Exercise 3.1: Models for the permittivity of metals (5-7 min)

For metals, the permittivity is generally a complex quantity ($\epsilon_M = \epsilon' + i\epsilon''$). A very simple model can be obtained from the fourth Maxwell-equation, the modified Ampere-law and assuming that the relation between $\vec{D}$ and $\vec{E}$ is described by the permittivity ϵ:

$$\nabla \times \vec{H} = \vec{j} + \epsilon\epsilon_0 \cdot \partial\vec{E}/\partial t$$

a) Get rid of the time-derivative by assuming a harmonic time dependence of the form $\exp(-i\omega t)$ and express current density with the conductivity by $\vec{j} = \sigma\vec{E}$. Identify the following relations:

$$\epsilon' = \epsilon$$

$$\epsilon'' = \frac{\sigma}{\omega\epsilon_0}$$

Drude described the conductivity in metals by free electrons that can follow variations of the electric field only up to a certain limiting frequency. Thus, the permittivity is determined by a relaxation time τ and the plasma frequency ω_p.

$$\epsilon_M = 1 - \frac{\omega_p{}^2}{\omega(\omega + i/\tau)}$$

The plasma frequency depends on the density of conduction-electrons N, their effective mass is m^* and their charge e by means of $\omega_p = \sqrt{e^2 N/m^*\epsilon_0}$.

b) Separate ϵ_M into real and imaginary parts and compare the high- and low-frequency limits of your result for the imaginary part with the result of the quasi-static case.

c) Find a data for a typical metal such as silver or aluminium, plot on a convenient scale and identify the transition between the models.

Task: Show only the key points of the derivation, minimise the use of formulae. Focus on the discussion of the data.

Exercise 3.2: Selective emitter (5-7 min)

In the development of c-Si solar cells, much of effort was devoted to the front contact. Highly diffused emitters like the phosphorous diffusion profiles shown in the course were already very early replaced by *passivated emitters*, and eventually further improved on by the introducing *selective emitters*.

a) Design a sketch of the front region of a c-Si solar cell, showing the *pn*-junction between wafer and the diffused region, the local contacts to the silver finger metallisation, and the passivated region between the fingers.

b) Using the diagram below,[1] explain the working principle of a passivated emitter. Discuss what motivated the development of passivated emitters.
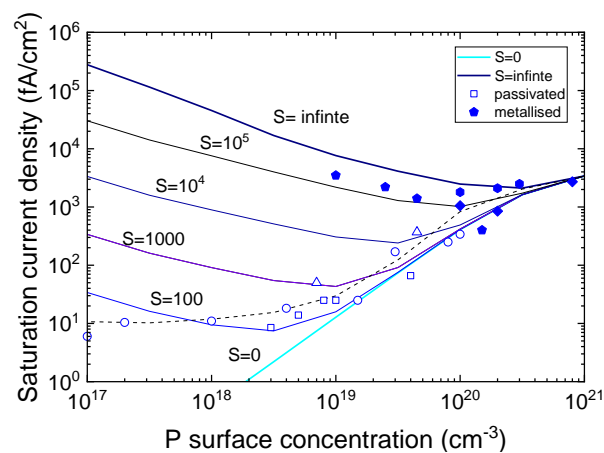
c) Assume a *passivated emitter* with reduced surface concentration of $N_D = 10^{19}$ cm^{-3}. Project the j_0 by using an area weighted sum of $j_{0,met}$ and $j_{0,pass}$, assuming that the silver fingers cover an area of 10%.

d) Explain the working principle of a *selective emitter* that combines highly doped regions below the fingers and lowly doped regions with passivation. Point out the additional improvement that is possible.

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[1] The symbols refer to experimental data digitized from King, TED (1980) and from Kerr, JAP (2001). The lines refer to a simple model with the geometry factor G_F, assuming constant donor density N_D equal to the surface concentration.

Exercise 3.3: Efficiency limit of crystalline silicon (3 people, 15-20 min)

In this group exercise for three people, you will derive the upper limit for crystalline silicon. The derivation splits into three tasks which you have to combine.

