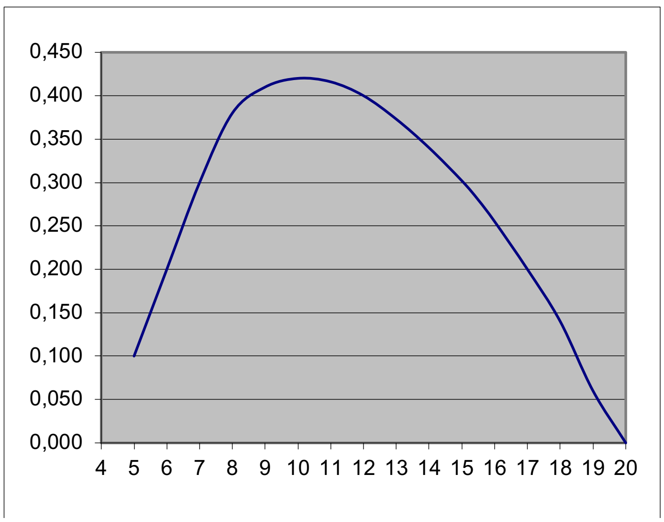
Determine operating parameters of a wind turbine, rotating at variable angular speed, from the Cp-λ characteristic

Consider a wind turbine designed to operate at variable angular speeds in order to maximize the energy extracted from the wind (C_p=const.=$C_{p\ max}$) between the *cut-in speed* ($v_{cut\text{-}in}$ = 5 m/s) and the *rated speed* (v_{rated}). The power from the rated speed onwards remains then constant (=rated power) up to the *maximal admissible cut-out speed* ($v_{cut\text{-}out}$= 17 m/s). The diameter of the wind turbine is 34 m and its rated power ($\dot{W}_{nom}$) 310 kW ; the C_p variation with λ is given below (remember that : $\dot{W} = \frac{1}{2} \cdot \rho \cdot \pi R^2 \cdot C_p \cdot v^3$; λ : tip speed ratio).

$C_{p\ max} = C_p$ (λ =10) = 0.42.



λ	5,0	6,0	7,0	8,0	9,0	10,0	11,0	12,0	13,0	14,0	15,0	15,5	16,0	17,0	18,0	19,0	20,0
C_p	0,10	0,20	0,30	0,38	0,41	0,42	0,416	0,40	0,373	0,340	0,302	0,28	0,255	0,20	0,14	0,06	0,0

The exploited wind regime (5-17 m/s, operating hours) is given below.

v[m/s]	5	6	7	8	9	10	11	12	13	14	15	16	17
t[h/yr]	1212	1200	1092	948	780	600	480	312	240	144	96	60	36

Air density: ρ = 1.22 kg/m^3.

Determine the *rated speed* (C_p = $C_{p\ max}$ at the rated power):

Rated speed: $v_{nom} = \{P_{nom}/\{(1/2)\cdot\rho\cdot\pi\cdot R^2\cdot C_p\}\}^{1/3}$
= {310'000 [W]/(0.5·1.22 [kg/m^3]·□π·(17 [m])^2·0.42)}$^{1/3}$ ={310'000 [W]/(553.8 [kg/m]·0.42)}$^{1/3}$ = **11.0 [m/s]**

Complete the table below (*T* is the couple (Torque) obtained from the wind):

$\lambda = \omega \cdot R/v$ $T = P/\omega$

v [m/s]	C_p [-]	P [kW]	ω [rad/s]	λ [-]	T [kN]	W [kWh]
5	0,420	29,08	2,94	10,00	9,89	35'240
6	0,420	50,24	3,53	10,00	14,24	60'292
7	0,420	79,78	4,12	10,00	19,38	87'125
8	0,420	119,10	4,71	10,00	25,31	112'903
9	0,420	169,57	5,29	10,00	32,03	132'266
10	0,420	232,61	5,88	10,00	39,54	139'566
11	0,420	309,60	6,47	10,00	47,85	148'609
12	0,324	310,00	5,15	7,30	60,17	96'720
13	0,255	310,00	5,01	6,60	61,91	74'400
14	0,204	310,00	4,97	6,05	62,32	44'640
15	0,166	310,00	4,99	5,70	62,09	29'760
16	0,137	310,00	5,05	5,40	61,38	18'600
17	0,114	310	5,14	5,14	60,32	11'160
					Total →	**991'282**

STEPS:

1. Maintain C_p constant (maximal, 0.42) from 5 to 11 m/s → compute P
2. Given (1), also λ is constant (=10) from 5 to 11 m/s
3. Power P is kept constant (310 kW) in the range from 11 to 17 m/s → compute C_p
4. Compute λ from 12 to 17 m/s by interpolating its value from the Cp- λ curve, choosing the lower λ values (power is limited by reducing the turbine rotation speed)
5. Compute ω from the tip speed ratio formula for the whole range 5-17 m/s
6. Compute Torque T from P and ω
7. Compute W from P times t (operating hours in the bottom table on previous page)

Observations:

1. The turbine rotation speed ω stays ~constant (5 rad/s) in the constant power range at 310 kW (11-17 m/s) – and thus also the torque. At speeds above the rated one (11 m/s), the power coefficient Cp is below maximum (mainly tip and wake losses), and turbine power is maintained constant.
2. At wind speeds v below the nominal one, the turbine rotation speed ω is reduced from the nominal one (from 5 to 3 rad/s), to maintain λ constant (10) at max Cp (0.42).
3. Over the whole range, ω varies between 3 and 6.5 rad/s, i.e. between roughly 0.5 and 1 full rotation/sec. (1 rotation = 2.Pi = 6.28 rad/s)
4. Most energy (W) is extracted at the nominal wind speed, or just below (9-11 m/s).