

Introduction II

Outlines

Introduction II

- Our galaxy, the Milky Way
- The Local Group and beyond
- Luminosity Distribution Function

- The Hubble-De Vaucouleurs Sequence
 - Elliptical galaxies
 - Spiral Galaxies
 - Lenticular Galaxies
 - Irregular galaxies

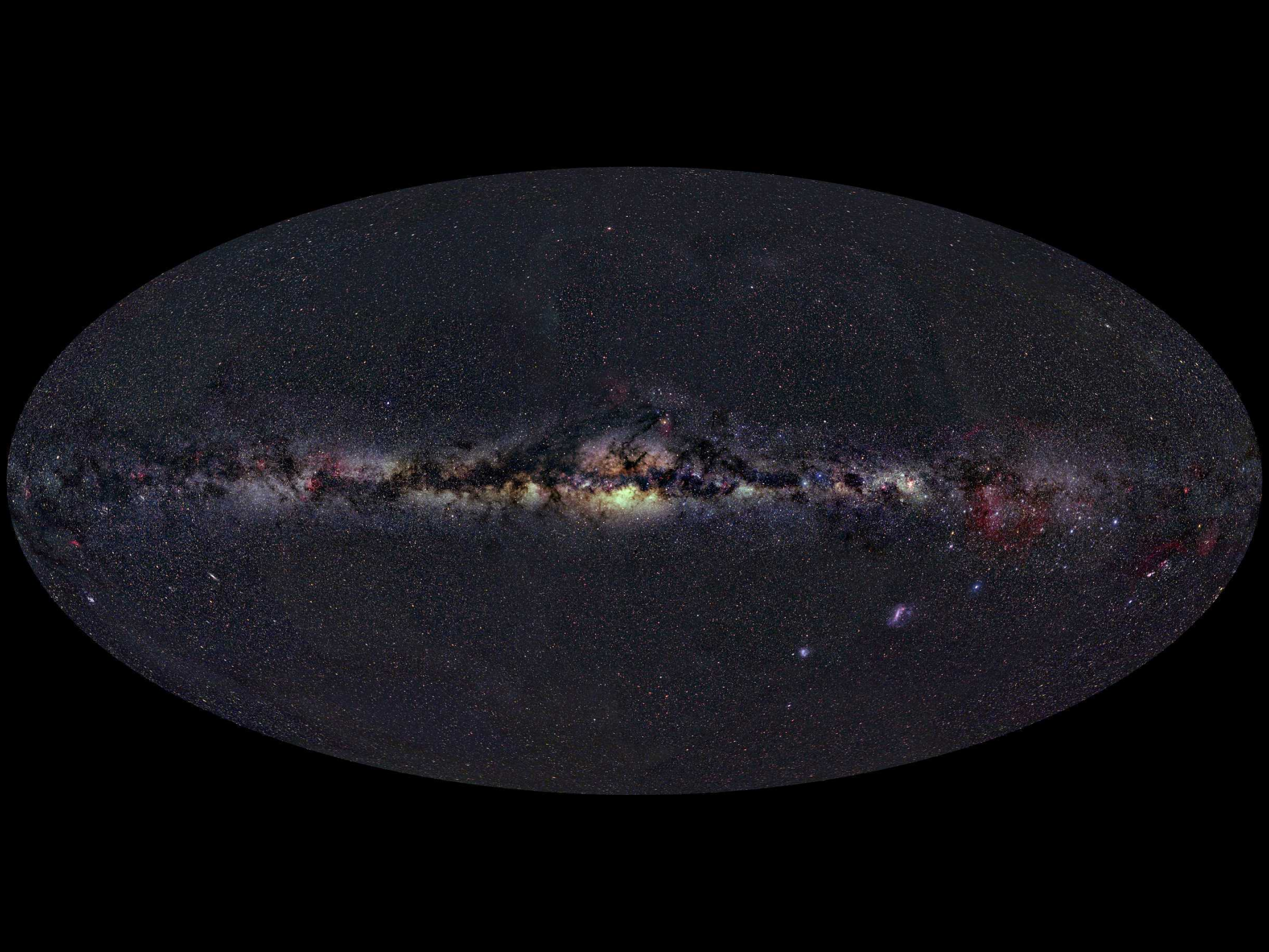
- The Hubble-Lemaître Law
- The Cosmic star formation history

The gravity : a long distance force

- collision-less systems
- the relaxation time

Introduction

Our galaxy
The Milky Way

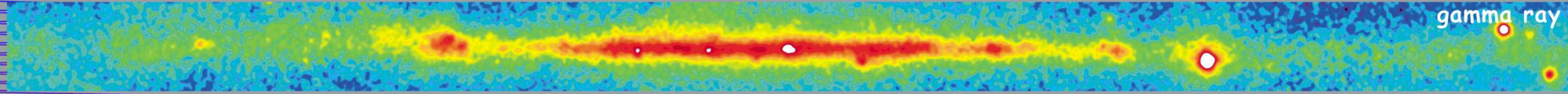
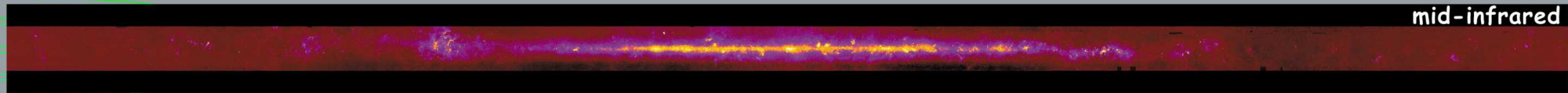
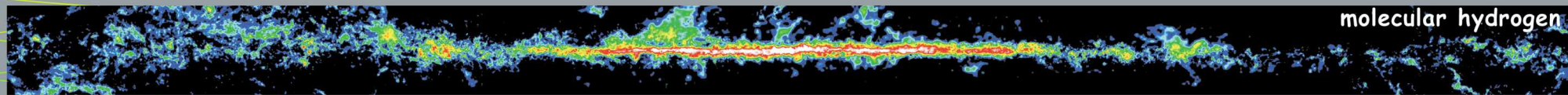
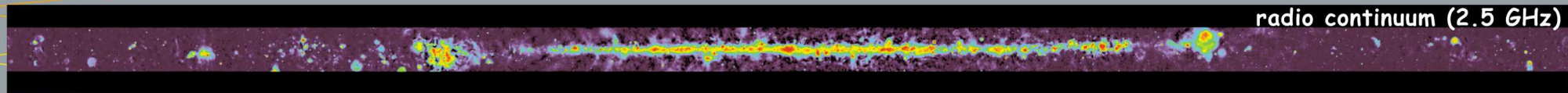
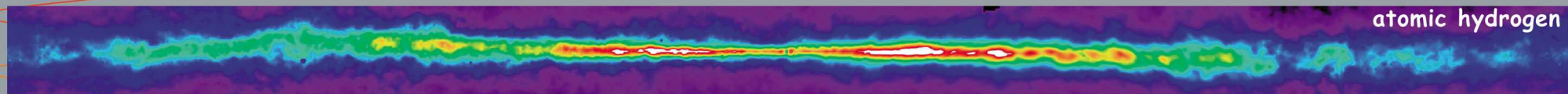
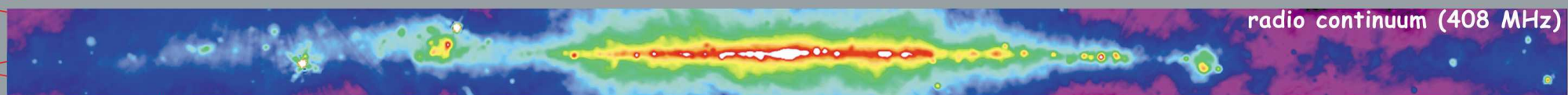


The Milky Way in different wavelength



The Milky Way in different wavelength



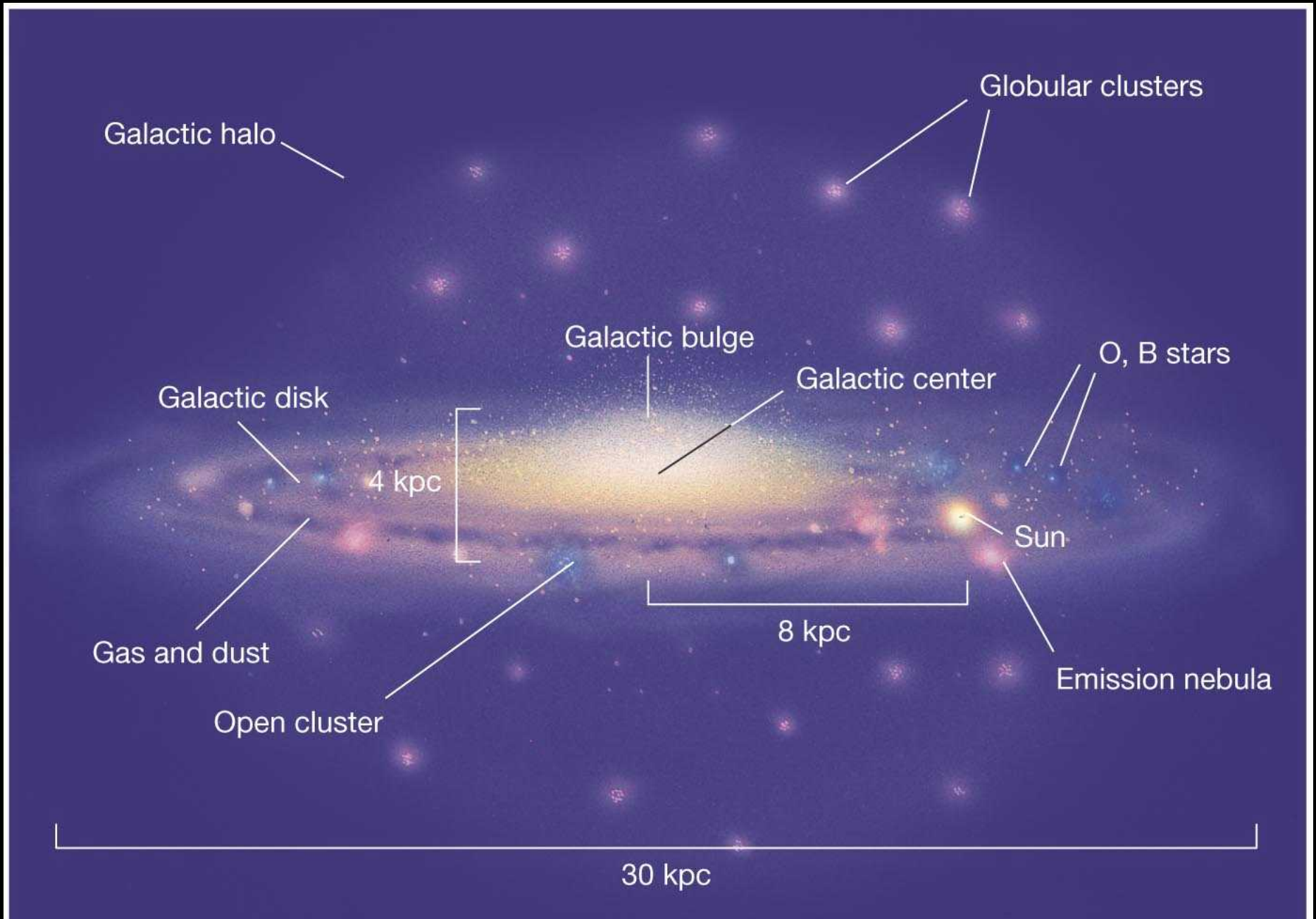


<http://adc.gsfc.nasa.gov/mw>



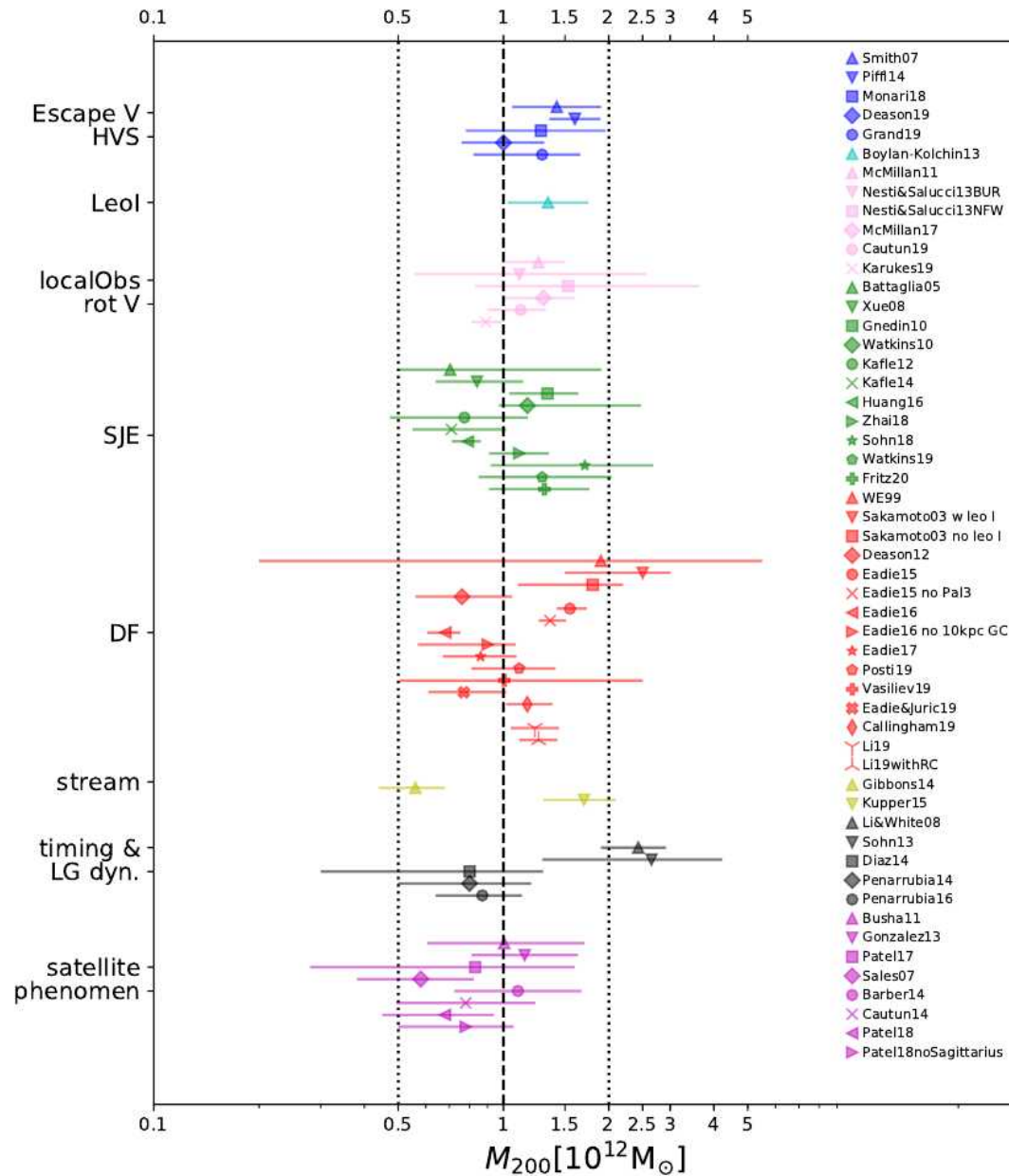
Multiwavelength Milky Way

Components of the WM



The Milky Way total (gravitational) mass

(Wang 2019, <https://arxiv.org/abs/1912.02599>)



Components of the WM



Diameter :

30 kpc

Total mass:

$10^{12} M_{\odot}$

Rotation :

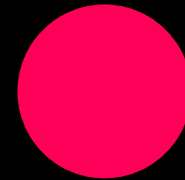
200 Myr (sun)

500 Myr (ext.)

Stellar component : bulge/bar

$0.5 \times 10^{10} M_{\odot}$

- old stars
- RMS vel ~ 150 km/s



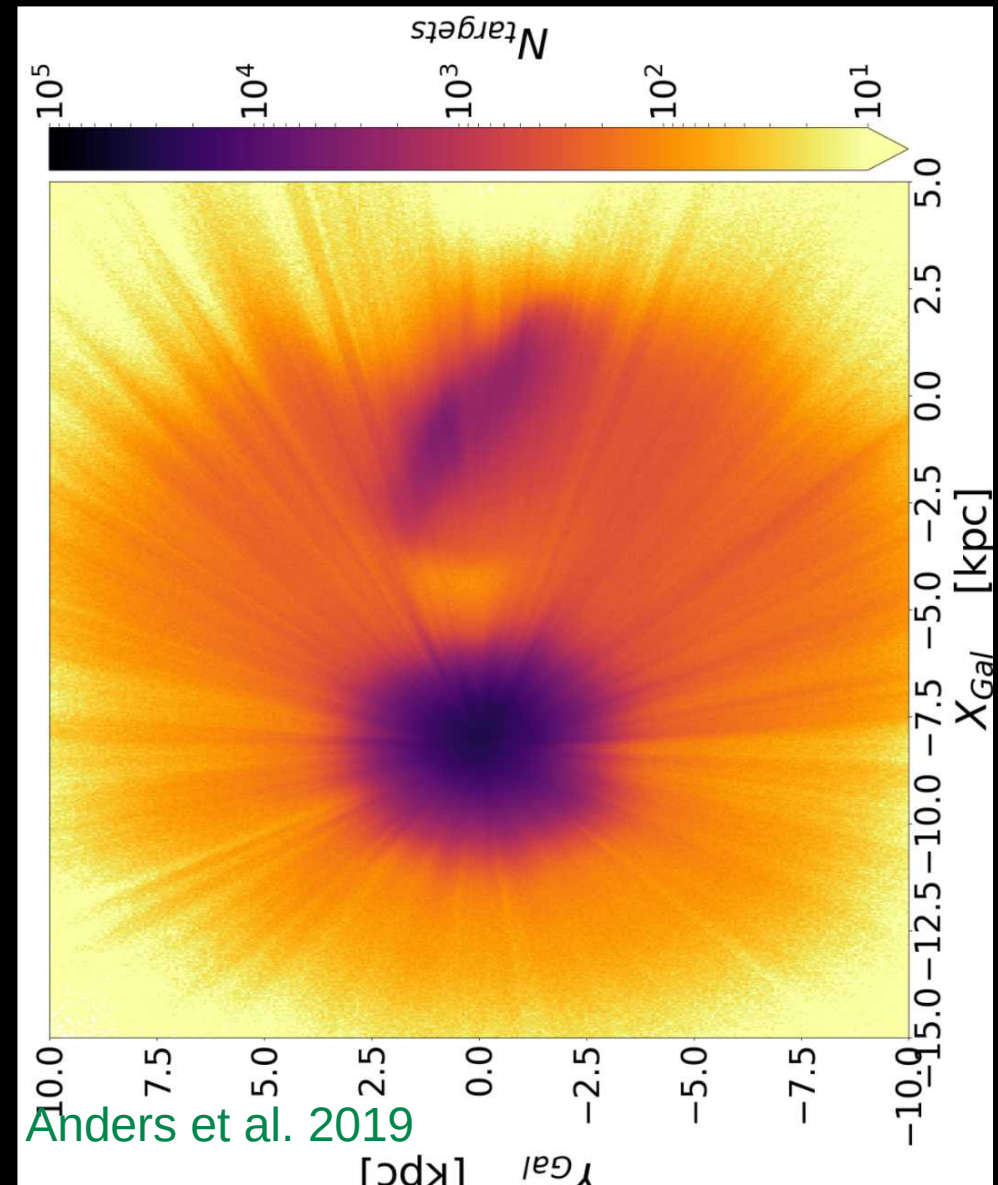
Stellar component : bulge/bar

$0.5 \times 10^{10} M_{\odot}$

265 millions of stars !