Basic idea: For high efficiency we have to create a high photocurrent, but this requires a large device thickness because silicon is a weak absorber. Consequently, the generation would spread over a large volume, resulting in a low injection level and thus a low V_{oc}. A thinner cell will generate a lower photocurrent, but it may have a higher V_{oc}. Construct a (shared) spreadsheet that finds the optimum of this trade-off. For the presentation, focus on explaining your reasoning and show a only minimum of formulae.

a) Determine the maximum photocurrent for a given cell thickness.
    Calculate the absorption with the multipass formula of Deckman, APL (1983) with the absorption coefficient tabulated in Green, SolMat (2008), assuming zero reflection at the front and zero absorption at the rear. Convolute the result with the AM1.5 spectrum
    https://rredc.nrel.gov/solar//spectra/am1.5/ASTMG173/ASTMG173.html and integrate in the relevant wavelength range for silicon. Repeat for different thicknesses, your result should look like the Lambertian shown in the course. Approximate the generation rate by a constant average through $G = j_{ph}/d$.
    Task: Create a table of cell thickness d vs. G for your colleague working on b).

b) Find a relation between the generation rate G and the injection level Δn.
    Assume you receive a table in which one column contains G. Using p-type material that is doped with $N_A = 10^{16}$ cm^{-3}, consult Richter, PRB (2012) for a parametrisation of radiative and Auger recombination mechanisms, e.g. their eq. (22). Noting that the generation rate is given by $G = \Delta n/\tau(\Delta n)$ and thus essentially a function of Δn, define a procedure to find the inverse function $\Delta n = \Delta n(G)$. To do so analytically, find the dominating recombination process and use a convenient approximation (hint: radiative and Auger processes transit from a constant value at low Δn into branches proportional to Δn^{-1} and Δn^{-2} at high Δn, respectively.). Alternatively, if you prefer working with the general description, you may define a look-up table.
    Task: Extend the table with a column that contains $\Delta n(G)$ for your colleague working on c).

c) Find the limiting efficiency.
    Assume you receive a table in which one column contains Δn. Find the implied V_{oc} which is equal to the quasi-Fermi level splitting $iV_{oc} = \Delta_{QFL} = kT/q \ln(np/n_i^2)$. Use the relations $n = n_0 + \Delta n = n_i^2/N_A$ and $p = p_0 + \Delta n = N_A + \Delta n$ with the doping concentration N_A given in part b). Next, consult Green, Solar Energy (1982) to relates the fill factor to the V_{oc}. In line with finding an upper limit for the efficiency, you may assume zero series-resistance and infinite parallel-resistance.
    Task: Extend the table by three more columns, containing iV_{oc}, the fill factor, and finally the projection of the efficiency given by $\eta = j_{ph} \cdot V_{oc} \cdot FF_0$.

All: Plot the efficiency η vs. the device thickness d and discuss the result.

Exercise 3.4: Schottky barrier (5-7 min)

The front contact in hetero-junction solar-cells is established between a highly n-doped ITO layer and a p-doped layer. In the depletion approximation, we assume that mobile charges recombine across the interface, leaving behind ionized cores. The depletion-regions are thus charged positively in the n-type ITO and negatively in the p-doped layer.
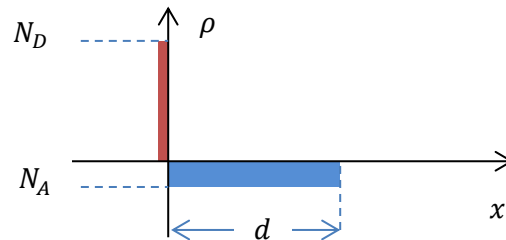
In the p-doped layer, we assume that the depletion zone extends over a width d that is less than the film thickness. Throughout this depleted region, we may assume a negative charge density equal to the acceptor concentration N_A. Since the ITO is highly doped, its depletion zone is very narrow and can be treated like a surface charge. The result is a one-sided p-n junction, similar to a Schottky-junction.

a) Applying the 1D Poission-equation $d^2\phi/dx^2 = qN_A/\epsilon\epsilon_0$, you can find a relation for the electric field E by recognizing that $d\Phi/dx = -E$. Integrate once and evaluate the boundary condition that the field vanishes at the edge of the depletion zone ($E(d) = 0$).

b) Find the electrostatic potential by carrying out a second integration. Determine the width of the depletion layer for a known height of the potential barrier V_b.

c) Find experimental data for barrier heights between metals and silicon, e.g. Schroder, TED (1984). Compare with theoretical values based on the work function.

Task: Show only the key results of the derivation with a minimum of formulae