{2I_\text{B1}(x)} \tag{6}$$

where $I_\text{B0}(x)$ and $I_\text{B1}(x)$ are the modified Bessel functions of the first kind of order 0 and 1 respectively. The ratio $\chi \triangleq x \cdot I_\text{B0}(x)/(2I_\text{B1}(x))$ can be found from the abacus or calculated as $\chi = 1.84$, resulting in a bias current of $I_\text{b} = 559.995\,\mu\text{A}$.

- Find the bias current I_b for the specified amplitude assuming the transistor is biased in strong inversion with $V_G - V_{T0} = 300\,\text{mV}$.

In strong inversion, the normalized critical overdrive voltage is given by

$$v_{gtcrit} \triangleq \frac{V_{Gcrit} - V_{T0}}{n \cdot U_\text{T}} = \sqrt{v_{gt}^2 - \frac{x^2}{2}} = 8.703 \tag{7}$$

where $v_{gt} \triangleq (V_G - V_{T0})/(n \cdot U_\text{T}) = 8.919$. The normalized critical current is then given by

$$i_{crit} \triangleq \frac{I_\text{crit}}{I_\text{spec}} = \left(\frac{v_{gtcrit}}{2}\right)^2 = 18.935. \tag{8}$$

where I_spec is the specific current given by

$$I_\text{spec} = \frac{2n \cdot U_\text{T} \cdot G_\text{mcrit}}{v_{gtcrit}} = 69.941\,\mu\text{A}. \tag{9}$$

The normalized bias current is finally given by

$$i_\text{b} \triangleq \frac{I_\text{b}}{I_\text{spec}} = i_{crit} + \frac{x^2}{8} = 19.885, \tag{10}$$

corresponding to a denormalized bias current

$$I_\text{b} = i_\text{b} \cdot I_\text{spec} = 1.391\,\text{mA}. \tag{11}$$

Problem 2 Complementary cross-coupled oscillator

2.1 Oscillator analysis

Fig. 2 shows a complementary cross-coupled oscillator. In the first part of the problem we will derive expressions for quantities that will be used in the second part. All transistors are biased in weak inversion and have transconductances equal to G_m. Quality factor of the inductor is Q_L.

- Draw the small signal equivalent circuit.

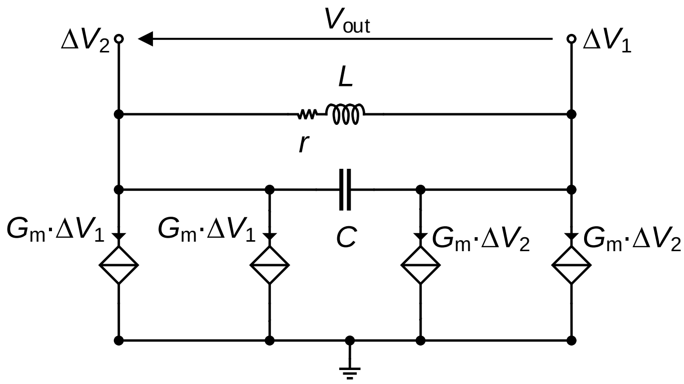

Figure 1: Complementary cross-coupled oscillator

- Derive the expression for the impedance seen from the inductor Z_c, and find $R_c = -\Re Z_c$ and $X_c = -\Im Z_c$.

  As can be seen from the Fig. 1 the complementary oscillator is practically equivalent to the NMOS one. The only difference is the total transconductance that is now equal to the sum of the transconductances of the NMOS and the PMOS transistors. It follows:

$$Z_c = \frac{1}{-G_{\mathrm{m}+}j\omega C}, \tag{12}$$

$$R_c = \frac{G_\mathrm{m}}{G_\mathrm{m}^2 + \omega^2 C^2}, \tag{13}$$

$$X_c = \frac{j\omega C}{G_\mathrm{m}^2 + \omega^2 C^2}. \tag{14}$$

- Derive the expression for the G_mcrit.

  To find the value of critical transconductance we can solve the equation:

$$\frac{X_c(\omega_0, G_\mathrm{mcrit})}{R_c(\omega_0, G_\mathrm{mcrit})} = Q_\mathrm{L}, \tag{15}$$

$$G_\mathrm{mcrit} = \frac{\omega_0 C}{Q_\mathrm{L}}. \tag{16}$$

- Derive the expression for the oscillation frequency ω_0.

  Oscillation frequency can be obtained from the equation:

$$R_c(\omega_0, G_\mathrm{mcrit}) = \omega_0 L, \tag{17}$$

$$\omega_0 = \frac{1}{\sqrt{LC\left(1 + \frac{1}{Q_L^2}\right)}} \tag{18}$$

2.2 Oscillator design

The derived expressions will now be used to design the oscillator with the following specifications:

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Again, assume that all transistors are biased in weak inversion.

- Find the inductance value for the given oscillation frequency.

$$L = \frac{1}{\omega_0^2 C \left(1 + \frac{1}{Q_L^2}\right)} = 8.709\,\text{nH} \tag{19}$$

- Find the value of G_mcrit.

$$G_\text{mcrit} = \frac{\omega_0 C}{Q_\text{L}} = 754\,\mu\text{S}. \tag{20}$$

- Calculate the bias current needed to achieve the desired amplitude of the output voltage V_out. You can assume here that the condition $V_\text{out} \gg 2nU_\text{T}$ is fulfilled.

  For the given amplitude $V_\text{out} = 325\,\text{mV}$ we can calculate:

$$x = \frac{V_\text{out}}{2nU_\text{T}} = 5, \tag{21}$$

  Due to the high amplitude of the voltage we can write:

$$\frac{G_\text{m(1)}}{G_\text{m}} = \frac{a_1}{x} = \frac{4}{\pi x} = 0.2547. \tag{22}$$

  For $G_\text{m(1)} = G_\text{mcrit}$ we have

$$I_\text{b} = 2nU_\text{T} G_\text{m} = 2nU_\text{T} \frac{\pi x}{4} G_\text{mcrit} = 192\,\mu\text{A} \tag{23}$$