<https://sci.esa.int/j/61461>

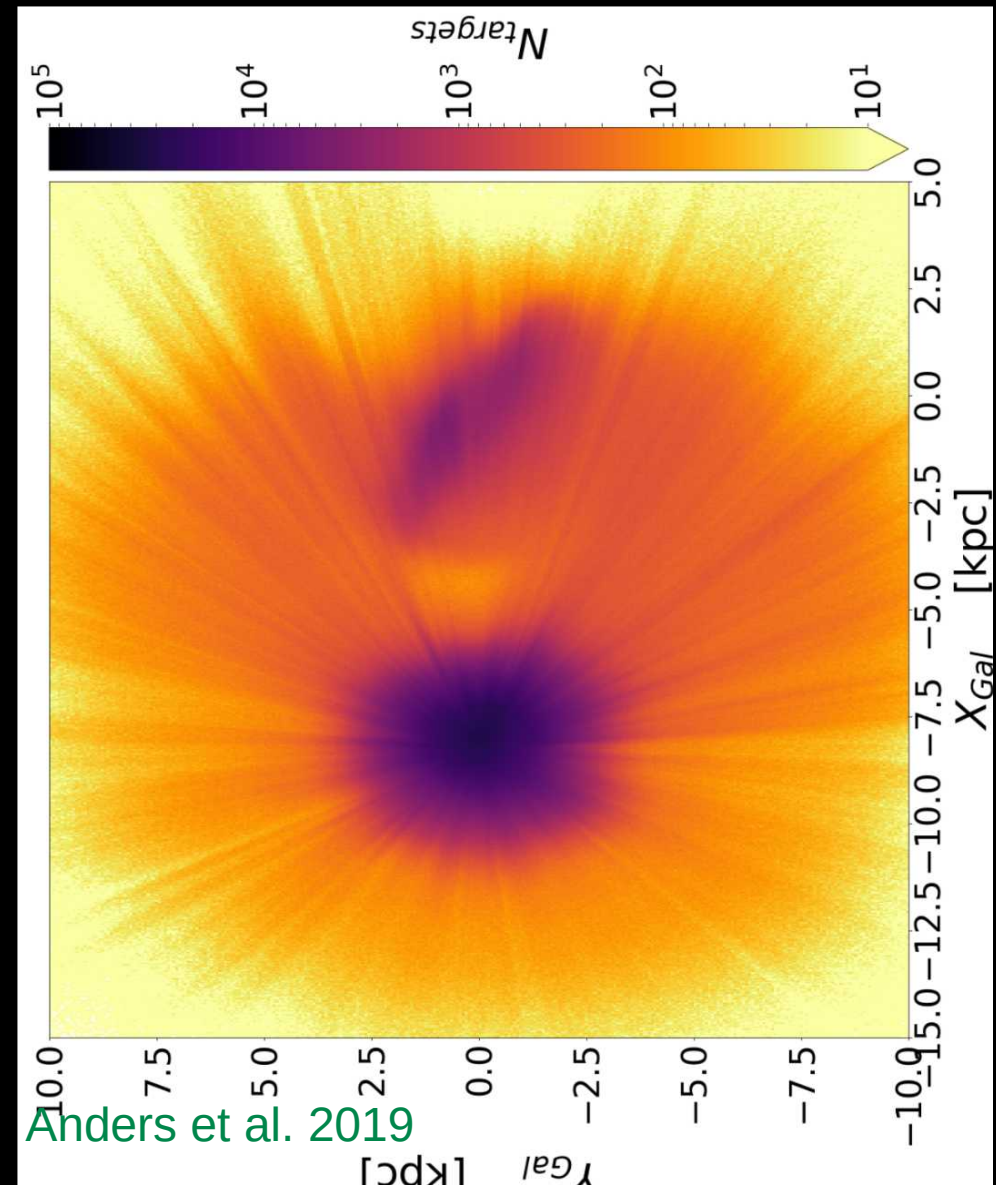
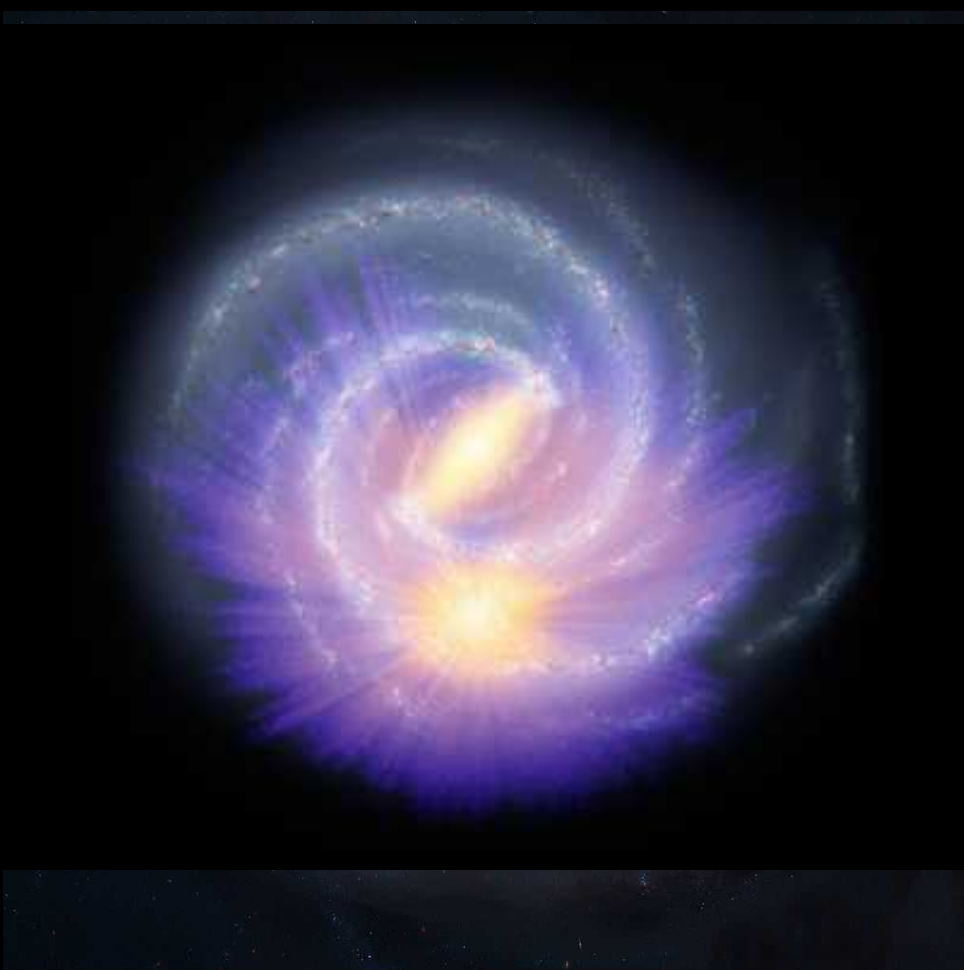


Anders et al. 2019

Stellar component : bulge/bar

$0.5 \times 10^{10} M_{\odot}$

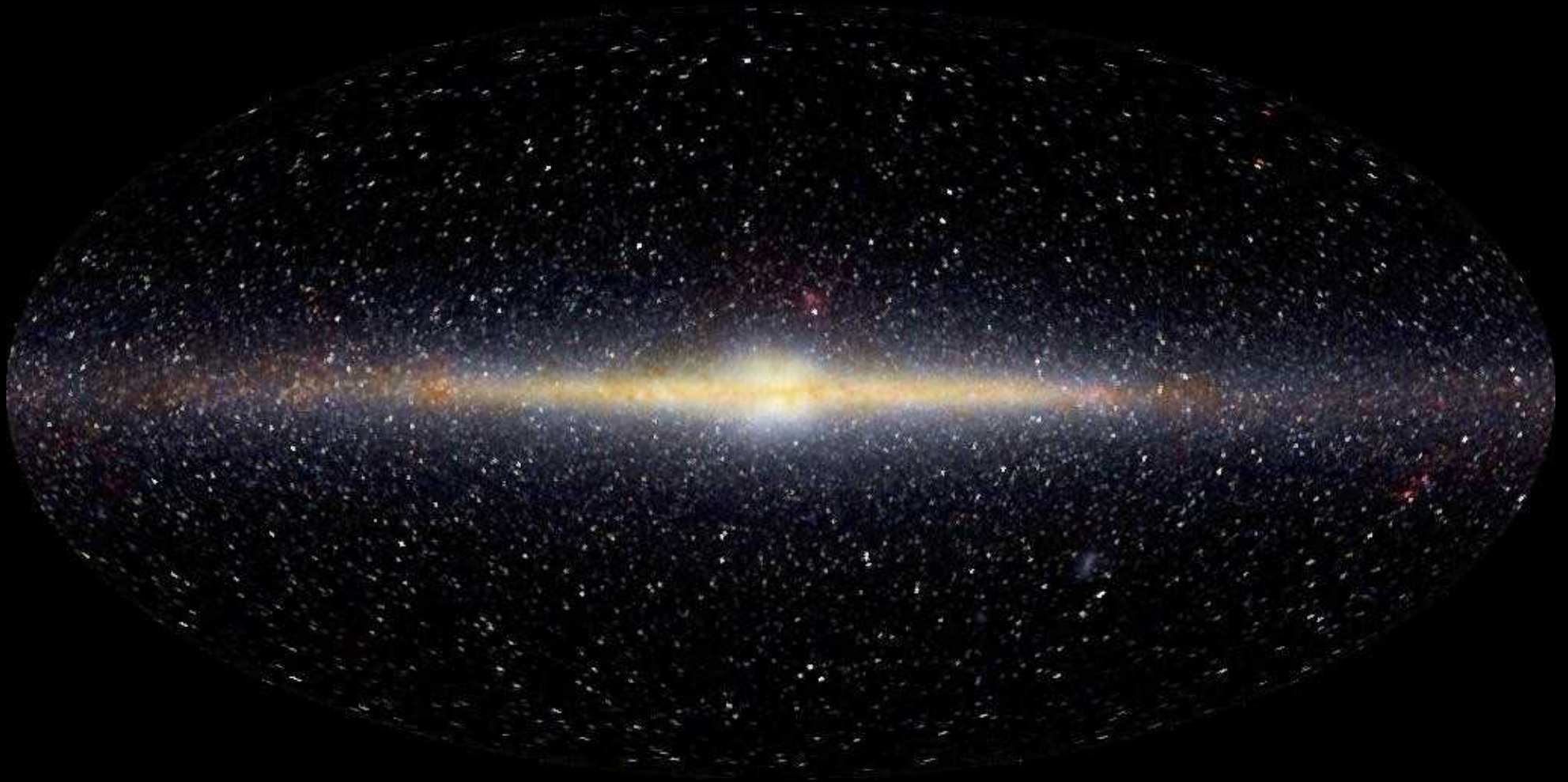
265 millions of stars !



Anders et al. 2019

<https://sci.esa.int/j/61461>

COBE satellite view of the MW in infrared light



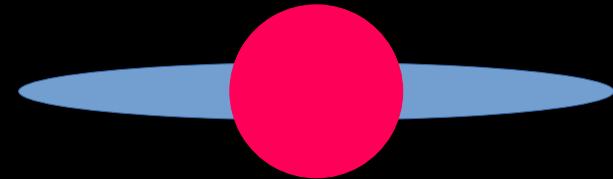
Robert Nemiroff (MTU) & Jerry Bonnell (USRA)

Stellar component : disk

$5 \times 10^{10} M_{\odot}$ (10 % of total)

thin disk:

- 90% of the stellar disk
- scale height : ~ 300 pc
- RMS vel ~ 50 km/s

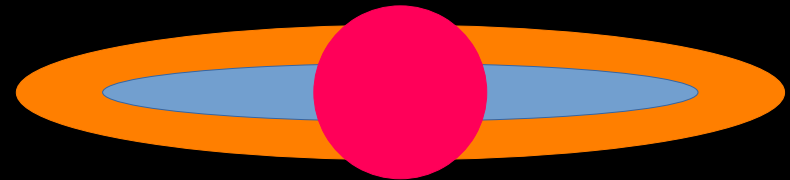


Stellar component : disk

$5 \times 10^{10} M_{\odot}$ (10 % of total)

thick disk:

- 10% of the stellar disk
- scale height : ~ 1 kpc
- RMS vel $> \sim 50$ km/s

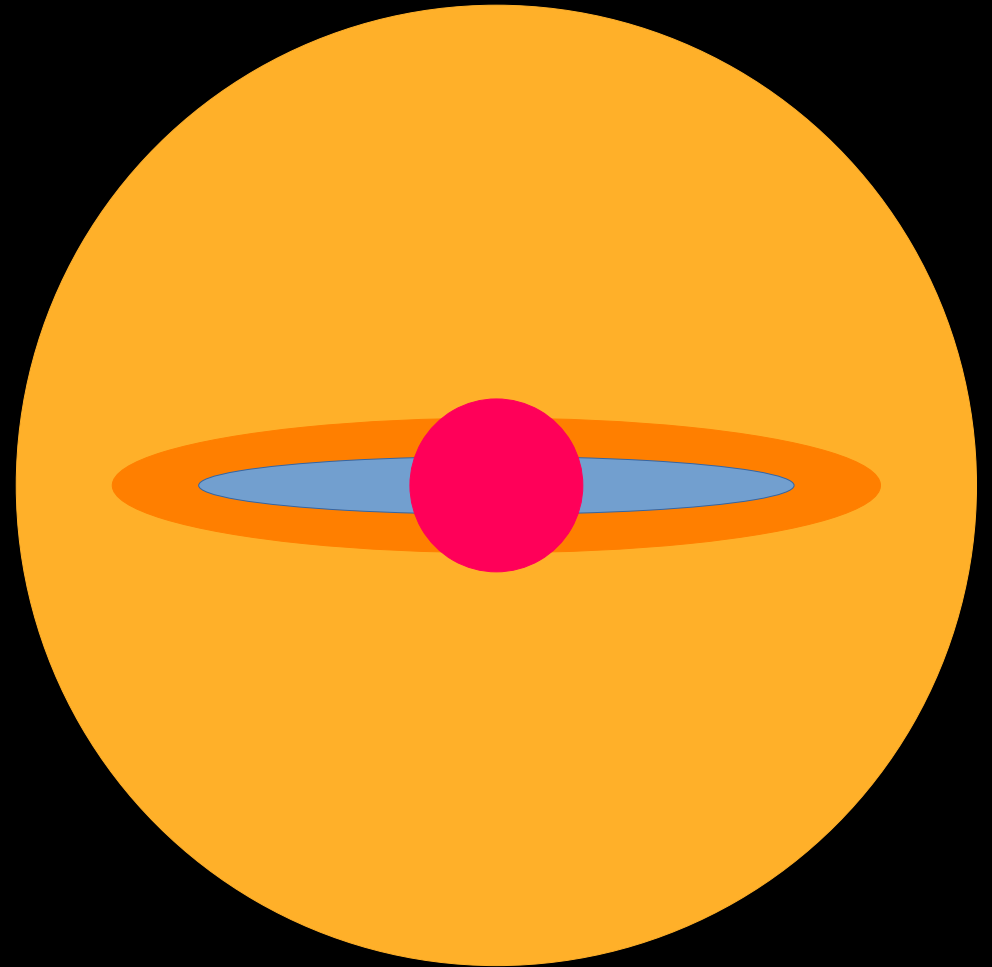


Stellar component : halo

$5 \times 10^8 M_{\odot}$ (1 % of stars)

- old stars

- no mean rotation

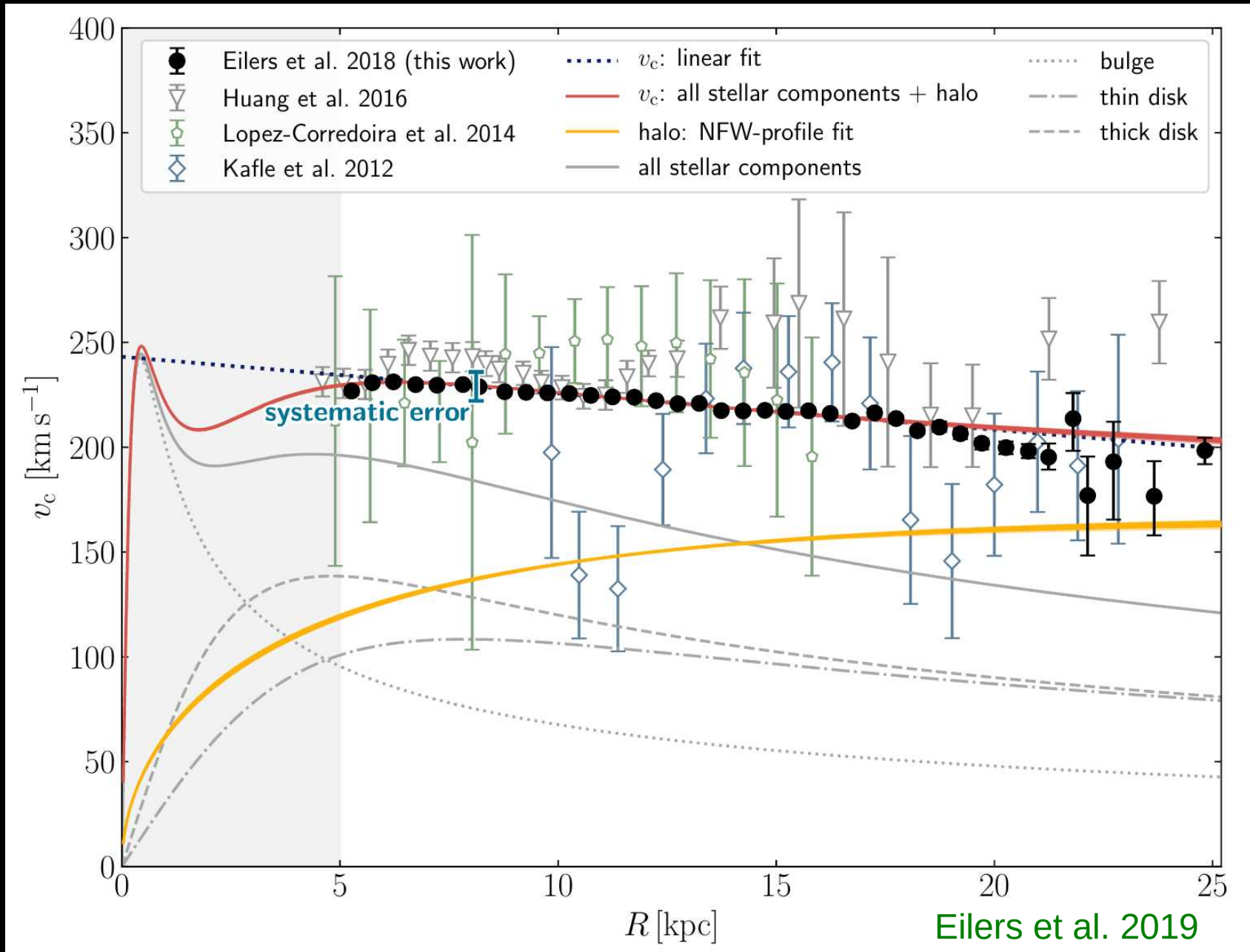


Gaseous component : disk, HVC

$10^9 M_{\odot}$ (0.1 %)

