Low-power radio design for the IoT - Exam Spring Semester 2021

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Instructions: Attempt all problems. Total allocated time is 3 hours. The points for each problem are indicated in square brackets. The maximum amount of points is 24.

Multiple Choice Questions [8]

Answer to the following questions (note that there is only one exact answer to each question):

1. Consider a cascade of two blocks. The first has power gain A_1 and noise factor F_1, while the following has A_2 and F_2. Assuming $A_1 = 6\,\text{dB}$, $A_2 = 30\,\text{dB}$, $F_1 = 1\,\text{dB}$ and $F_2 = 10\,\text{dB}$, the total noise factor of the cascade then is: [1]

   (a) 11 dB (b) 5.5 dB (c) 1 dB (d) 2.5 dB

2. Considering the effect of velocity saturation, what is the dependence of the maximum transit frequency ω_{tpeak} on the channel length L of a MOSFET biased in saturation and in strong inversion? [1]

   (a) ω_{tpeak} is proportional to L
   (b) ω_{tpeak} is inversely proportional to L
   (c) ω_{tpeak} is proportional to $\sqrt{L}$
   (d) ω_{tpeak} does not depend on L

3. The amplification chain consists of two amplifiers. For the first amplifier the voltage gain is $G_1 = 10\,\text{dB}$ and $IIP3_1 = 1\,\text{V}$ and for the second amplifier voltage gain is $G_2 = 16\,\text{dB}$ and $IIP3_2 = 2\,\text{V}$. What is the $IIP3$ of the chain, assuming that amplifiers are narrow-band? [1]

   (a) 0.535 V (b) 0.196 V (c) 0.167 V (d) 0.387 V

4. What is the conversion gain of a single balanced mixer if the transistors are biased in weak inversion and normalized LO amplitude is $v_{LO} \triangleq \dfrac{V_{LO}}{2\,n\,U_T} \ll 1$? [1]

   (a) $v_{LO}/4$ (b) $v_{LO}/2$ (c) $2/\pi$ (d) $v_{LO}/(2\sqrt{IC})$

5. For a common source amplifier, accounting for the channel thermal noise and the gate resistance noise, one of the following statements is valid: [1]

   (a) F_{min} decreases with the operating frequency
   (b) $F_{min} = 1$
   (c) F_{min} decreases with the transit frequency f_t
   (d) $F_{min} = 0$

6. Consider an heterodyne receiver. Choosing a higher intermediate frequency results in: [1]

   (a) More demanding image rejection and channel selection
   (b) Less demanding image rejection and channel selection
   (c) More demanding image rejection but eased channel selection
   (d) Eased image rejection but more demanding channel selection

7. A two tone signal at frequencies ω_1 and ω_2 with power of -10 dBm at each frequency is supplied to the input of a nonlinear amplifier. At the output, one measures a power of 40 dBm at the fundamentals and -25 dBm at $2\omega_1 - \omega_2$. What is the input third-order intercept point (IIP3) of this amplifier? [1]

   (a) 42.5 dBm (b) 22.5 dBm (c) 55 dBm (d) 25.5 dBm

8. We want to select a signal centered at 100 kHz while rejecting the image frequency at -100 kHz. To this purpose we design a complex bandpass filter centered at 100 kHz with a -3 dB bandwidth of 20 kHz. Which of the following transfer function describes the most suitable filter? [1]

   (a) $H(j\omega) = \dfrac{1}{1 + \dfrac{j\omega}{2\pi\,120\,\text{kHz}}}$
   (b) $H(j\omega) = \dfrac{1}{1 - j5 + \dfrac{j\omega}{2\pi\,10\,\text{kHz}}}$
   (c) $H(j\omega) = \dfrac{1}{1 - j10 + \dfrac{j\omega}{2\pi\,10\,\text{kHz}}}$
   (d) $H(j\omega) = \dfrac{1}{1 + \dfrac{j\omega}{2\pi\,20\,\text{kHz}}}$

Problem 1: Wideband LNA [11]

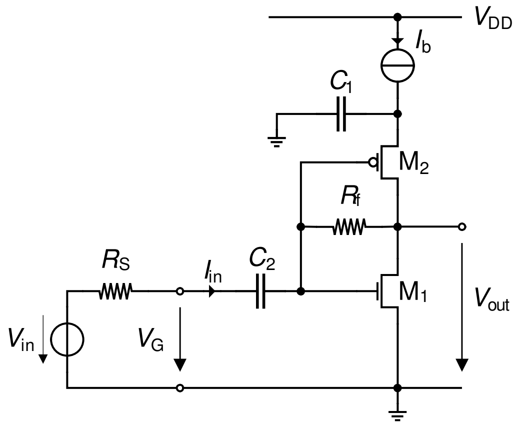

Figure 1: Current-reuse wideband LNA.

