

ΠCAA lecture 3: quiz solutions

- 1) a) the chain is finite, irreducible but periodic with period 2
 \Rightarrow admits a unique stat. dist., but no limiting dist.
- b) the chain is finite, irreducible and aperiodic (self-loop)
 \Rightarrow admits a unique limiting and stat. dist.
- c) only state 2 is recurrent \Rightarrow the chain admits a unique limiting and stat. dist. (concentrated in state 2)
- d) the chain has two recurrent classes $\{1\}$ & $\{3\}$
 \Rightarrow the chain admits multiple stat. dist. (& no limiting dist.)

1) e) The chain is finite, irreducible and aperiodic
($\gcd(2,3)=1$) \Rightarrow it admits a unique limiting
& stat. dist.

f) The chain is finite, irreducible but periodic with
period 3 \Rightarrow it admits a unique stat. dist.,
but no limiting dist.

2) a) Both statements a1) & a2) are correct:

a1) a finite chain does not have null-recurrent states

a2) use the first theorem seen in the lecture

- 2) b) This statement is wrong: finite & irreducible implies positive-recurrent, so by the first theorem, the chain admits a unique stationary distribution, but not necessarily a limiting distribution
- c) This statement is wrong: the chain could be made of two positive-recurrent classes, for example
- d) Any chain can always be decomposed into equivalent classes; if the chain admits a stationary distribution, this means one of these equivalent classes must have a non-zero weight with this distribution \Rightarrow the states in this class are positive-recurrent; the statement is correct.