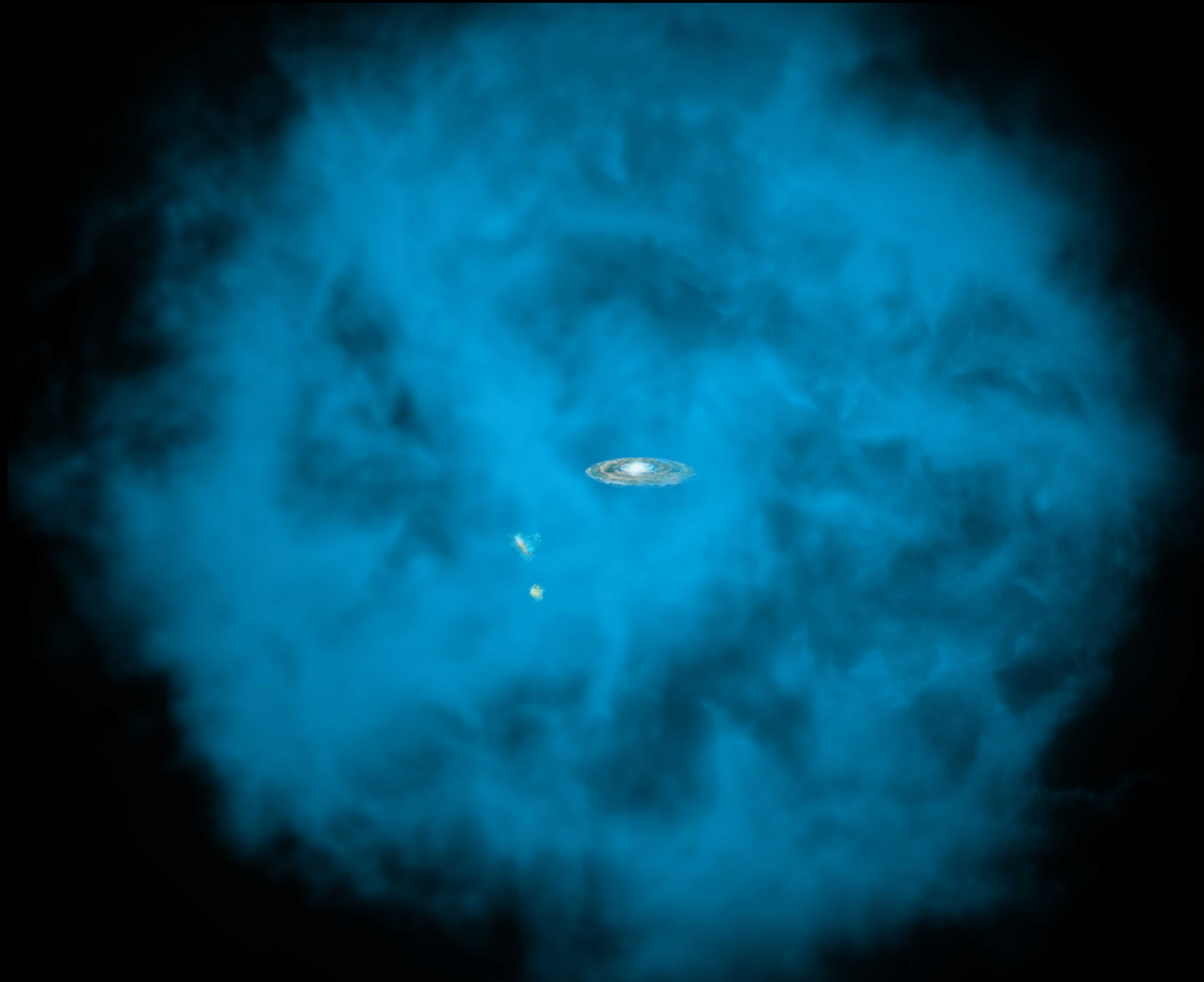


The circular rotation curve of the MW



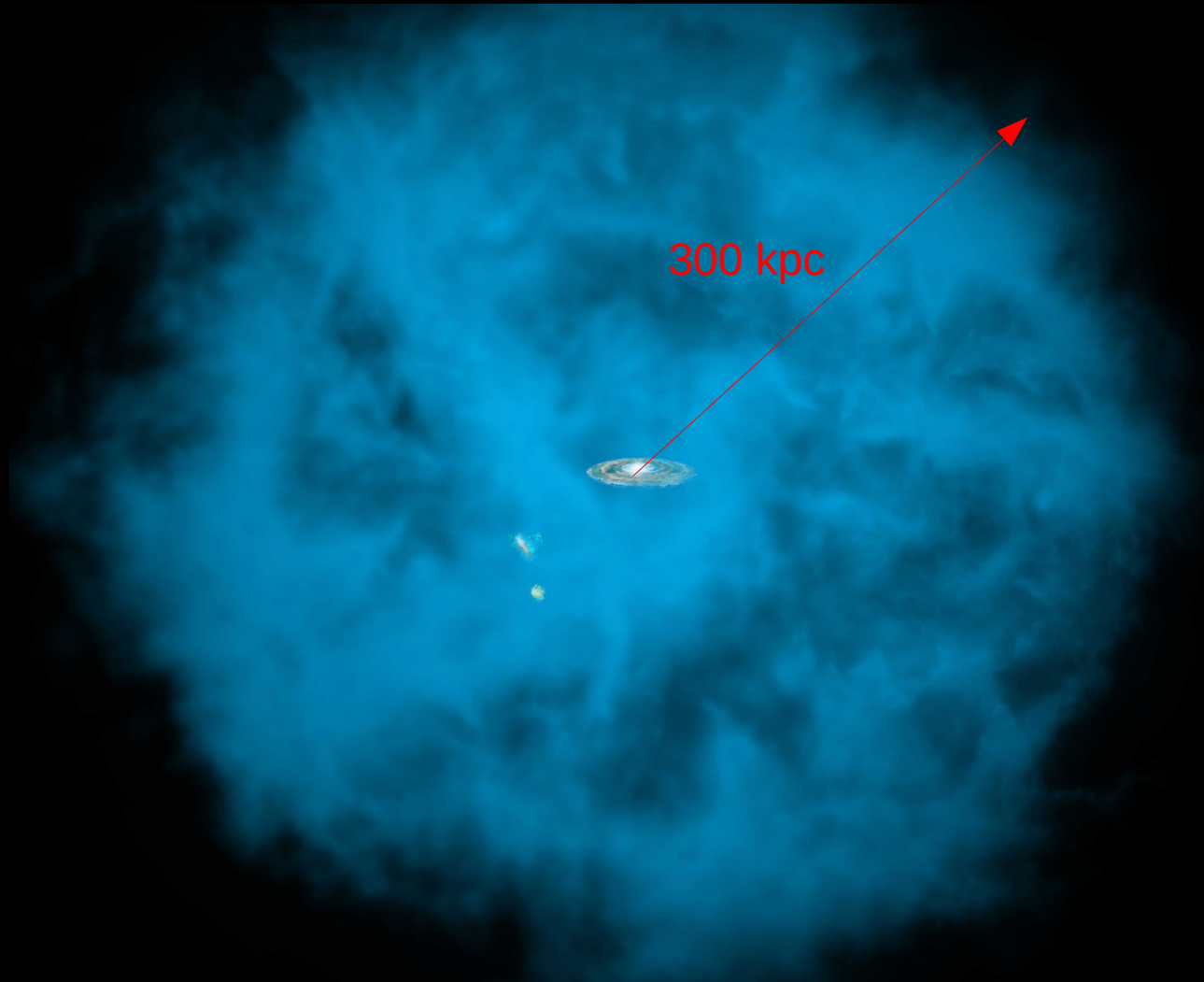
dark component : dark matter halo

about 90% of the total mass, $10^{12} M_{\odot}$



dark component : dark matter halo

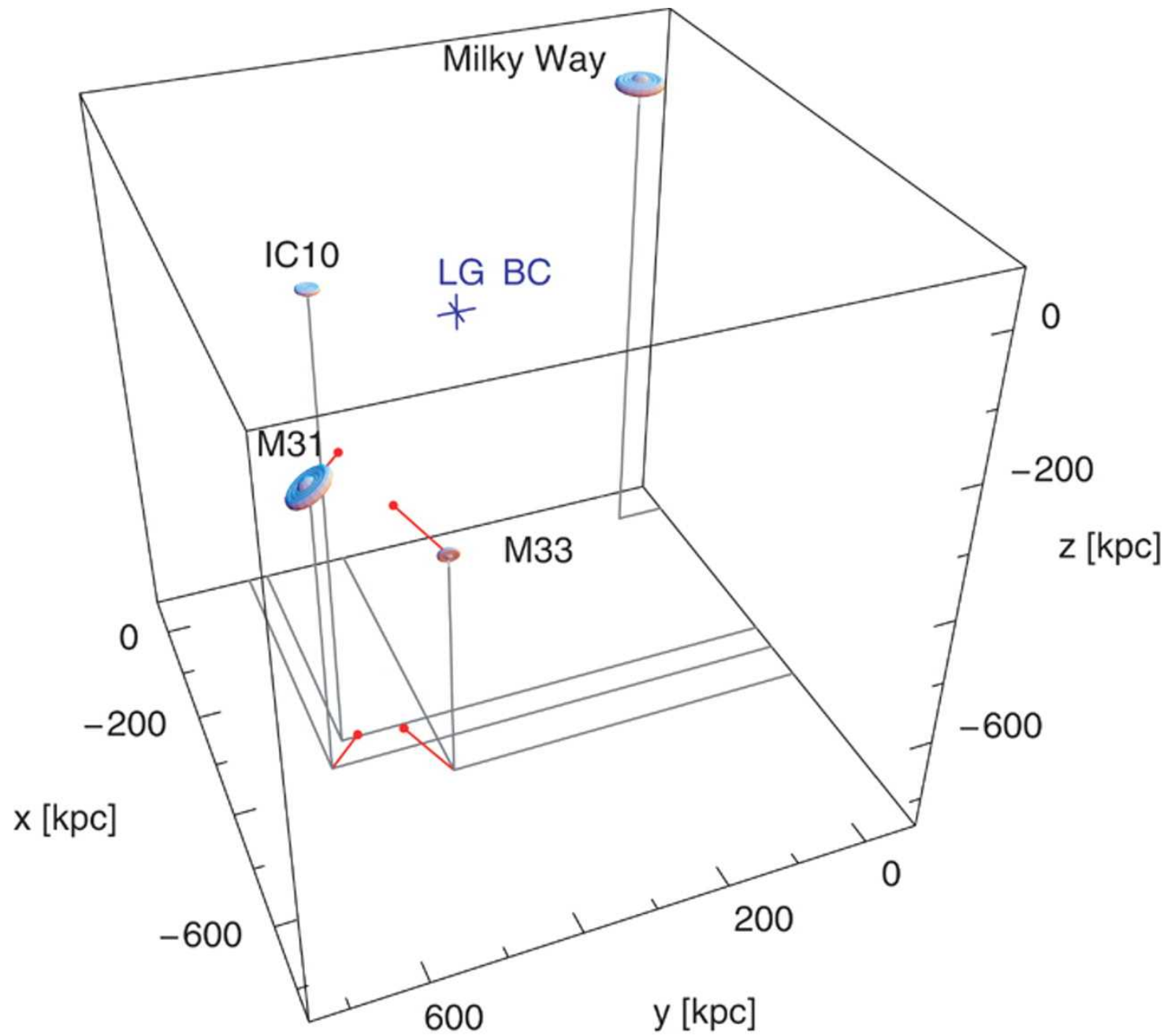
about 90% of the total mass, $10^{12} M_{\odot}$



Observation of Galaxies

The Local Group

~ 3 Mpc



M31 : The Andromeda Galaxy



distance 770 kpc, total mass $\sim 10^{12} M_{\odot}$

M33 : The Triangulum Galaxy



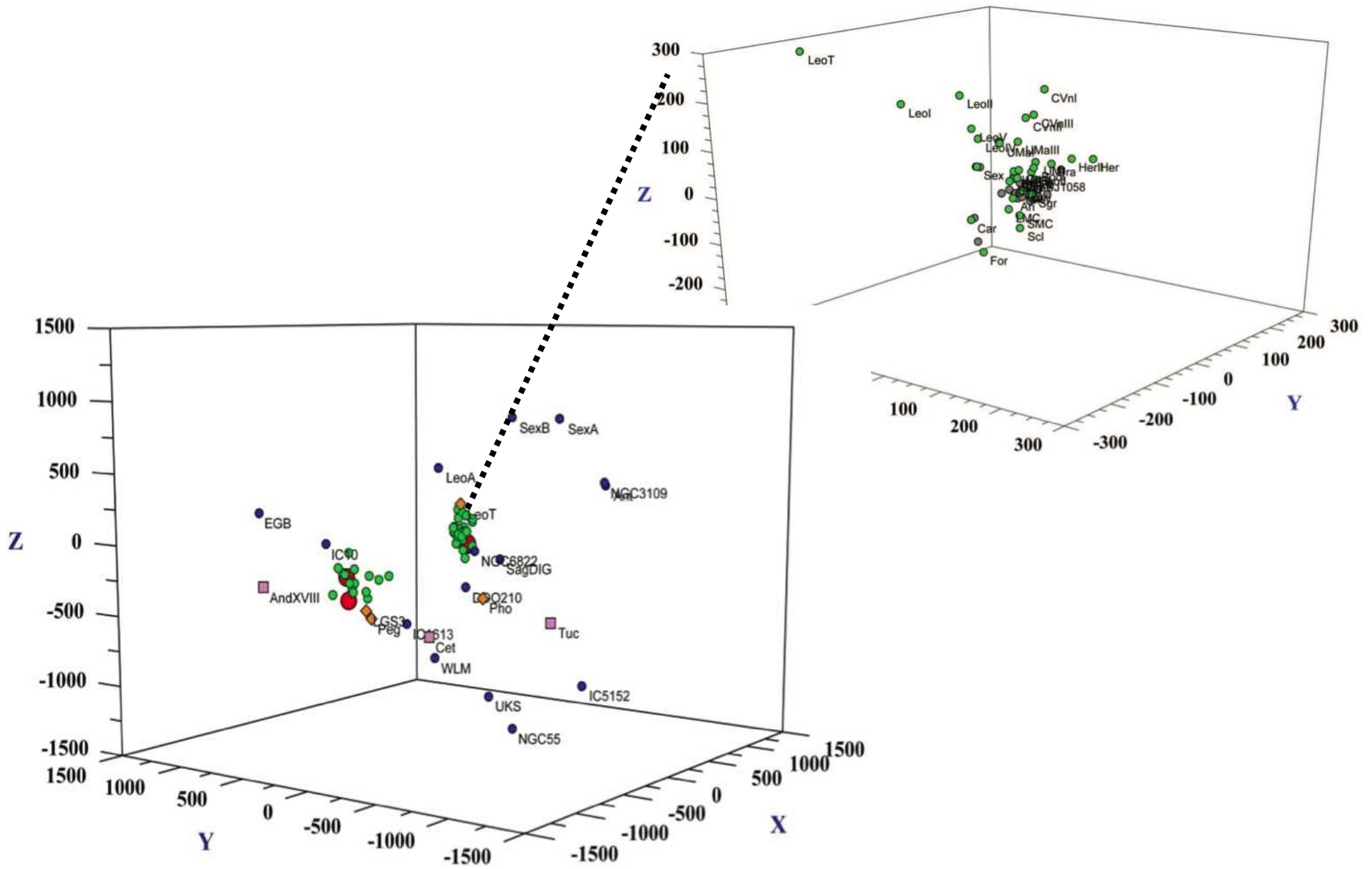
distance 847 kpc, total mass $6 \times 10^{10} M_{\odot}$

IC 10 : an irregular galaxy

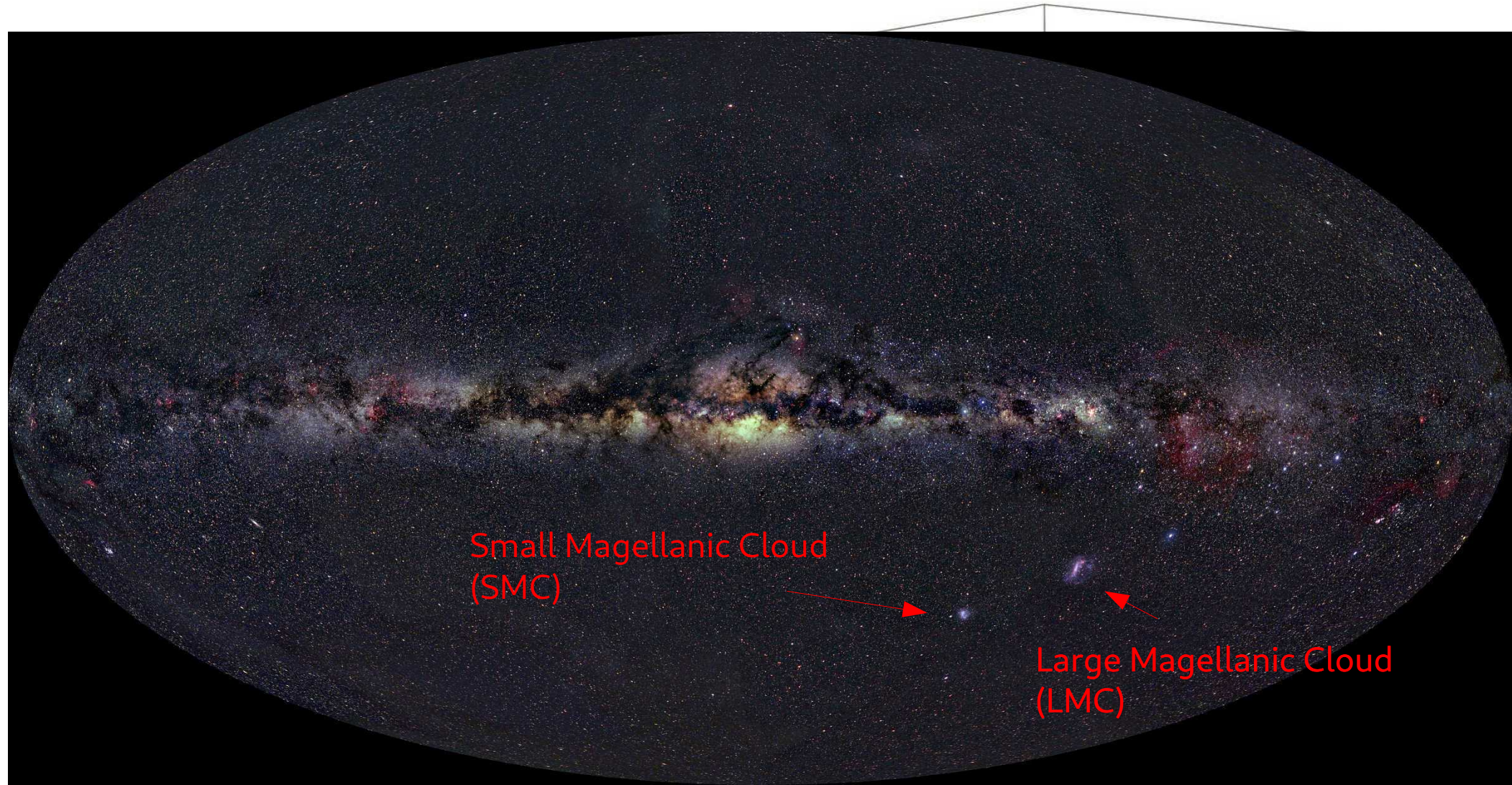


distance 660 kpc, total mass $\sim 2 \times 10^9 M_{\odot}$

+ about 130 satellite dwarfs...



+ about 130 satellite dwarfs...