1.1 Small-signal Analysis [6]

- Draw the equivalent small-signal schematic of the circuit given in Fig. 1 assuming the transistors are biased in saturation. Assume that the capacitors C_1 and C_2 can be considered as shorts at the frequency of operation. [2]
- Calculate the input impedance $Z_{in} \triangleq \Delta V_G / \Delta I_{in}$ assuming that the output conductances of M_1 and M_2 can be neglected. [1]
- What is the condition for impedance matching? [1]
- Calculate the small-signal voltage gain $A_v \triangleq \Delta V_{out} / \Delta V_{in}$. [1]
  Simplify then the resulting expression assuming that $G_{mi} \cdot R_f \gg 1$ and $G_{mi} \cdot R_S \gg 1$ for $i = 1, 2$.
- What is the value of A_v assuming impedance matching at the input? Simplify then the resulting expression assuming that $R_f \gg R_S$. [1]

1.2 Noise Analysis [5]

- Draw the equivalent small-signal circuit including all the noise sources. [1]
- Calculate the input-referred thermal noise resistance R_{neq}.
  To simplify, assume that both transistors M_1 and M_2 have identical noise excess factors ($\gamma_{nD1} = \gamma_{nD2} = \gamma_{nD}$). Simplify then the resulting expression assuming that $G_{mi} \cdot R_S \gg 1$ for $i = 1, 2$ and $R_f \gg R_S$. [2]
- Calculate the noise factor F. Simplify then the resulting expression assuming that $G_{mi} \cdot R_S \gg 1$ for $i = 1, 2$ and $R_f \gg R_S$.
- Calculate the noise factor F assuming impedance matching. [1]
  Simplify then the resulting expression assuming that $R_S \ll R_f$. [1]

Problem 2: Colpitts Oscillator [5]

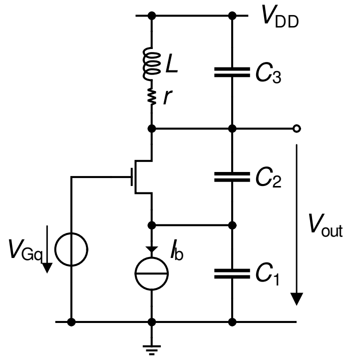

Figure 2: Colpitts oscillator.

Design the Colpitts oscillator shown in Fig. 4 for the following specifications:

$$f_0 = 2.4\,\text{GHz}, \quad C_2 = 1\,\text{pF}, \quad C_3 = 1\,\text{pF}, \quad Q_L = 10, \quad \hat{V}_{out} = 100\,\text{mV}$$

- Find the inductance value. [1]
- Find the critical source transconductance value. [1]
- Find the critical current value assuming the transistor is biased in weak inversion (take $n = 1.3$). [1]
- Find the bias current I_b for the specified amplitude assuming the transistor is biased in weak inversion. [2]

Solutions to Exam Spring Semester 2021

Multiple Choice Questions [8]

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Problem 1: Wideband LNA [11]

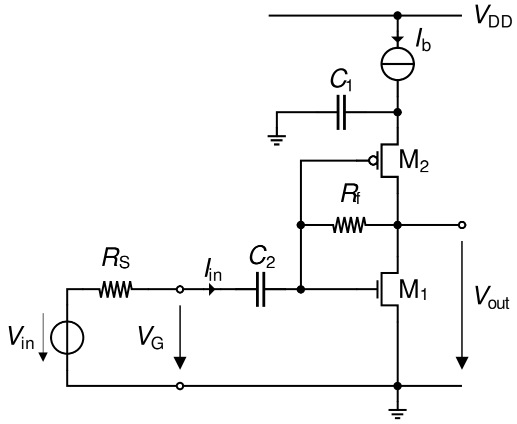

Figure 1: Current-reuse wideband LNA.

1.1 Small-signal Analysis [6]

- Draw the equivalent small-signal schematic of the circuit given in Fig. 1 assuming the transistors are biased in saturation. Assume that the capacitors C_1 and C_2 can be considered as shorts at the frequency of operation. [2]

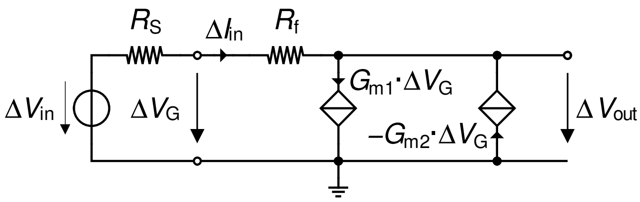

Figure 2: Wideband LNA.

