MCAA lecture 7 : quiz

1) When μ and ν are two distributions on $S = \{0,1\}$, which of the following equalities hold?

a) $\| \mu - \nu \|_{TV} = |\mu_0 - \nu_0| + |\mu_1 - \nu_1|$

b) $\| \mu - \nu \|_{TV} = |\mu_0 - \nu_0|$

c) $\| \mu - \nu \|_{TV} = |\mu_1 - \nu_1|$

d) $\| \mu - \nu \|_{TV} = \sup_{i \in \{0,1\}} |\mu_i - \nu_i|$

e) $\| \mu - \nu \|_{TV} = \frac{1}{2} \sup_{x_0, x_1 \in [-1, +1]} |\mu_0 x_0 + \mu_1 x_1 - \nu_0 x_0 - \nu_1 x_1|$

f) $\| \mu - \nu \|_{TV} = \sup_{x_0 \in [-1, +1]} |\mu_0 x_0 - \nu_0 x_0|$

Subsidiary question: Is it possible that $\| \mu - \nu \|_{TV} = 1$? when?

2) For the Markov chain with state space $S = \{0, 1\}$
and transition matrix $P = \begin{pmatrix} 1-p & p \\ q & 1-q \end{pmatrix}$, we have (see hw 2):

$$p_{00}(n) = \frac{q + p(1-p-q)^n}{p+q} \qquad \text{and} \qquad \pi_0 = \frac{q}{p+q}$$

For which values of the parameters p, q does it hold that
$\| P_0^n - \pi \|_{TV} = 0 \quad \forall n \geq 1$?

a) $p = q = \frac{1}{2}$ only   b) all values of p, q such that $p = q$
c) all values of p, q such that $p+q = 1$ d) all $p \& q$

Subsidiary question: Are there values of p, q for
which you observe a cut-off phenomenon ?