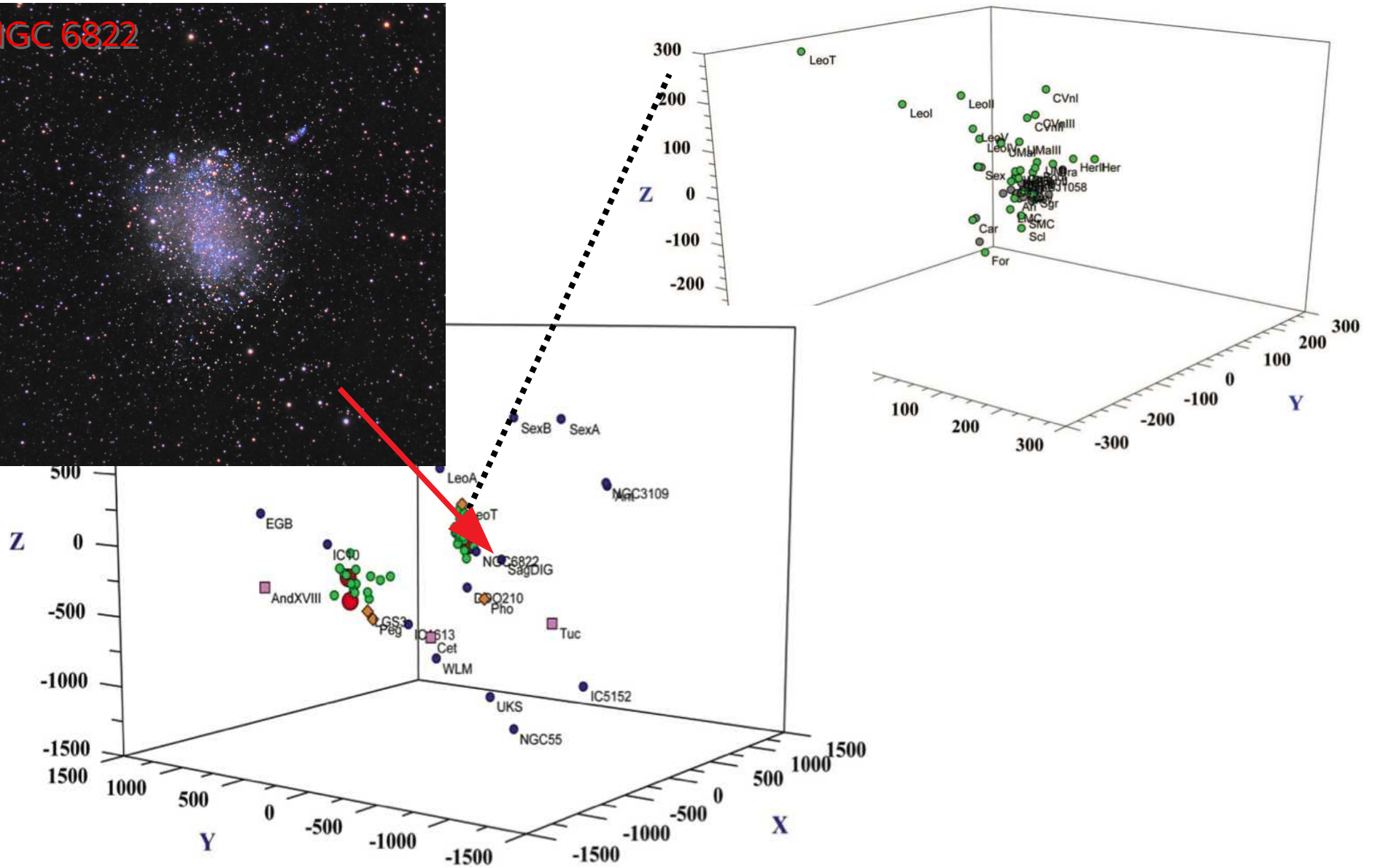
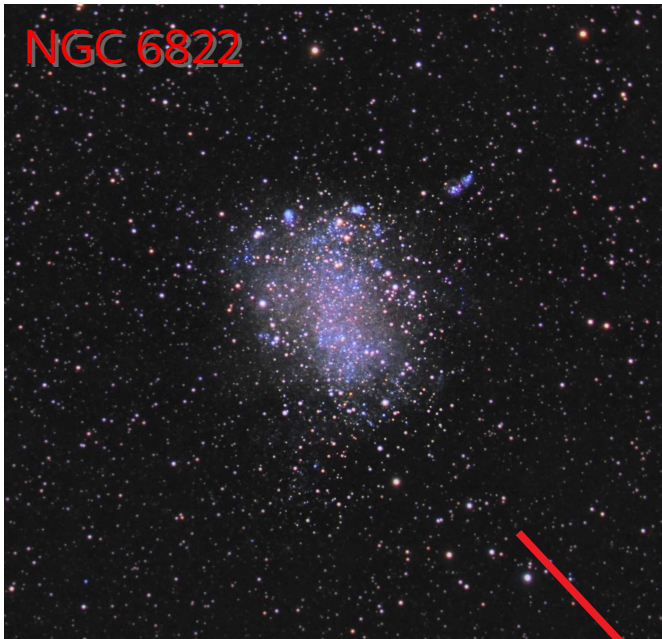


Small Magellanic Cloud
(SMC)

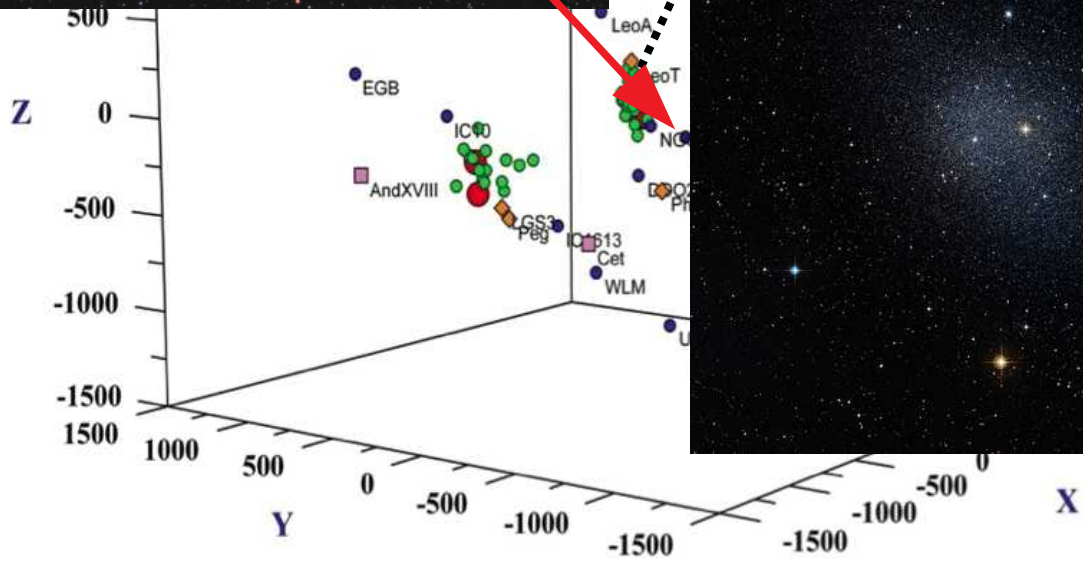
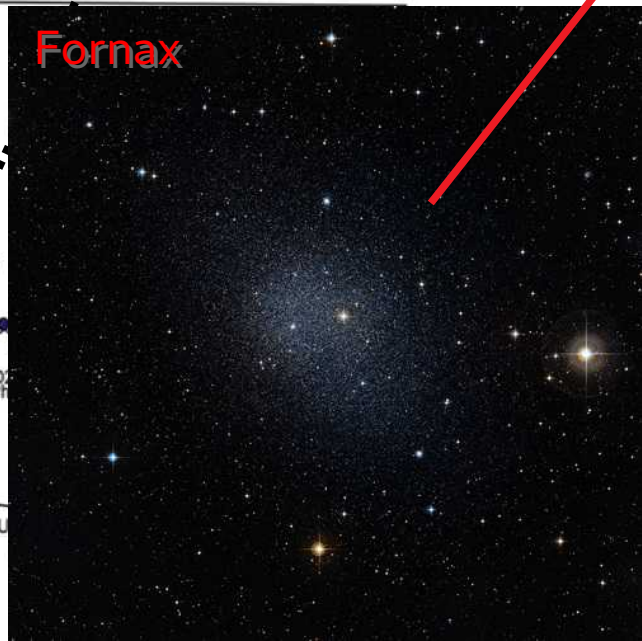
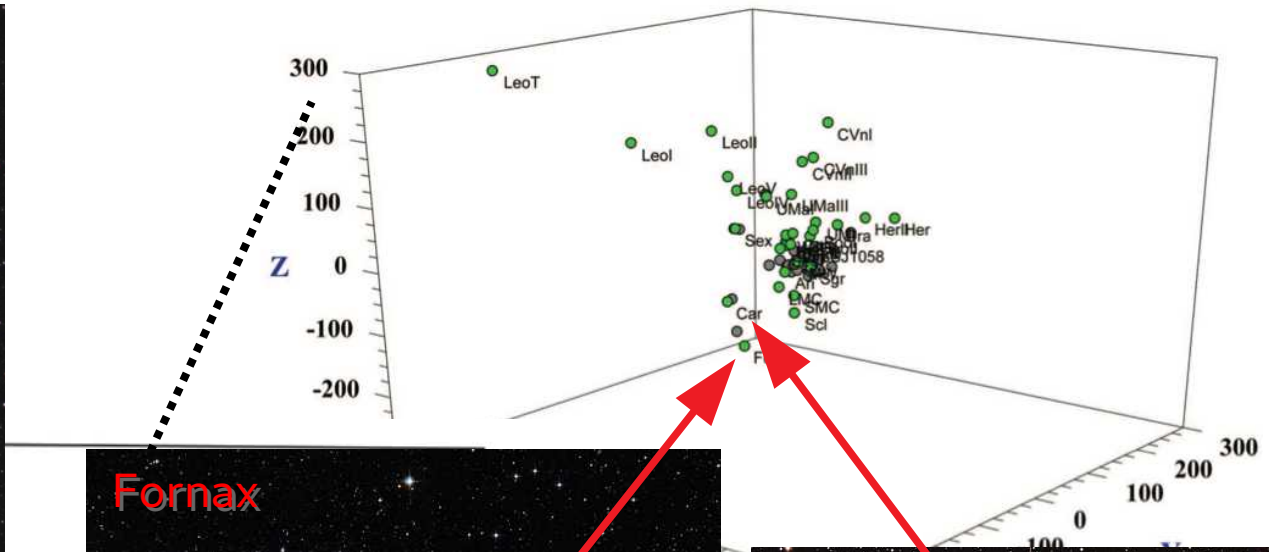
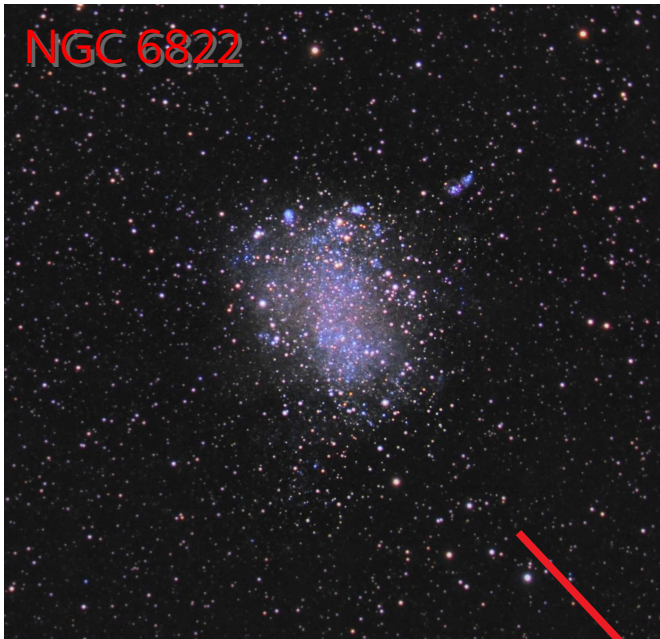
Large Magellanic Cloud
(LMC)



+ about 130 satellite dwarfs...



+ about 130 satellite dwarfs...