The equivalent small-signal circuit corresponding to the current-reuse wideband LNA shown in Fig. 1 is given in Fig. 2. It shows that both VCCS G_{m1} and G_{m2} come in parallel and are controlled by the same voltage. They can hence be combined into a single VCCS having a transconductance $G_m \triangleq G_{m1} + G_{m2}$.

- Calculate the input impedance $Z_{in} \triangleq \Delta V_G/\Delta I_{in}$ assuming that the output conductances of M_1 and M_2 can be neglected. [1]

The input impedance Z_{in} is simply given by

$$Z_{in} \triangleq \frac{\Delta V_G}{\Delta I_{in}} = \frac{1}{G_m}. \tag{1}$$

- What is the condition for impedance matching? [1]

Impedance matching is met $Z_{in} = R_S$. From (1) we get:

$$G_{m\text{-}match} = \frac{1}{R_S}. \tag{2}$$

- Calculate the small-signal voltage gain $A_v \triangleq \Delta V_{out}/\Delta V_{in}$.
  Simplify then the resulting expression assuming that $G_{mi} \cdot R_f \gg 1$ and $G_{mi} \cdot R_S \gg 1$ for $i = 1, 2$. [1]

The voltage gain is given by

$$A_v \triangleq \frac{\Delta V_{out}}{\Delta V_{in}} = -\frac{G_m \cdot R_f - 1}{G_m \cdot R_S + 1} \cong -\frac{R_f}{R_S} \tag{3}$$

for $G_m \cdot R_f \gg 1$ and $G_m \cdot R_S \gg 1$.

- What is the value of A_v assuming impedance matching at the input? Simplify then the resulting expression assuming that $R_f \gg R_S$. [1]

From (2) and (3), we get

$$A_{v\text{-}match} = -\frac{1}{2} \cdot \left(\frac{R_f}{R_S} - 1\right), \tag{4}$$

which for $R_f \gg R_S$ reduces to

$$A_{v\text{-}match} \cong -\frac{R_f}{2R_S}. \tag{5}$$

1.1.1 Noise Analysis **[5]**

- Draw the equivalent small-signal circuit including all the noise sources. [1]

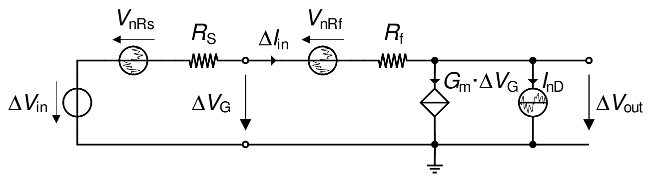

Figure 3: Current-reuse wideband LNA equivalent small-signal circuit including all the thermal noise sources.

The equivalent small-signal circuit including the noise sources is reproduced in Fig. 3 where I_{nD} includes both noise sources of transistors M_1 and M_2.

- Calculate the input-referred thermal noise resistance R_{neq}. To simplify, assume that both transistors M_1 and M_2 have identical noise excess factors ($\gamma_{nD1} = \gamma_{nD2} = \gamma_{nD}$). Simplify then the resulting expression assuming that $G_{mi} \cdot R_S \gg 1$ for $i = 1, 2$ and $R_f \gg R_S$. [2]

The input-referred noise resistance R_{neq} is given by

$$\begin{aligned} R_{neq} &= R_S + \left(\frac{G_m \cdot R_S + 1}{G_m \cdot R_f - 1}\right)^2 \cdot R_f + \left(\frac{R_f + R_S}{G_m \cdot R_f - 1}\right)^2 \cdot G_{nD} \\ &\cong R_S + \left(\frac{R_S}{R_f}\right)^2 \cdot R_f + \left(\frac{1}{G_m}\right)^2 \cdot G_{nD} \\ &= R_S + \frac{R_S}{R_f} \cdot R_S + \frac{\gamma}{G_m}, \end{aligned} \tag{6}$$

where $G_{nD} \triangleq G_{nD1} + G_{nD2}$ and

$$\gamma \triangleq \frac{\gamma_{nD1} \cdot G_{m1} + \gamma_{nD2} \cdot G_{m2}}{G_m} = \frac{\gamma_{nD1} \cdot G_{m1} + \gamma_{nD2} \cdot G_{m2}}{G_{m1} + G_{m2}}. \tag{7}$$

