

## Renewable Energy: Energy Storage solution

In this exercise, you will learn about energy storage solutions.

---

### 1. Application of Flywheels in Cars

(a) Kinetic Energy:  $E_{kin} = \frac{1}{2}M \cdot \nu^2 \approx 320 \text{ kJ} \approx 0.089 \text{ kWh}$

(b) Losses due to air drag:  $P_{air} = F_{air} \cdot \nu = \frac{1}{2}\rho_{air} \cdot c_d \cdot A_{front} \cdot \nu^3 \approx 4.5 \text{ kW}$

(c) Necessary  $E_{flywheel} = \frac{1}{\eta}(E_{kin} + P_{air} \cdot \frac{d_{range}}{\nu}) \approx 35.1 \text{ MJ} \approx 9.8 \text{ kWh}$

(d) In a car, there is only space for wheels with a radius R of up to 70 cm. Therefore R is set to 70 cm.

The maximal angular frequency is  $\omega = \frac{2}{R} \sqrt{\frac{\sigma_{CFP}}{\rho_{CFP}} K} \approx 2568 \text{ rad/s} \approx 24525 \text{ U/min}$

Comment: This is a rather high value, which probably causes additional losses due to aerodynamic and bearing drag.

The rotational energy of a disc with radius R and constant thickness D is

$$E_{flywheel} = \frac{1}{2}\Theta \cdot \omega^2 = \frac{1}{2}\omega^2 \int_V r^2 \cdot \rho_{CFP} \cdot dV = \frac{1}{2}\omega^2 \cdot 2\pi \cdot D \cdot \rho_{CFP} \int_0^R r^3 \cdot dr = \frac{\pi}{4}\rho_{CFP} \cdot \omega^2 \cdot D \cdot R^4$$

According to (c), each flywheel has to store  $E_{flywheel} = 17.55 \text{ MJ}$ . So now, the thickness D of one flywheel can be calculated:

$$D = \frac{4E_{flywheel}}{\pi \cdot \rho_{CFP} \cdot \omega^2 \cdot R^4} \approx 9.4 \text{ mm}$$

The mass of both flywheels is accordingly  $m = 2\rho_{CFP} \cdot \pi \cdot R^2 \cdot D \approx 43.4 \text{ kg}$

(e) The pair of flywheels should store the kinetic energy of a car moving at a speed of 120 km/h:

$$2E_{flywheel} = E_{kin} = \frac{1}{2}M \cdot \nu^2 \approx 720 \text{ kJ} \approx 0.20 \text{ kWh}$$

Losses due to air resistance are neglected here. There is less space for a supplementary device. As a consequence, the radius of the flywheels R is set to 30 cm.

The maximal angular frequency is  $\omega = \frac{2}{R} \sqrt{\frac{\sigma_{CFP}}{\rho_{CFP}} K} \approx 6000 \text{ rad/s} \approx 57000 \text{ U/min}$

Thickness of each flywheel  $D = \frac{4E_{flywheel}}{\pi \cdot \rho_{CFP} \cdot \omega^2 \cdot R^4} \approx 1.1 \text{ mm}$

The mass of both flywheels is accordingly  $m = 2\rho_{CFP} \cdot \pi \cdot R^2 \cdot D \approx 0.89 \text{ kg}$

2. Pumped air storage:

- (a) Uncompressed air:  $p_0 \approx 1 \text{ bar} \approx 100 \text{ kPa}, T_0 \approx 25 \text{ }^\circ\text{C}$   
 Compressed air (gas tank):  $p_1 \approx 300 \text{ bar} \approx 30 \text{ MPa}, T_1 = T_0 \approx 25 \text{ }^\circ\text{C}$   
 Released air:  $p_2 = p_0 \approx 1 \text{ bar} \approx 100 \text{ kPa}, T_2 < T_0$   
 Isothermal process:  $p \cdot V = n \cdot R \cdot T = \text{const.}$  or  $V(p) = \frac{n \cdot R \cdot T}{p}$   
 Adiabatic process:  $p \cdot V^\kappa = \text{const.}$  or  $V(p) = V_1 \cdot \left(\frac{p_1}{p}\right)^{1/\kappa} = \frac{n \cdot R \cdot T_1}{p_1} \cdot \left(\frac{p_1}{p}\right)^{1/\kappa}$