Ma

MW – Andromeda collision

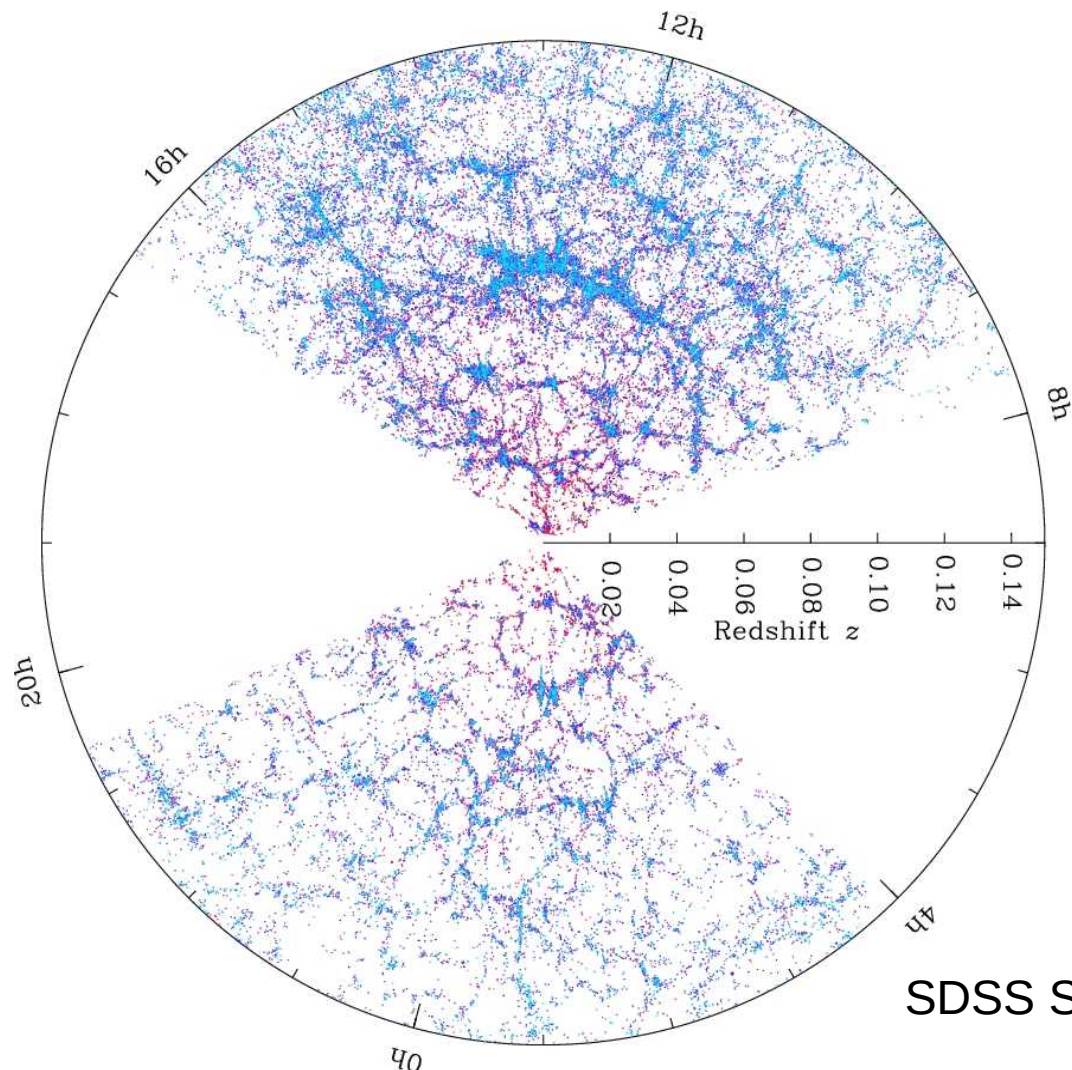
Temps = 4815 Millions d'années



<https://youtu.be/6aqsvkMp4ns>

Observation of Galaxies

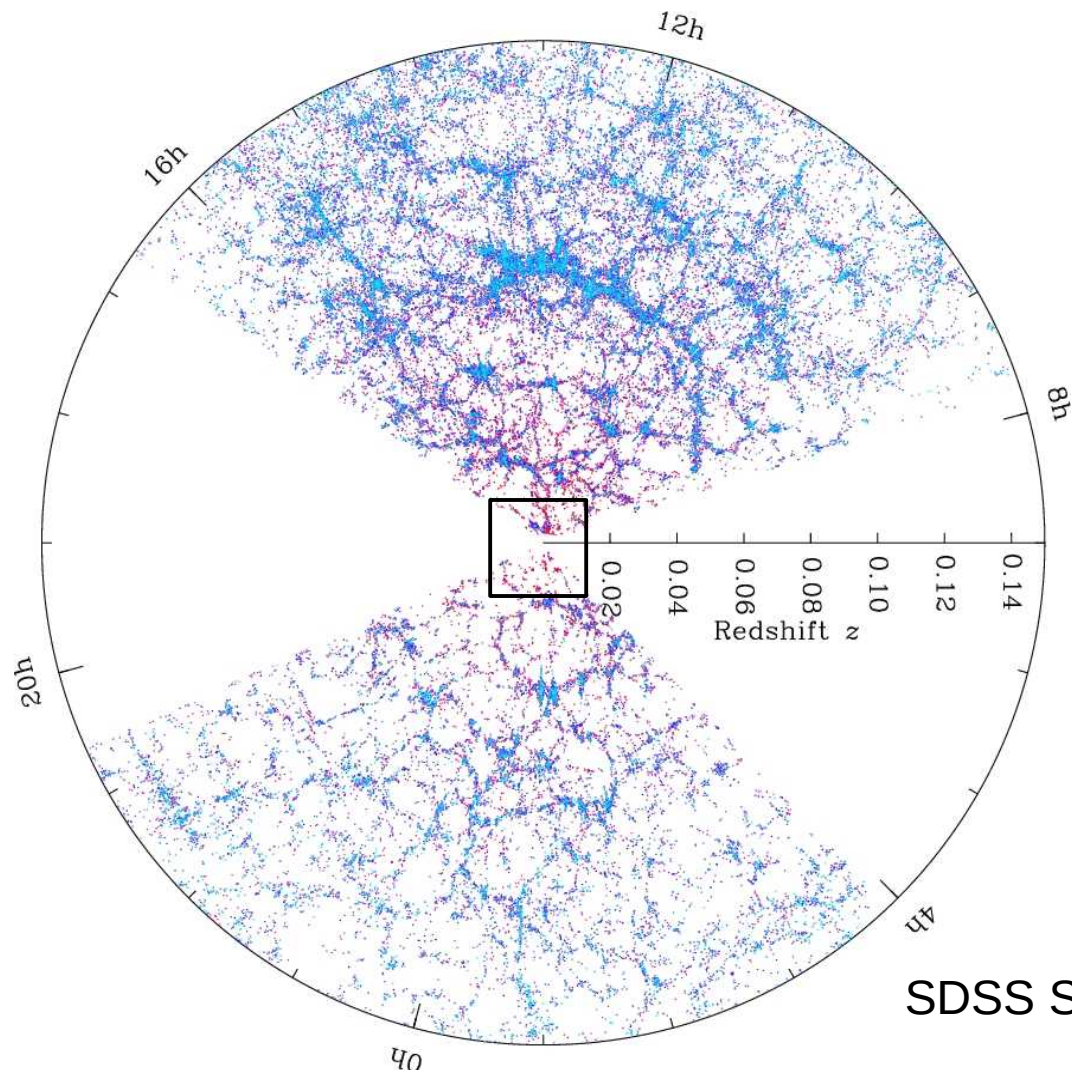
Beyond the LG... the LV



SDSS Sloan Digital Sky Survey

Observation of Galaxies

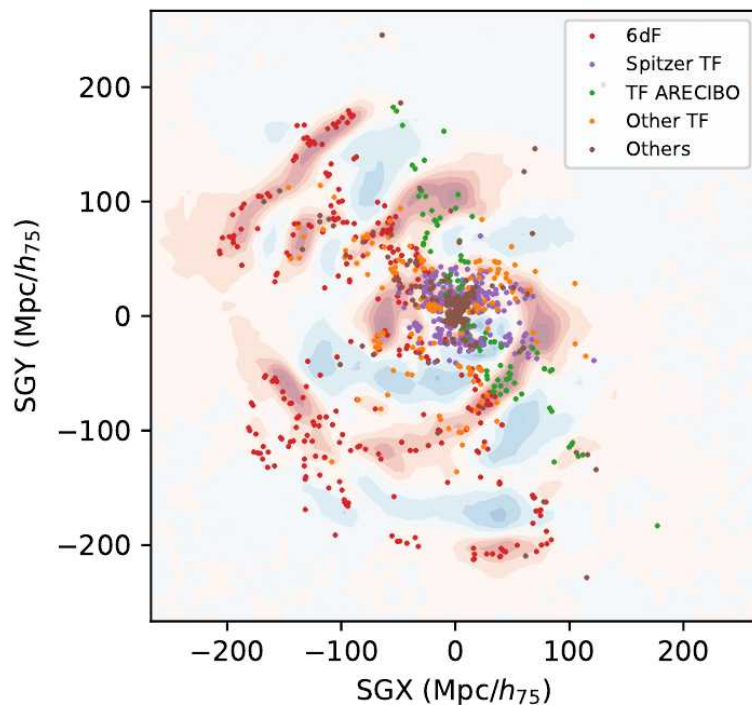
Beyond the LG... the LV



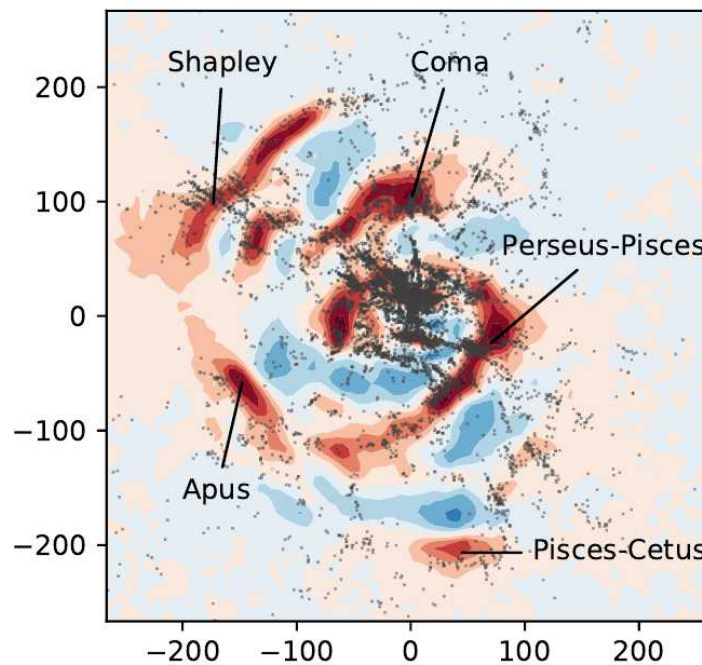
~500 galaxies
with $D < 10$ Mpc

SDSS Sloan Digital Sky Survey

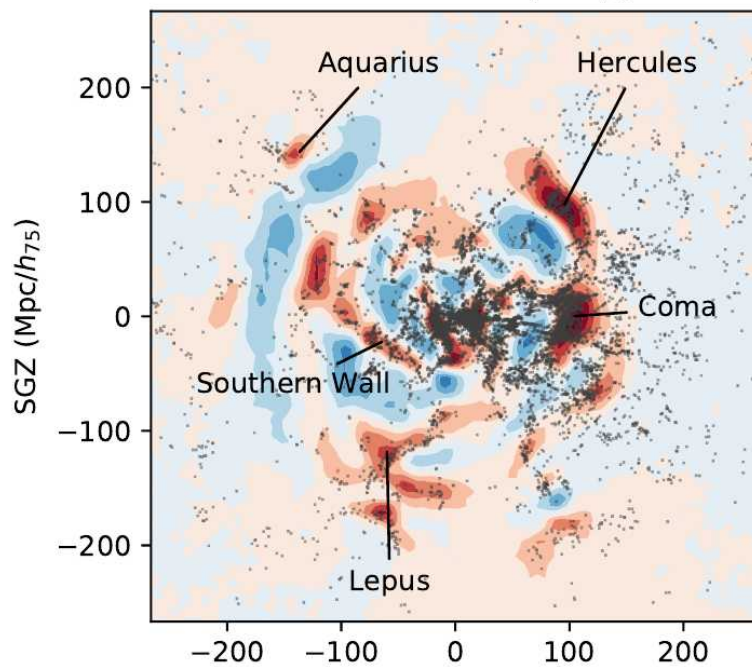
SGZ = 0 ± 5 Mpc/ h_{75}



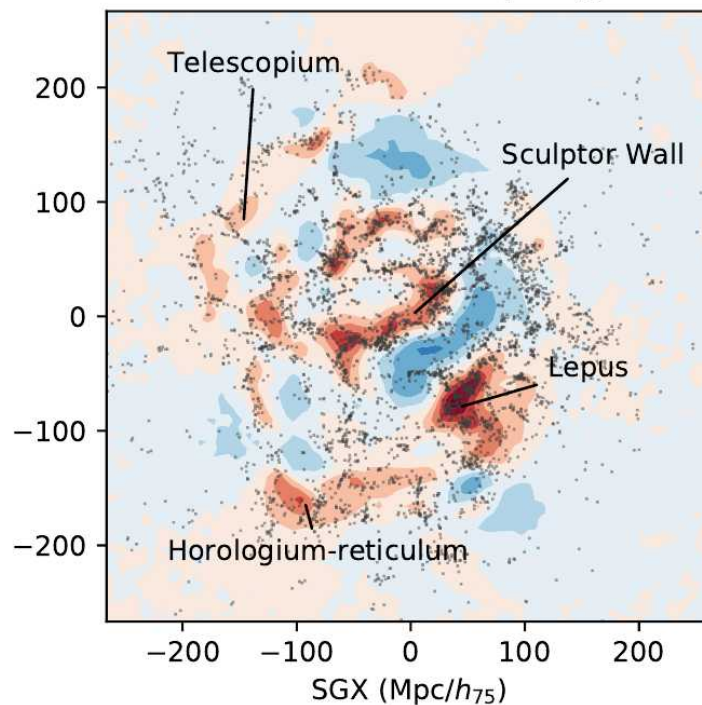
SGZ = 0 ± 5 Mpc/ h_{75}



SGX = 0 ± 5 Mpc/ h_{75}



SGY = -93 ± 5 Mpc/ h_{75}



The most detailed 3D view of the Universe SDSS/eBOSS



Observation of Galaxies

Luminosity Distribution Function

Luminosity distribution function

Luminosity Function: Schechter law (1976)

number of galaxies in the luminosity range $[L, L+dL]$

$$\Phi(L) dL = \Phi_{\star} \left(\frac{L}{L_{\star}} \right)^{\alpha} \exp(-L/L_{\star}) \frac{dL}{L_{\star}}$$

with $\alpha \approx -1.1$
 $L_{\star} \approx 2.9 \times 10^{10} L_{\odot}$

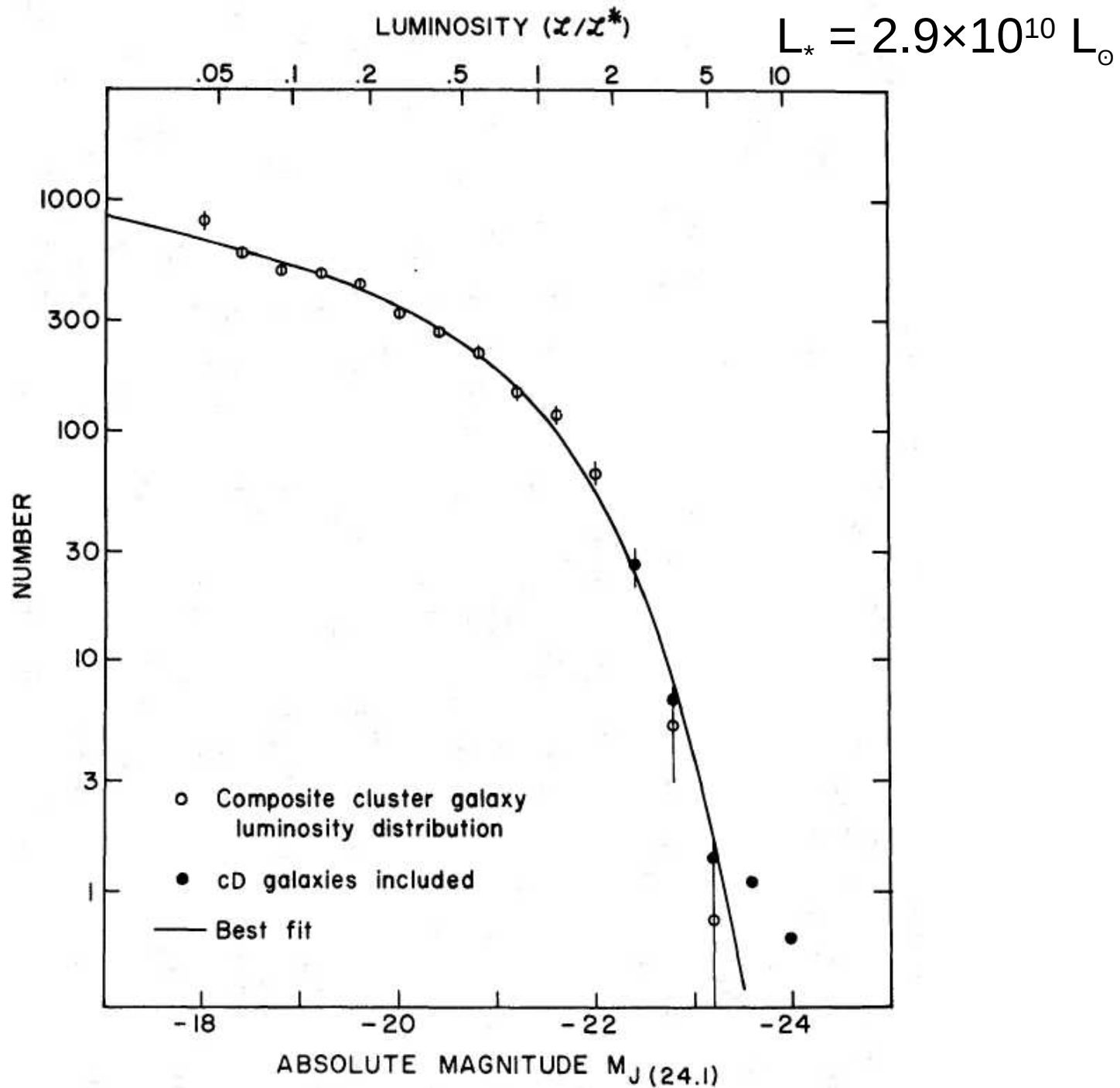
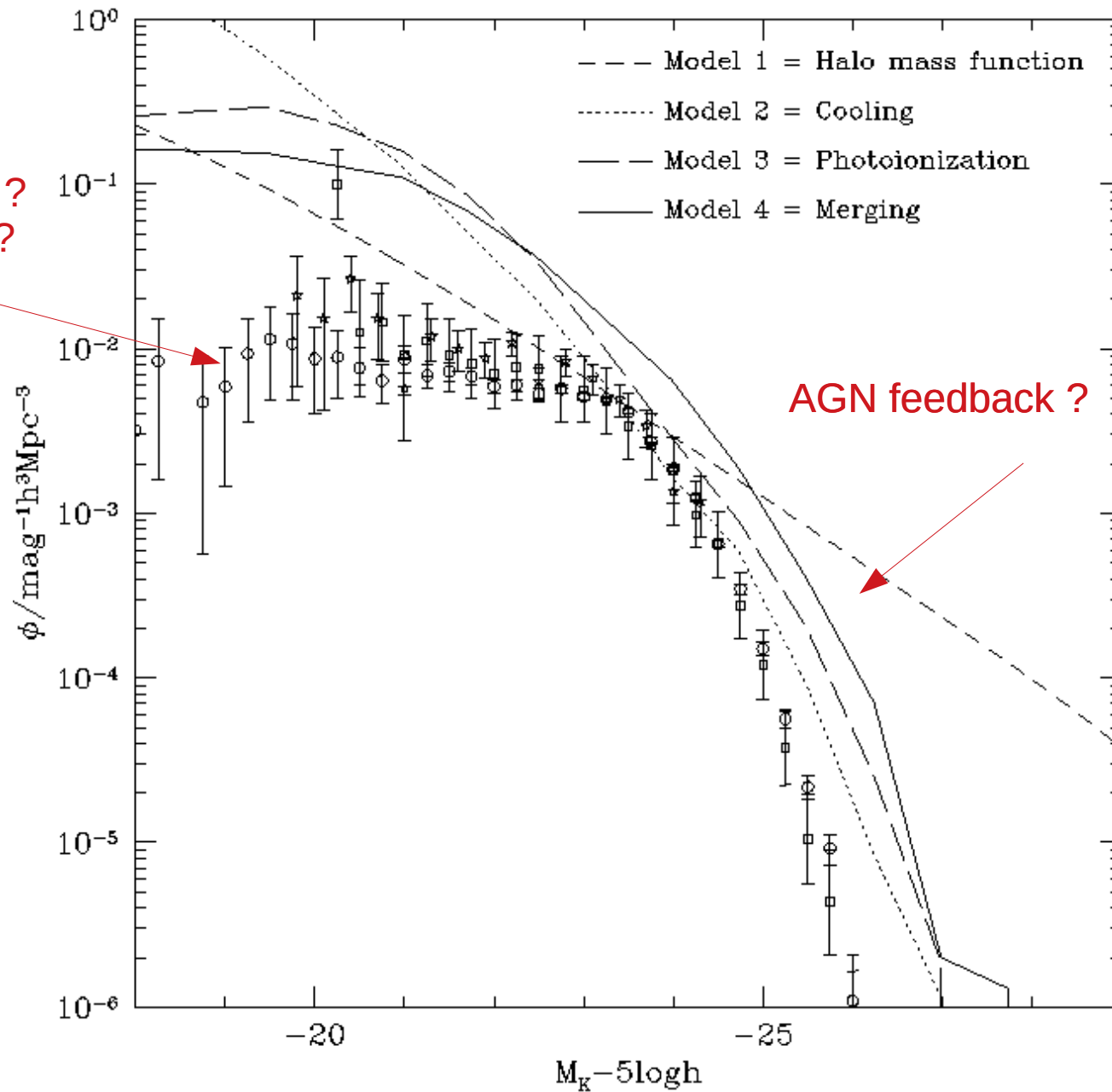


FIG. 2.—Best fit of analytic expression to observed composite cluster galaxy luminosity distribution. Filled circles show the effect of including cD galaxies in composite.

Stellar feedback ?
UV-background ?