Assuming $\gamma_{nD1} = \gamma_{nD2}$, (7) reduces to

$$\gamma \cong \gamma_{nD1} = \gamma_{nD2}. \tag{8}$$

- Calculate the noise factor F. Simplify then the resulting expression assuming that $G_{mi} \cdot R_S \gg 1$ for $i = 1, 2$ and $R_f \gg R_S$. [1]

The noise factor is then given by

$$F = 1 + \left(\frac{G_m \cdot R_S + 1}{G_m \cdot R_f - 1}\right)^2 \cdot \frac{R_f}{R_S} + \left(\frac{R_f + R_S}{G_m \cdot R_f - 1}\right)^2 \cdot \frac{G_{nD}}{R_S} \cong 1 + \frac{R_S}{R_f} + \frac{\gamma}{G_m \cdot R_S}. \tag{9}$$

- Calculate the noise factor F assuming impedance matching. Simplify then the resulting expression assuming that $R_S \ll R_f$. [1]

Assuming impedance matching, expression (9) simplifies to

$$\begin{aligned} F_{match} &= 1 + \frac{4}{(R_f/R_S - 1)^2} \cdot \frac{R_f}{R_S} + \left(\frac{R_f + R_S}{R_f/R_S - 1}\right)^2 \cdot \frac{G_{nD}}{R_S} \\ &= 1 + \frac{4}{(R_f/R_S - 1)^2} \cdot \frac{R_f}{R_S} + \left(\frac{R_f/R_S + 1}{R_f/R_S - 1}\right)^2 \cdot \gamma \\ &\cong 1 + \frac{4R_S}{R_f} + \gamma. \end{aligned} \tag{10}$$

Problem 2: Colpitts Oscillator [5]

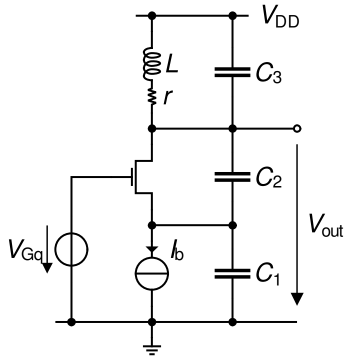

Figure 4: Colpitts oscillator.

Design the Colpitts oscillator shown in Fig. 4 for the following specifications:

$$f_0 = 2.4\,\text{GHz}, \quad C_2 = 1\,\text{pF}, \quad C_3 = 1\,\text{pF}, \quad Q_L = 10, \quad \hat{V}_{out} = 100\,\text{mV}$$

- Find the inductance value. [1]

To find the inductance value we first need to know the value of C_1. We know that G_{mscrit} and hence the power consumption are minimum for $C_1 = C_2$, hence $C_1 = 1\,\text{pF}$. The inductance is approximately given by

$$L \cong \frac{1}{\omega_0^2 \cdot (C_3 + C_{12})} \tag{11}$$

where $\omega_0 = 2\pi f_0$ and $C_{12} = C_1 \cdot C_2/(C_1 + C_2) = 0.5\,\text{pF}$. This leads to $L_a = 2.932\,\text{nH}$.

- Find the critical source transconductance value. [1]

The critical source transconductance is given by

$$G_{mscrit} \cong \frac{\omega_0}{Q_L} \cdot (C_1 + C_2) \cdot \left(1 + \frac{C_3}{C_{12}}\right) = 9.048\,\text{mA/V}. \tag{12}$$

- Find the critical current value assuming the transistor is biased in weak inversion (take $n = 1.3$). [1]

The critical current assuming the transistor is biased in weak inversion is then given by

$$I_{crit} = G_{mscrit} \cdot U_T = 234.11\,\mu\text{A}, \tag{13}$$

where $U_T \triangleq kT/q = 25.875\,\text{mV}$ is the thermodynamic voltage.

- Find the bias current I_b for the specified amplitude assuming the transistor is biased in weak inversion. [2]

The amplitude ΔV_{out} is specified at the output (drain of M1), whereas the control voltage corresponds to the source. The amplitude at the source is therefore given by

$$A = \frac{C_2}{C_1 + C_2} \cdot \Delta V_{out} = 50\,\text{mV}. \tag{14}$$

The normalized amplitude $x \triangleq A/U_T = 1.932$. The bias current is then given by

$$I_b = I_{crit} \cdot \chi \tag{15}$$

with χ defined as

$$\chi \triangleq \frac{x \cdot I_{B0}(x)}{2 I_{B1}(x)}. \tag{16}$$

where $I_{B0}(x)$ and $I_{B1}(x)$ are the modified Bessel functions of the first kind of order 0 and 1 respectively. The factor χ can be found from the abacus or calculated as $\chi = 1.408$, resulting in a bias current of $I_b = 329.56\,\mu\text{A}$.