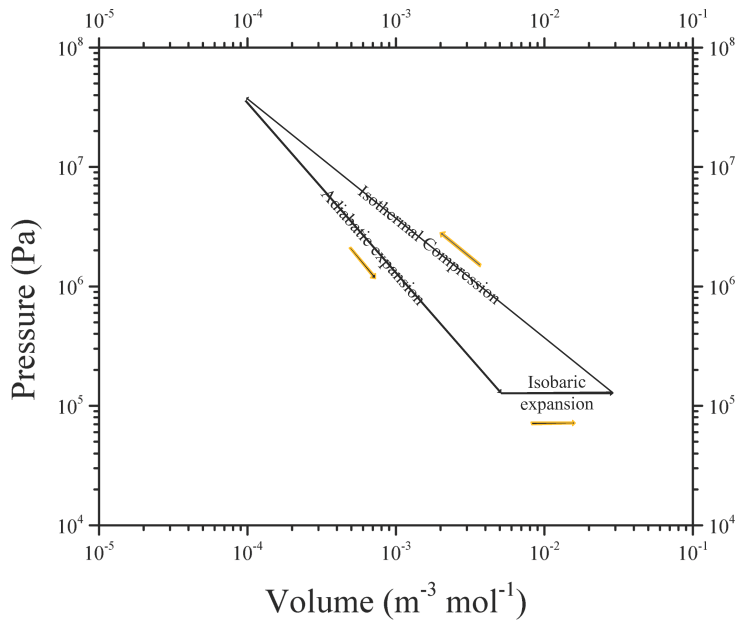


Figure 1: P-V diagram

- (b) Isothermal compression work:

$$W_{comp} = - \int_0^1 p \cdot dV = - \int_{p_0}^{p_1} p \frac{dV}{dp} \Big|_{isothermal} dp$$

$$= nRT_0 \int_{p_0}^{p_1} \frac{dp}{p} = nRT_0 \cdot \ln\left(\frac{p_1}{p_0}\right) \approx 14.1 \text{ kJ/mol}$$

Adiabatic expansion work:

$$W_{exp1} = - \int_1^2 p \cdot dV = - \int_{p_1}^{p_0} p \frac{dV}{dp} \Big|_{adiabatic} dp$$

$$= \frac{p_1^{1/\kappa} \cdot V_1}{\kappa} \int_{p_1}^{p_0} p^{-1/\kappa} \cdot dp = \frac{p_1^{1/\kappa} \cdot V_1}{\kappa} \frac{\kappa}{\kappa - 1} (p_0^{\frac{\kappa-1}{\kappa}} - p_1^{\frac{\kappa-1}{\kappa}}) = nRT_1 \frac{p_1^{\frac{1-\kappa}{\kappa}}}{\kappa - 1} (p_0^{\frac{\kappa-1}{\kappa}} - p_1^{\frac{\kappa-1}{\kappa}}) =$$

$$\frac{nRT_1}{\kappa - 1} \left( \left( \frac{p_0}{p_1} \right)^{\frac{\kappa-1}{\kappa}} - 1 \right) \approx -5.0 \text{ kJ/mol}$$

Isobaric expansion work:

$$W_{exp2} = - \int_2^0 p \cdot dV = -p_0(V_0 - V_2) = nRT_0 \left( \left( \frac{p_0}{p_1} \right)^{\frac{\kappa-1}{\kappa}} - 1 \right) \approx -2.0 \text{ kJ/mol}$$

Losses:  $W_{losses} = W_{comp} - W_{exp1} - W_{exp2} \approx 7 \text{ kJ/mol}$

$$\text{Efficiency: } \eta = \frac{W_{exp1} + W_{exp2}}{W_{comp}} \approx 50\%$$

(c) From Problem 1c:

$$\text{Energy needed for 120 km: } E_{drive} = P_{air} \cdot \frac{d_{range}}{\nu} \approx 24.2 \text{ MJ}$$

Released work from pumped air storage:  $W_{released} = W_{exp1} + W_{exp2} \approx 7.0 \text{ kJ/mol}$

$$\rightarrow \text{Minimal amount of air : } n = \frac{E_{drive}}{W_{released}} \approx 3478 \text{ mol}, V_{air} = \frac{R \cdot T_1}{p_1} \frac{E_{drive}}{W_{released}} \approx 0.287 \text{ m}^3$$

There should be enough space in a car for a 300 litre tank.

3. Pumped water storage:

(a) Potential energy of 1 m<sup>3</sup> water:  $E_{pot} = m \cdot g \cdot \Delta h = 1000 \cdot 9.81 \cdot 1000 \approx 9.81 \text{ MJ}$

Annual production of 100 MW<sub>p</sub> PV plant:

$$E_{prod} = \eta \cdot P_p \cdot t = 0.15 \cdot 10^8 \cdot 365 \cdot 24 \cdot 3600 \approx 4.7 \cdot 10^{14} \text{ J}$$

$$\text{Amount of water: } V_{water} = \eta_{pump} \cdot \frac{E_{prod}}{E_{pot}} = 0.85 \cdot \frac{4.7 \cdot 10^{14}}{9.8 \cdot 10^6} \text{ m}^3 \approx 4.1 \cdot 10^7 \text{ m}^3$$