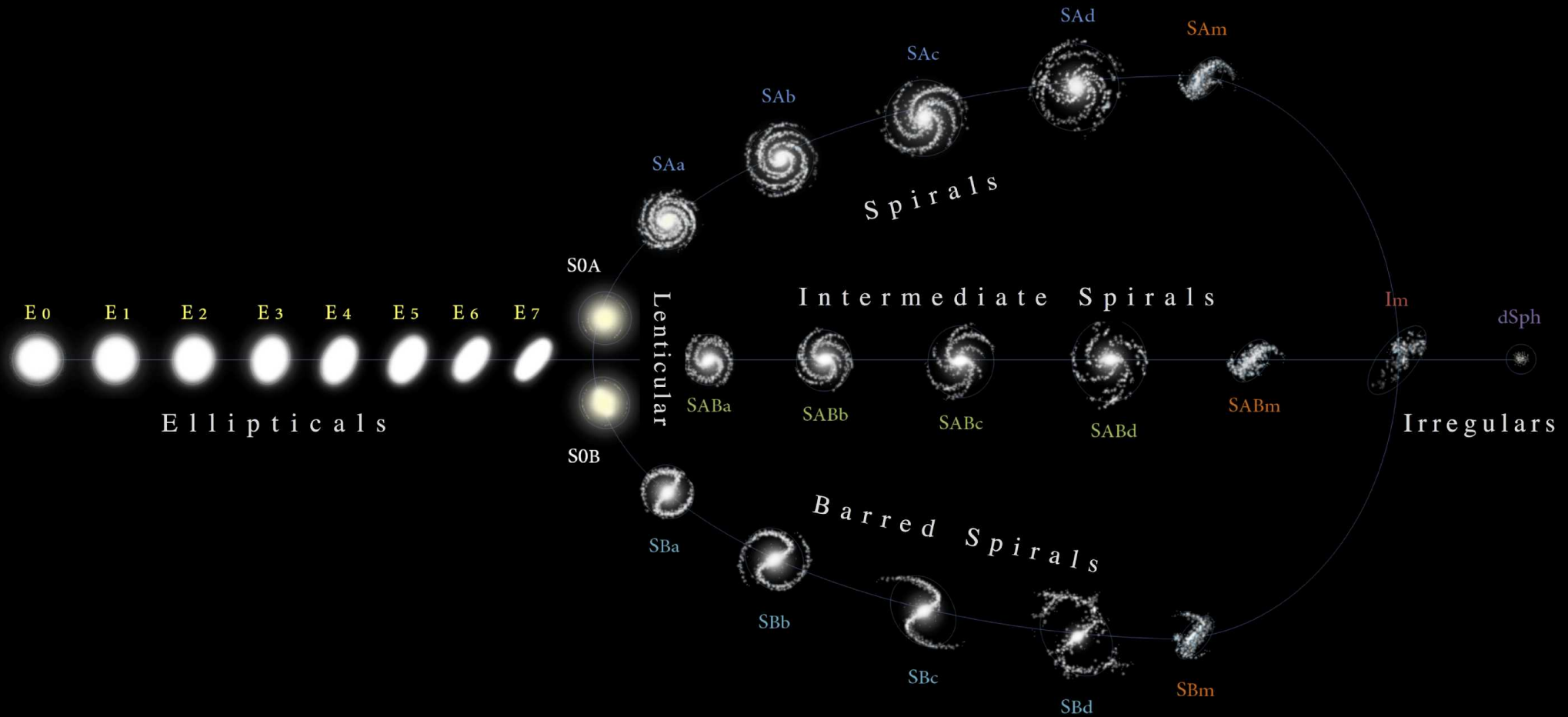


Observation of Galaxies

The Hubble-De Vaucouleurs Sequence

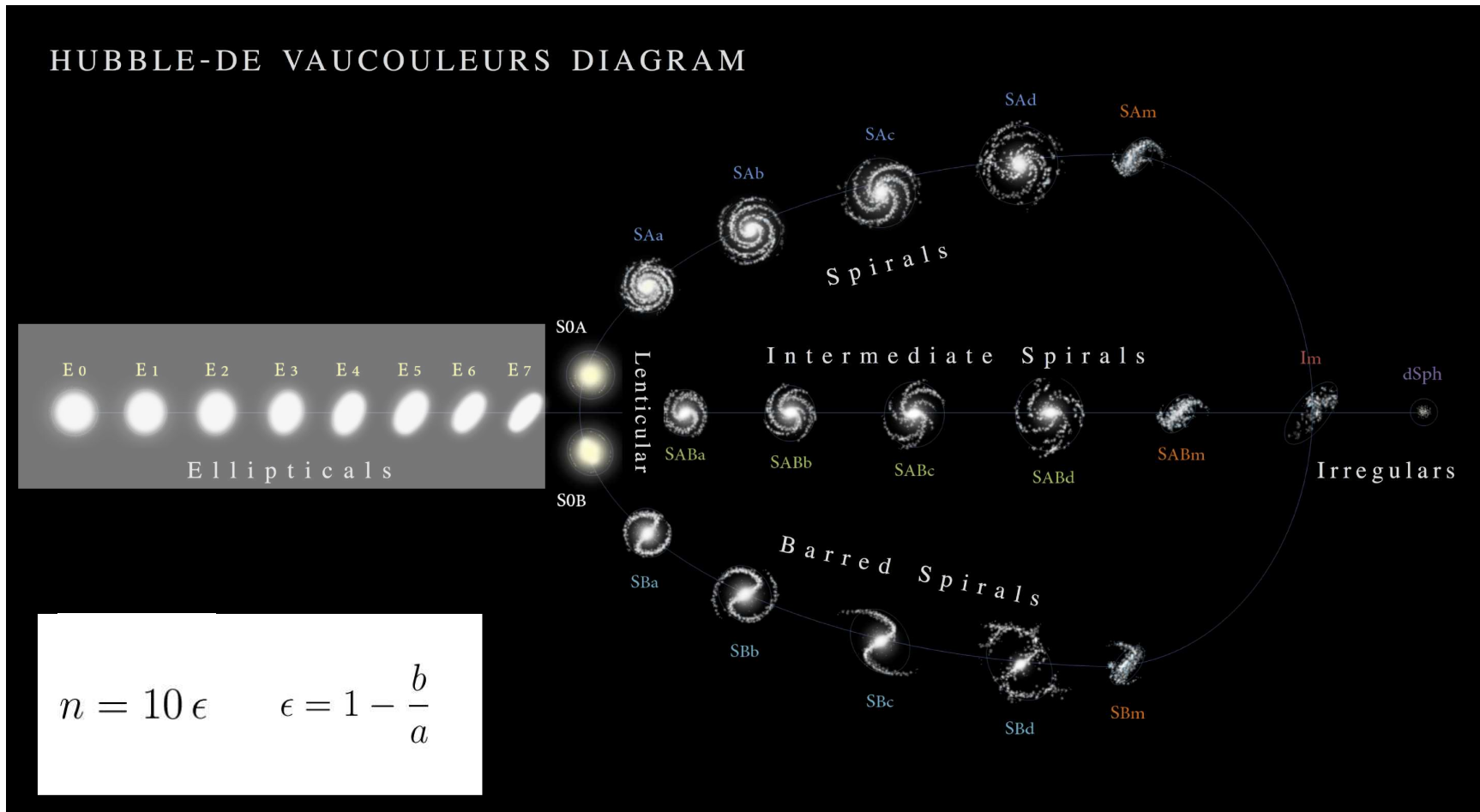
The Hubble-De Vaucouleurs Sequence

HUBBLE-DE VAUCOULEURS DIAGRAM



Observation of Galaxies

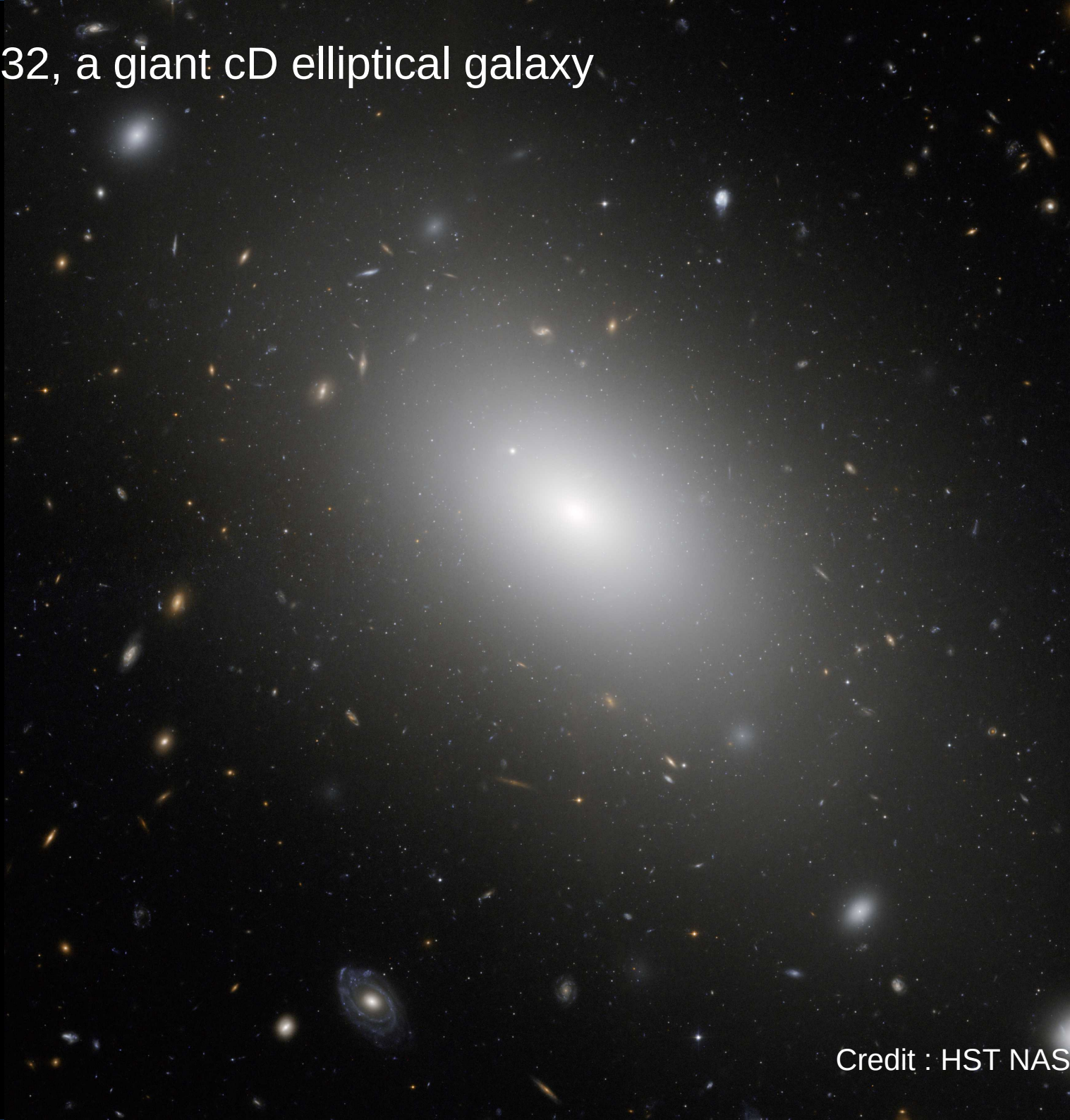
Elliptical Galaxies



M87 (cD or BCG, bright cluster galaxy) and several other ellipticals



NGC 1132, a giant cD elliptical galaxy



Credit : HST NASA/ESA

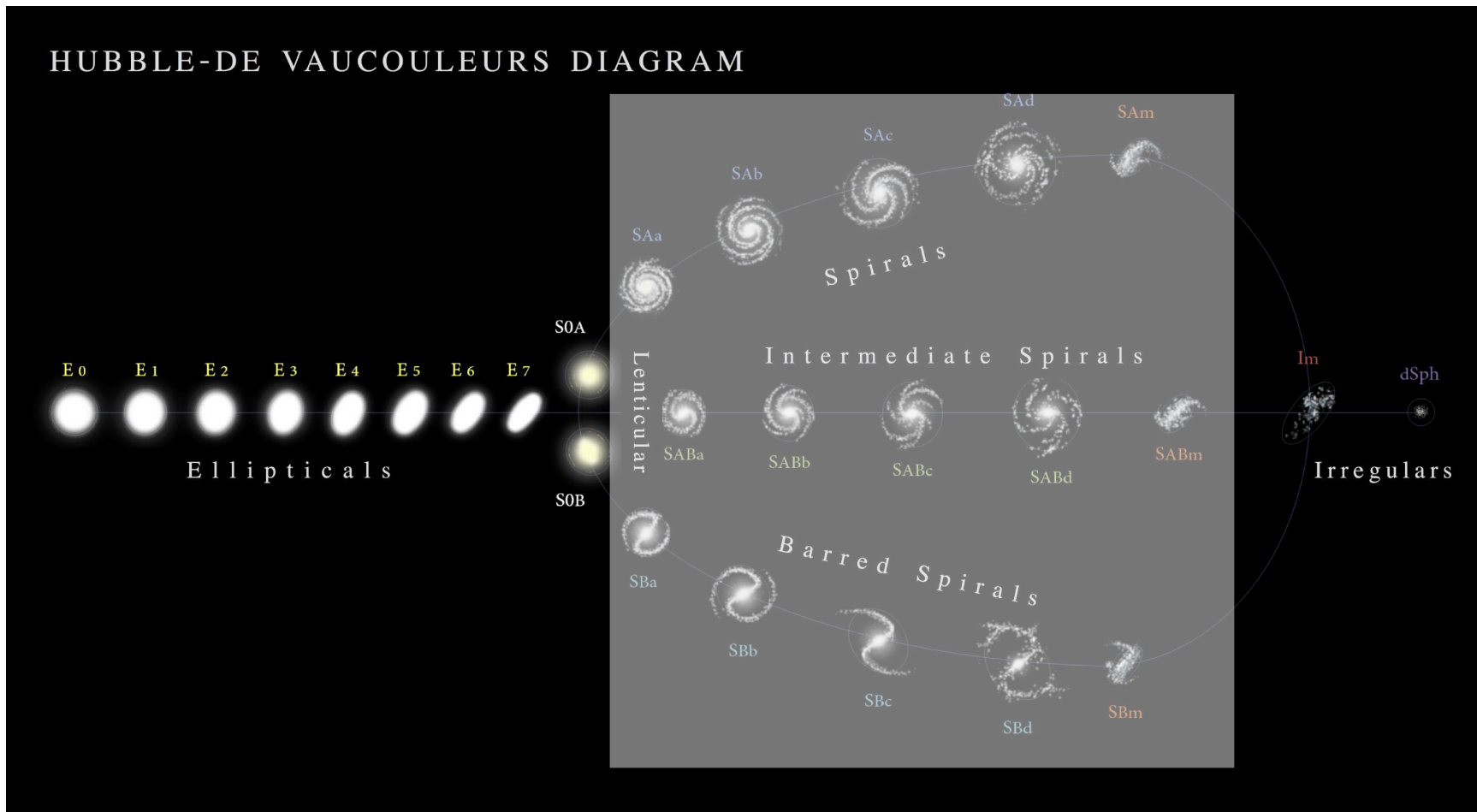
NGC 1316



Credit : ESO VLT

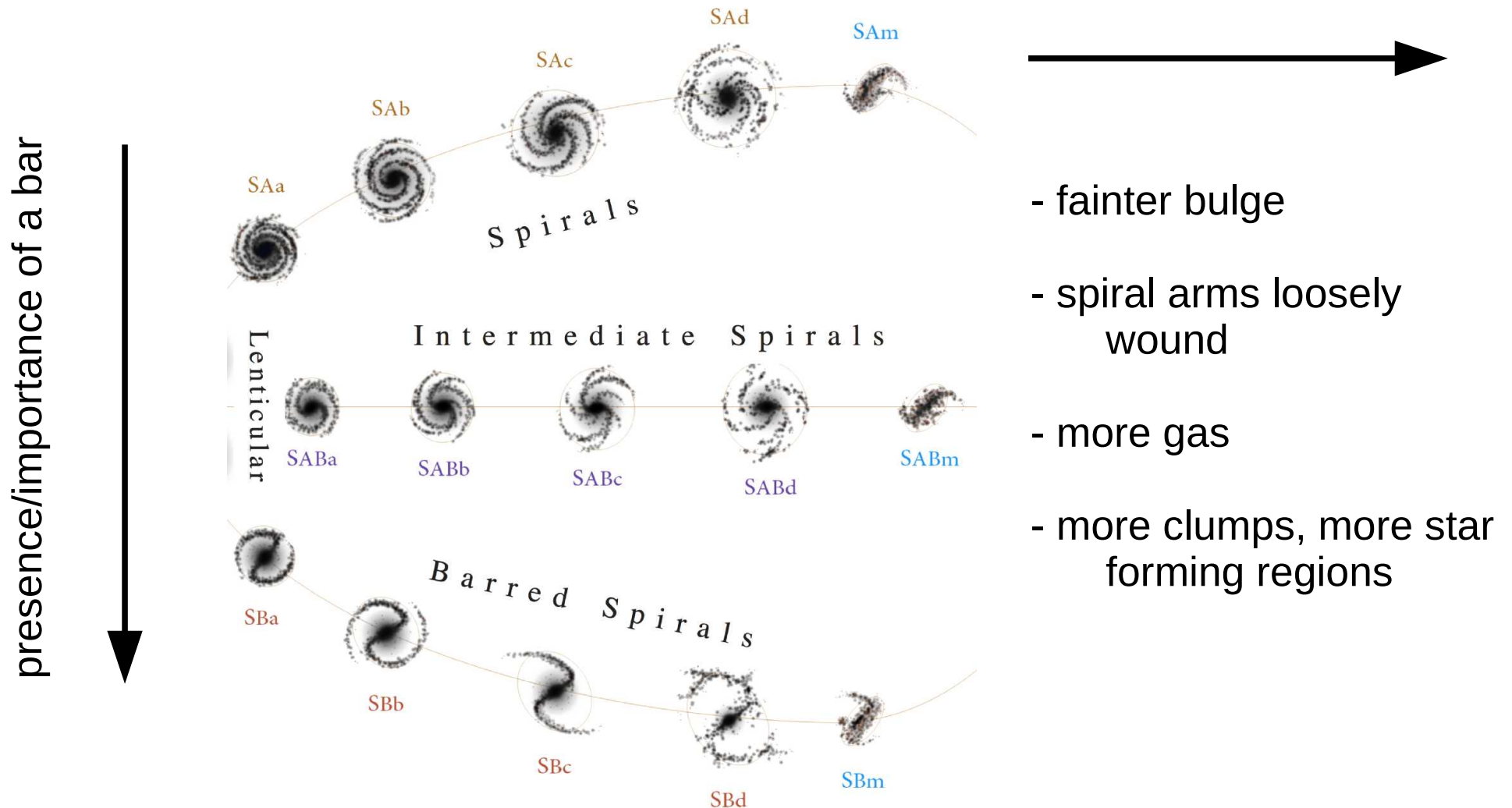
Observation of Galaxies

Spiral Galaxies



Spiral Galaxies

The relative importance of bulges with respect to disks is a classification criteria of spiral galaxies.

