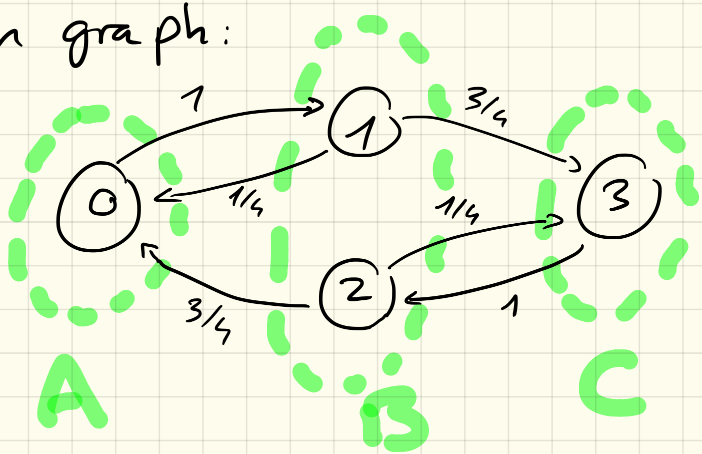
Comment: Aggregating the states of a Markov chain into "superstates" can lead to a new process which is not a Markov chain.

Here is an example: Consider first the Markov chain $(X_n, n \geq 0)$ with state space $S = \{0, 1, 2, 3\}$ and transition graph:



Consider now the process $(Y_n, n \geq 0)$ with state space $S' = \{A, B, C\}$ and the correspondence:

$$Y_n = A \longleftrightarrow X_n = 0, \quad Y_n = B \longleftrightarrow X_n = 1 \text{ or } 2, \quad Y_n = C \longleftrightarrow X_n = 3$$

The process $(Y_n, n \geq 0)$ is not a Markov chain:

$$\mathbb{P}(Y_{n+1} = A \mid Y_n = B, Y_{n-1} = A) = \frac{1}{4}$$

while

$$\mathbb{P}(Y_{n+1} = A \mid Y_n = B, Y_{n-1} = C) = \frac{3}{4}$$

The problem is that in the first case, $Y_n = B$ means actually $X_n = 1$ (because $Y_{n-1} = A$, i.e. $X_{n-1} = 0$), while in the second case, it means $X_n = 2$ (because $Y_{n-1} = C$, i.e. $X_{n-1} = 3$).