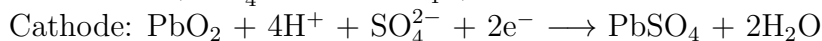
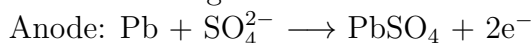
(b) Annual production of 100 MW<sub>av</sub> PV plant:

$$E_{prod} = P_{av} \cdot t = 10^8 \cdot 365 \cdot 24 \cdot 3600 \approx 3.2 \cdot 10^{15} \text{ J}$$

$$\text{Amount of water: } V_{water} = \eta_{pump} \cdot \frac{E_{prod}}{E_{pot}} = 0.85 \cdot \frac{3.2 \cdot 10^{15}}{9.8 \cdot 10^6} \text{ m}^3 \approx 2.7 \cdot 10^8 \text{ m}^3$$

4. Batteries:

(a) for the discharge:



(b) Equation for the electrochemical equilibrium:  $U^0 = \Delta E^0 = -\frac{\Delta G^0}{z \cdot F}$   
 $\Delta G^0$  for Pb-Acid and F are given, it is possible to see from point a) that  $z=2$ .

$$\rightarrow U^0 = 2.047 \text{ V}$$

If a 24 V battery is required, a series of at least 12 Pb-Acid cells is needed  $\rightarrow$  =c.a 24.56 V

- (c) How many moles of Pb got converted? (= moles of PbSO<sub>4</sub> formed on the anode only)

$$n_{C_d} = m_{C_d} / M_{C_d} = \frac{11.6g}{207.2g/mol} = 0.056 \text{ mol}$$

With the help of the Faraday constant (which defines the mol-specific charge of matter), we can now calculate the overall charge in Anode side we get, when the 56 mmol are converted. Note from the half-cell reaction, that there are 2 electrons involved when 1 Pb is converted.

$$F = \frac{Q_0}{z \cdot n} \quad Q_0 = F \cdot z \cdot n = 96485 \text{ A.s/mol} \cdot 2 \cdot 0.056 \text{ mol} = 10803 \text{ C}$$

To determine the time it will take to recharge the battery, we divide the charge by the given current:

$$10803 \text{ A.s} / 1.5 \text{ A} = 7202.2 \text{ s} = 2.0 \text{ h}$$

- (d) For obtaining the mass specific charge Q in Ah/kg we use the Faraday law again. Note, that all the charge-carrying species (educts, left side of the overall reaction equation) are involved in the calculation by their molar masses:

$$Q = \frac{z \cdot F}{\sum_i M_i}; \sum_i M_i = 1 \cdot M(\text{Pb}) + 1 \cdot M(\text{PbO}_2) + 2 \cdot M(\text{H}_2\text{SO}_4)$$

From the given molar masses for Pb,O,S,H to be 207.2, 16, 32, 1 g/mol respectively, it is possible to obtain:  $\sum_i M_i = 642.4 \text{ g/mol}$

Having in mind that z is still 2, the specific charge now calculates to  $Q = 300.39 \text{ C/g} = 83.44 \text{ Ah/kg}$ .

The energy density can be obtained from the charge density (= mass specific charge) by multiplying by the reversible cell voltage, since voltage U[V].current I[A] = Power P[W] and Power P[W].time t[h] = Energy E[Wh]:

$E = Q \cdot U^0$ ; using  $U^0$  from above = 2.047 V, it follows:  $E = 170.8 \text{ Wh/kg}$ .

- (e) i. Equation for the electrochemical equilibrium:  $U^0 = \Delta E^0 = -\frac{\Delta G^0}{z \cdot F}$ ,  
 $\rightarrow U^0 = 4.20 \text{ V}$ .

- ii.  $Q = \frac{z \cdot F}{\sum_i M_i}; \sum_i M_i = 1 \cdot M(\text{LiC}_6) + 1 \cdot M(\text{CoO}_2) = 169.8 \text{ g/mol}; z=1$   
 $\rightarrow Q_{\text{Li-ion}} = 157.84 \text{ Ah/kg} \rightarrow U_{\text{Li-ion}}^0 \rightarrow E_{\text{Li-ion}} = 662.54 \text{ Wh/kg}$   
compare:  
 $\rightarrow Q_{\text{Pb-Acid}} = 83.44 \text{ Ah/kg} \rightarrow U_{\text{Pb-Acid}}^0 \rightarrow E_{\text{Pb-Acid}} = 170.8 \text{ Wh/kg}$

- iii. reason 1): reversible cell voltage has doubled
- reason 2): less weight of the charged electrode and electrolyte
- both parameters bring big advantage in salability of a battery system