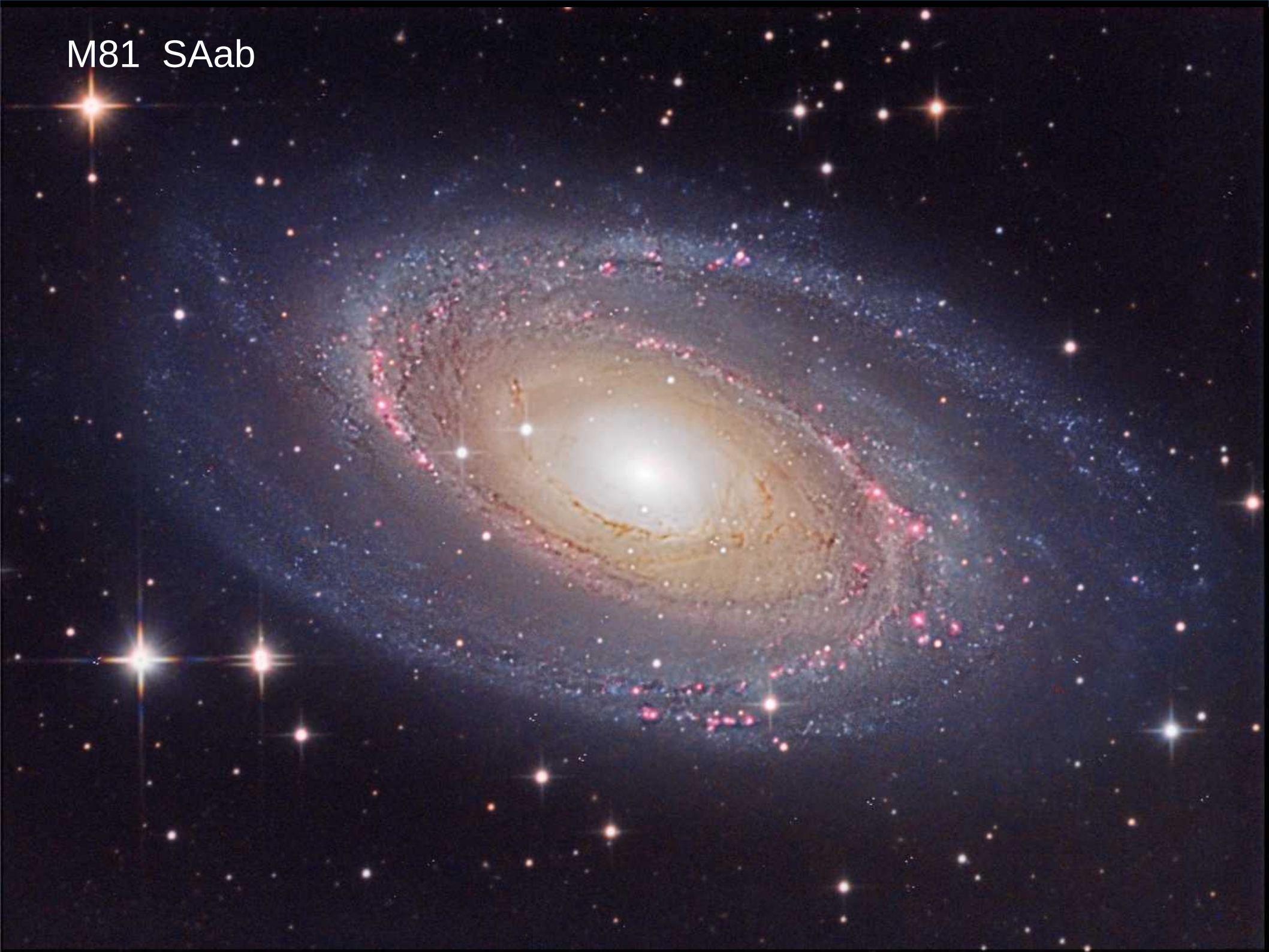


SAa

Sombrero Galaxy • M104



M81 SAab



M33 SAcd



M100 SABbc



M101 SABc



NGC 1365 SBb



NGC 1300 SBb

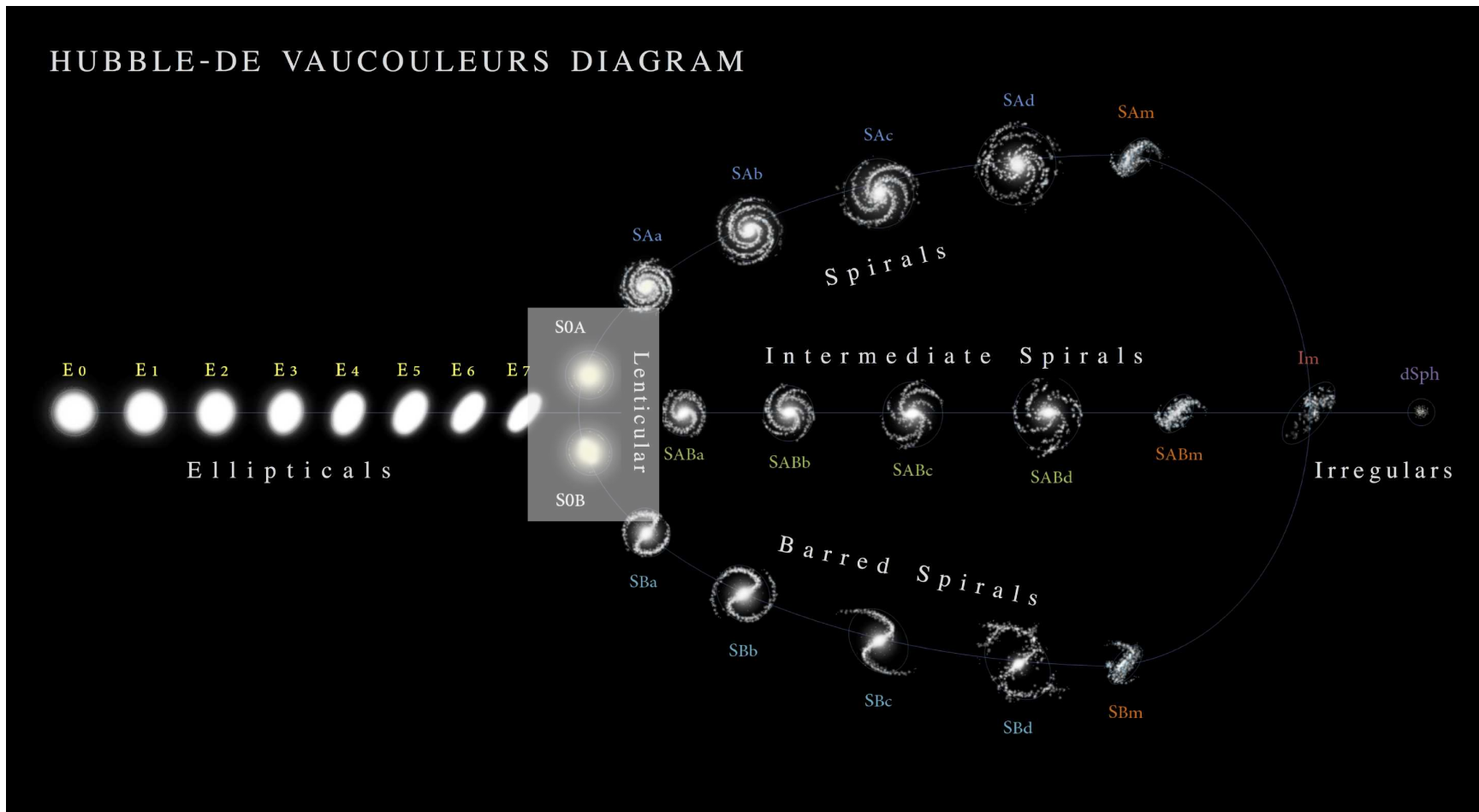


M109 SBc

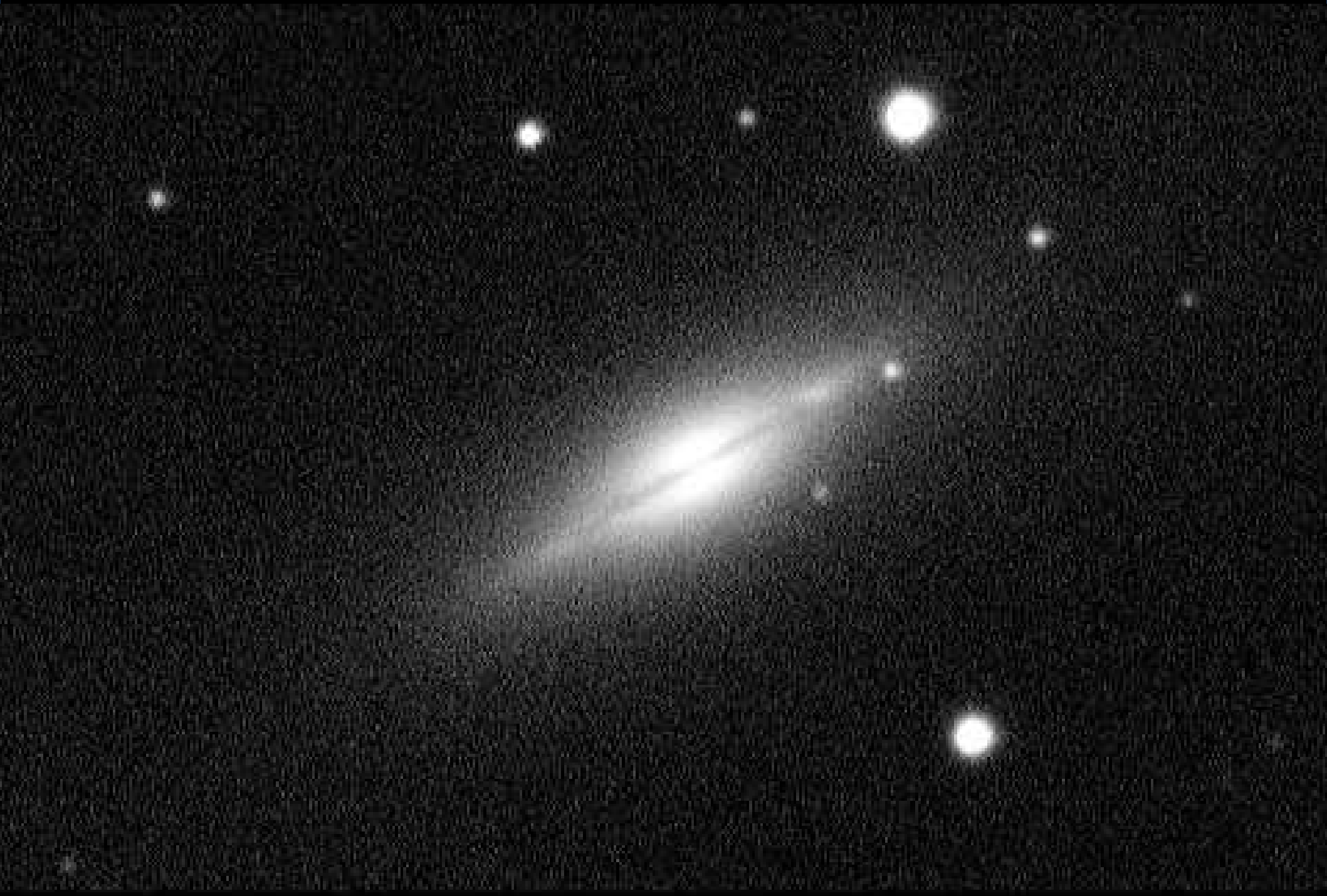


Observation of Galaxies

Lenticular Galaxies



The lenticular galaxy NGC 5866

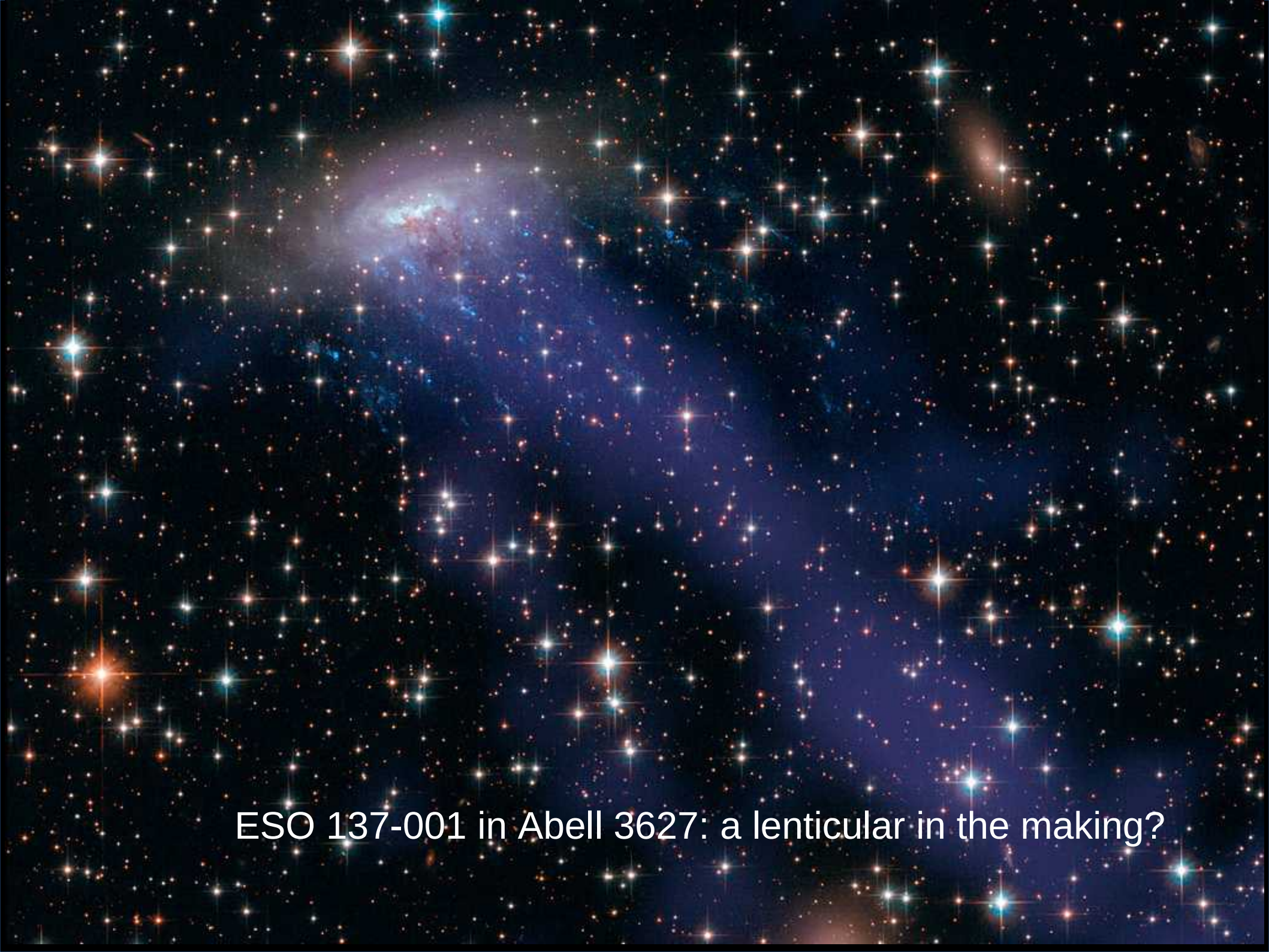


The lenticular galaxy NGC 1381



NGC 7049

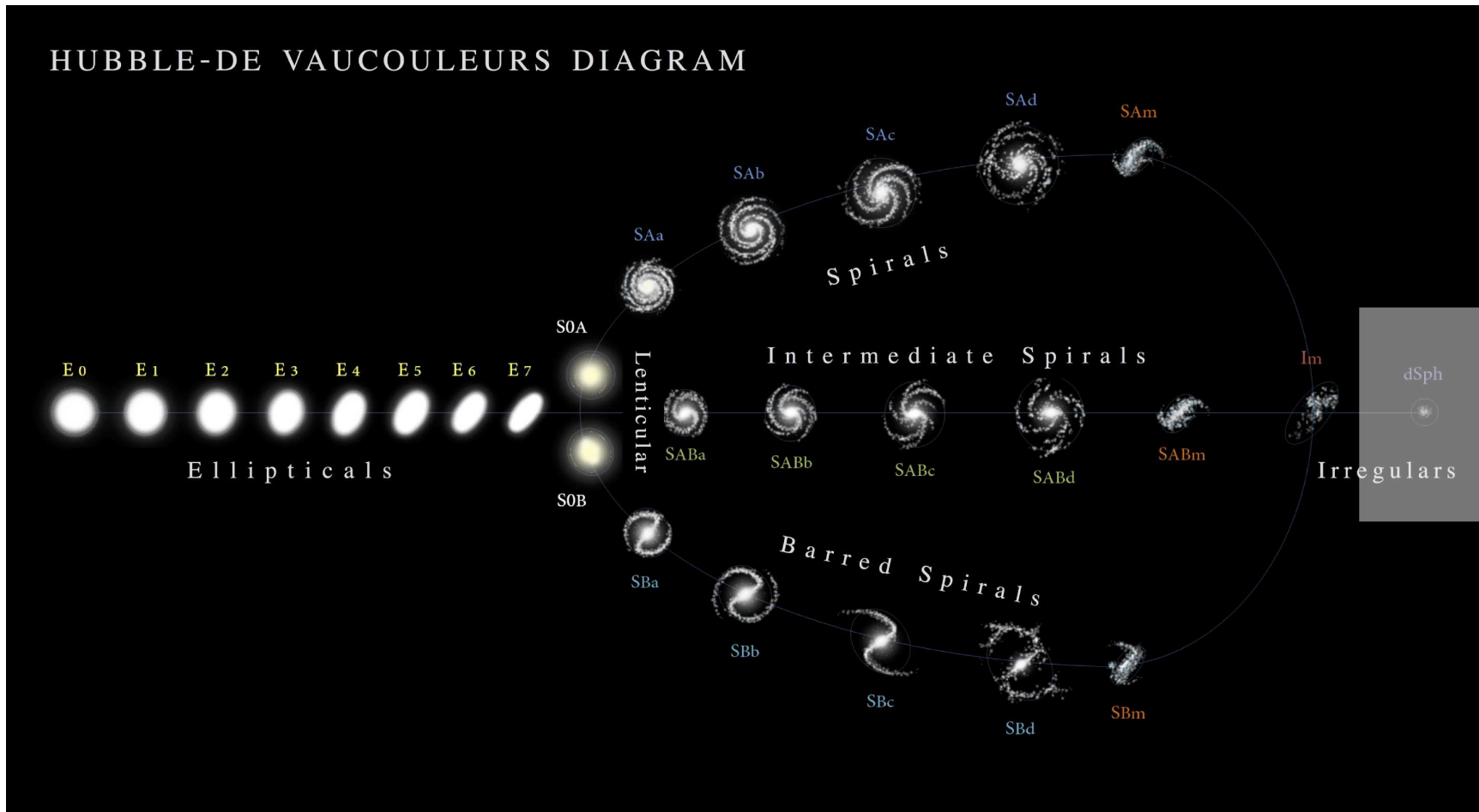




ESO 137-001 in Abell 3627: a lenticular in the making?

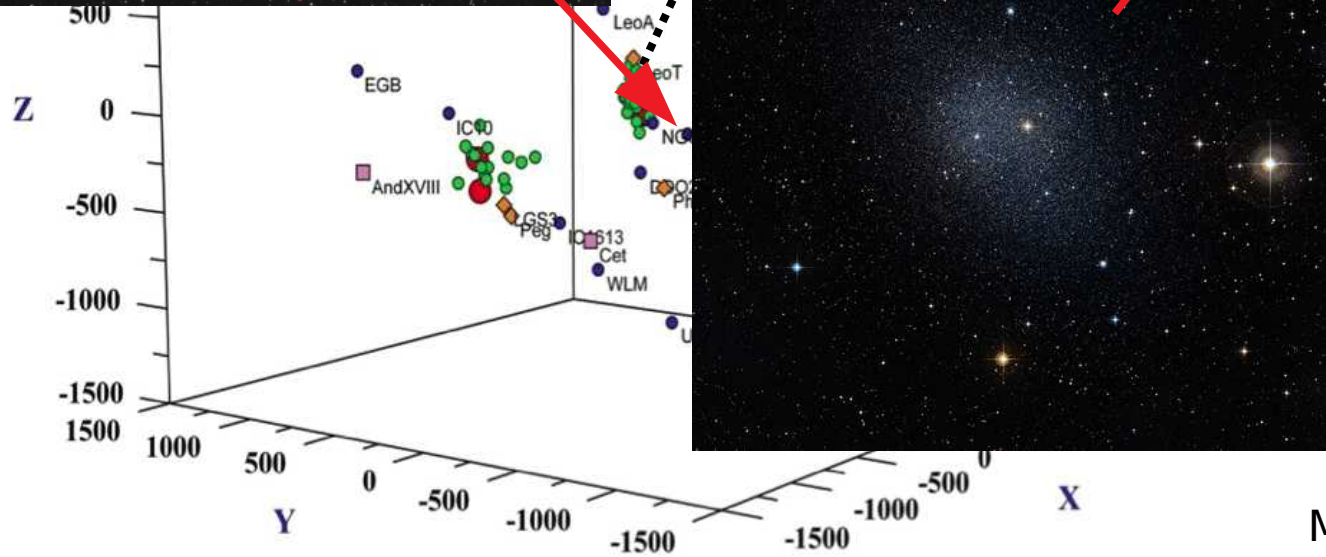
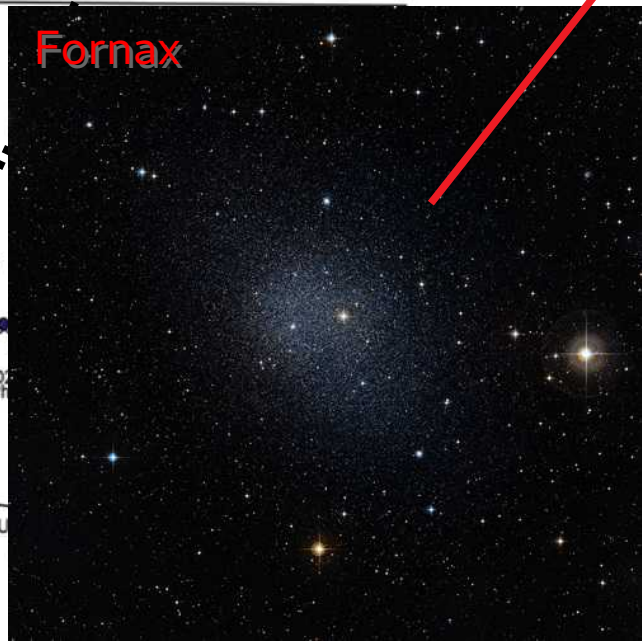
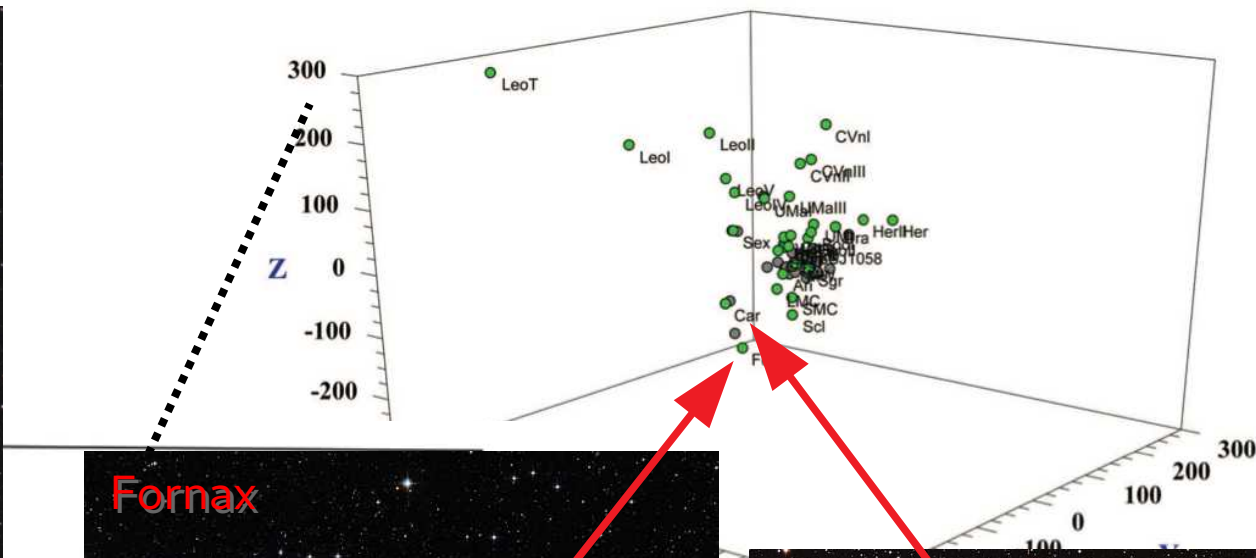
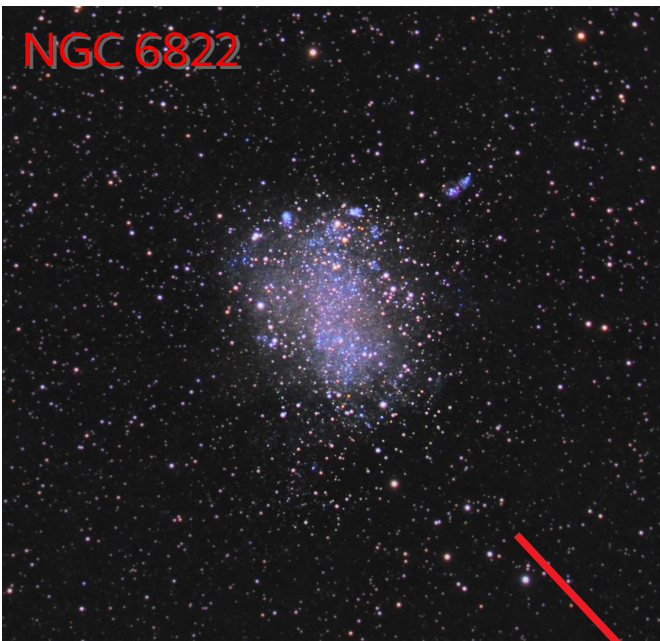
Observation of Galaxies

Dwarf spheroidal (dSph) + ultra-faint dwarfs (UFDs)



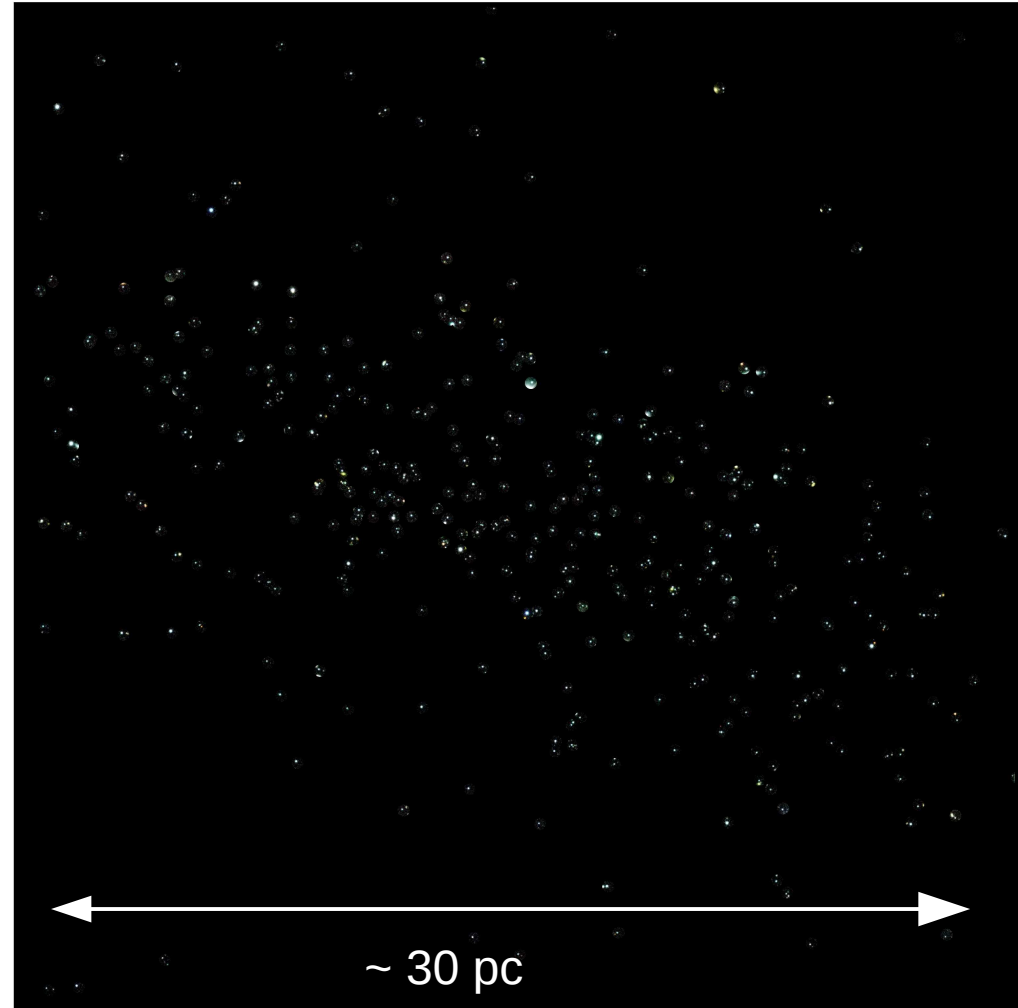
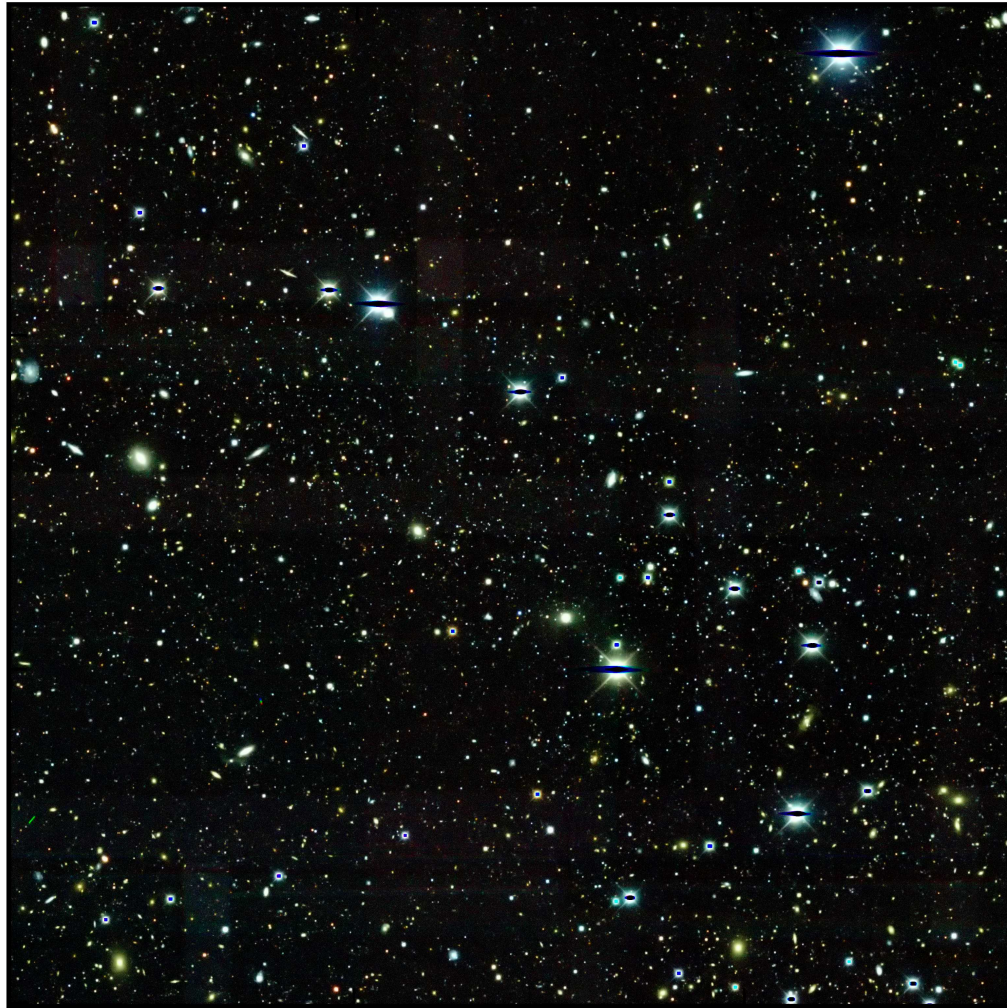
Leo I dwarf galaxy (dSph)





Reticulum II (UFD)

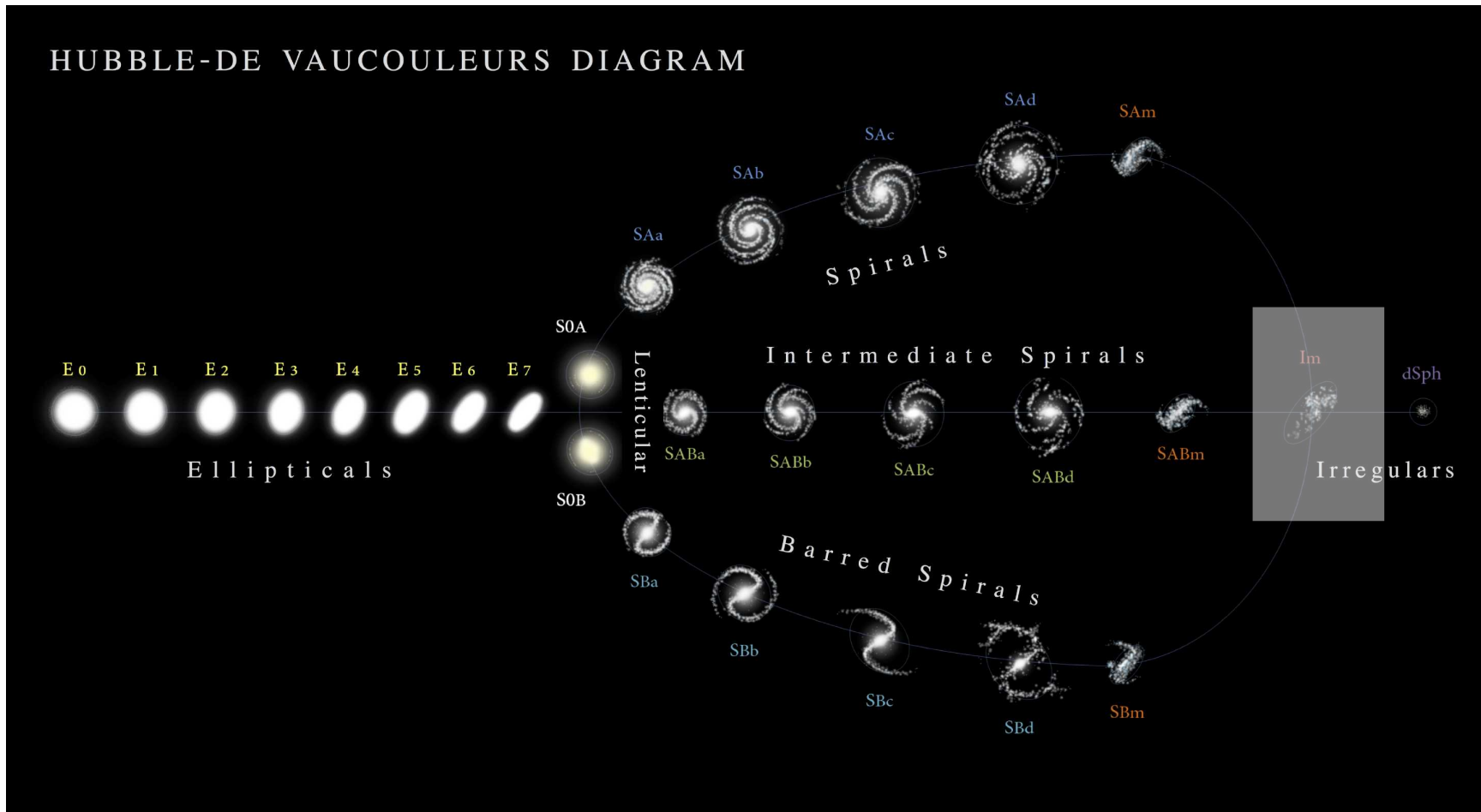
$$L_V \cong 1000L_\odot$$



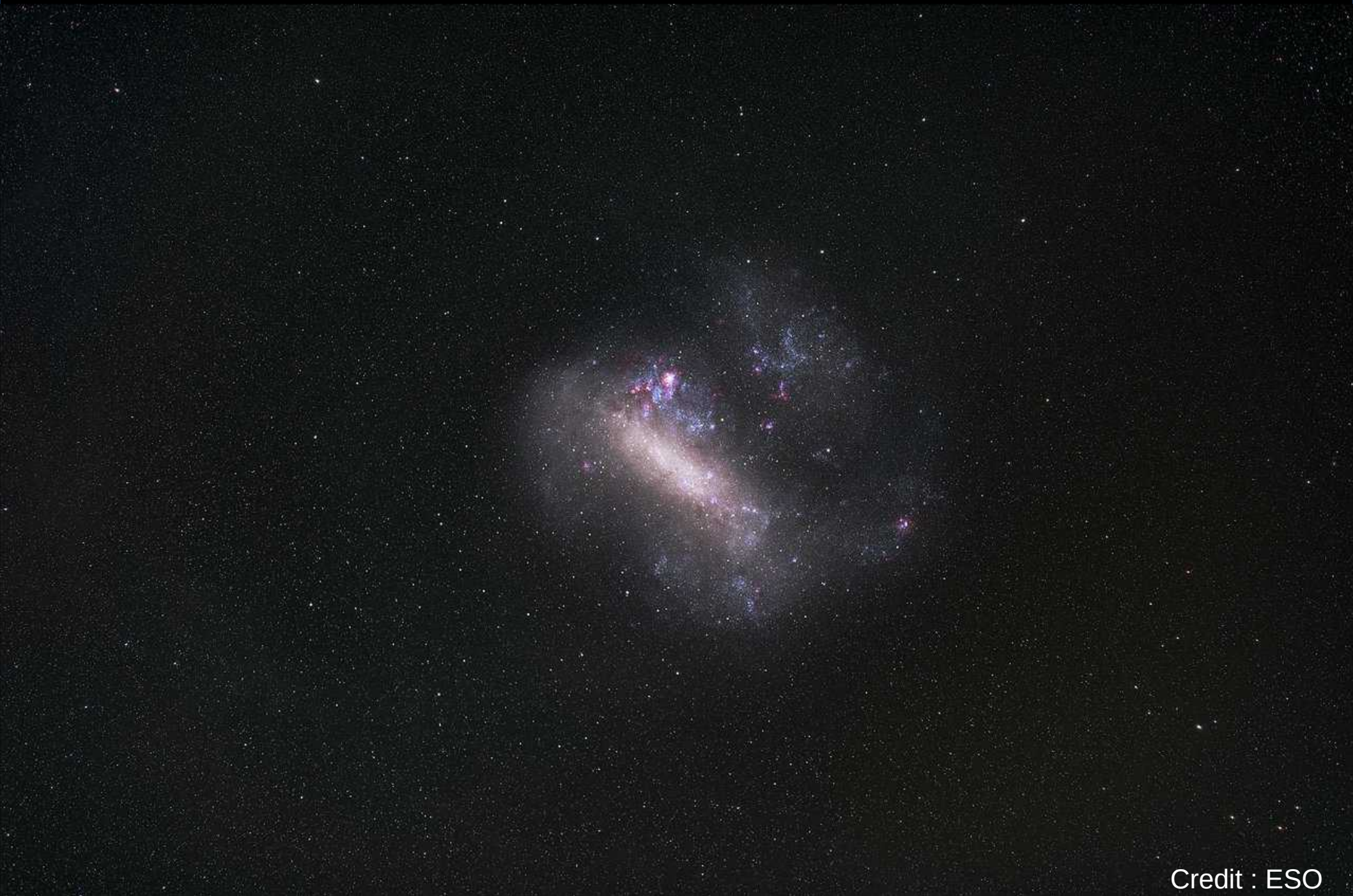
Dark Energy Survey

Observation of Galaxies

Irregular Galaxies

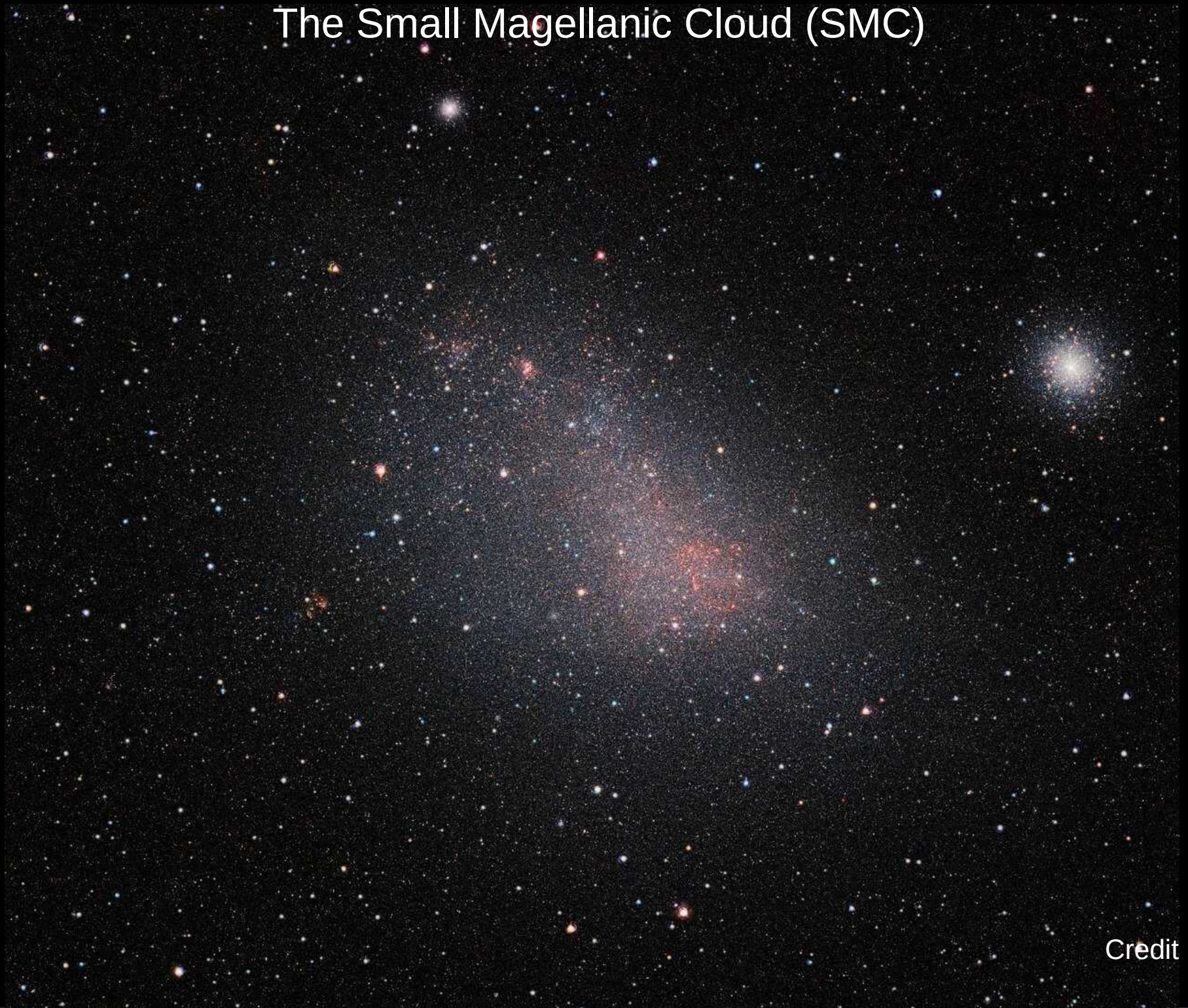


The Large Magellanic Cloud (LMC)



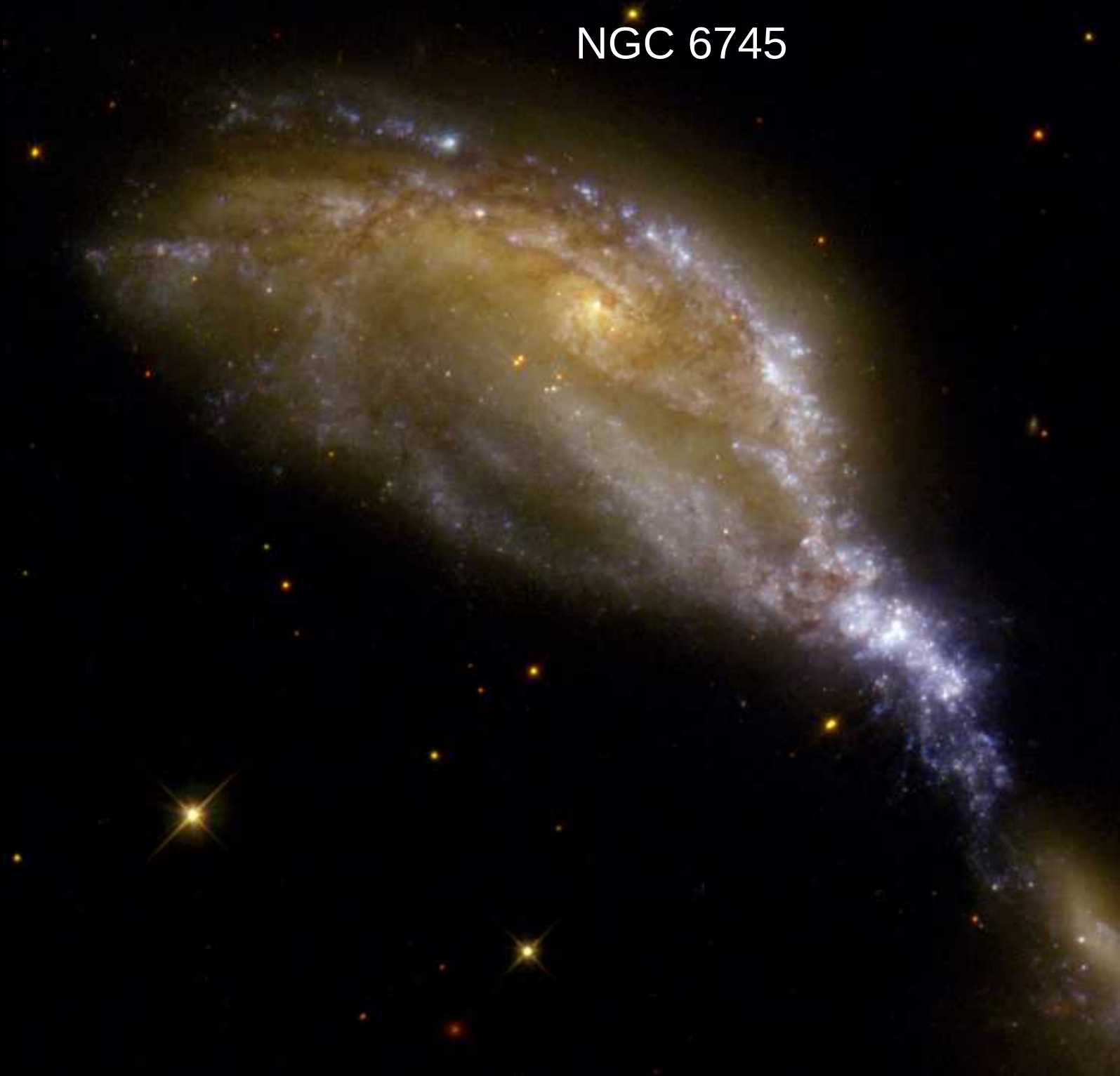
Credit : ESO

The Small Magellanic Cloud (SMC)



Credit : ESO

NGC 6745

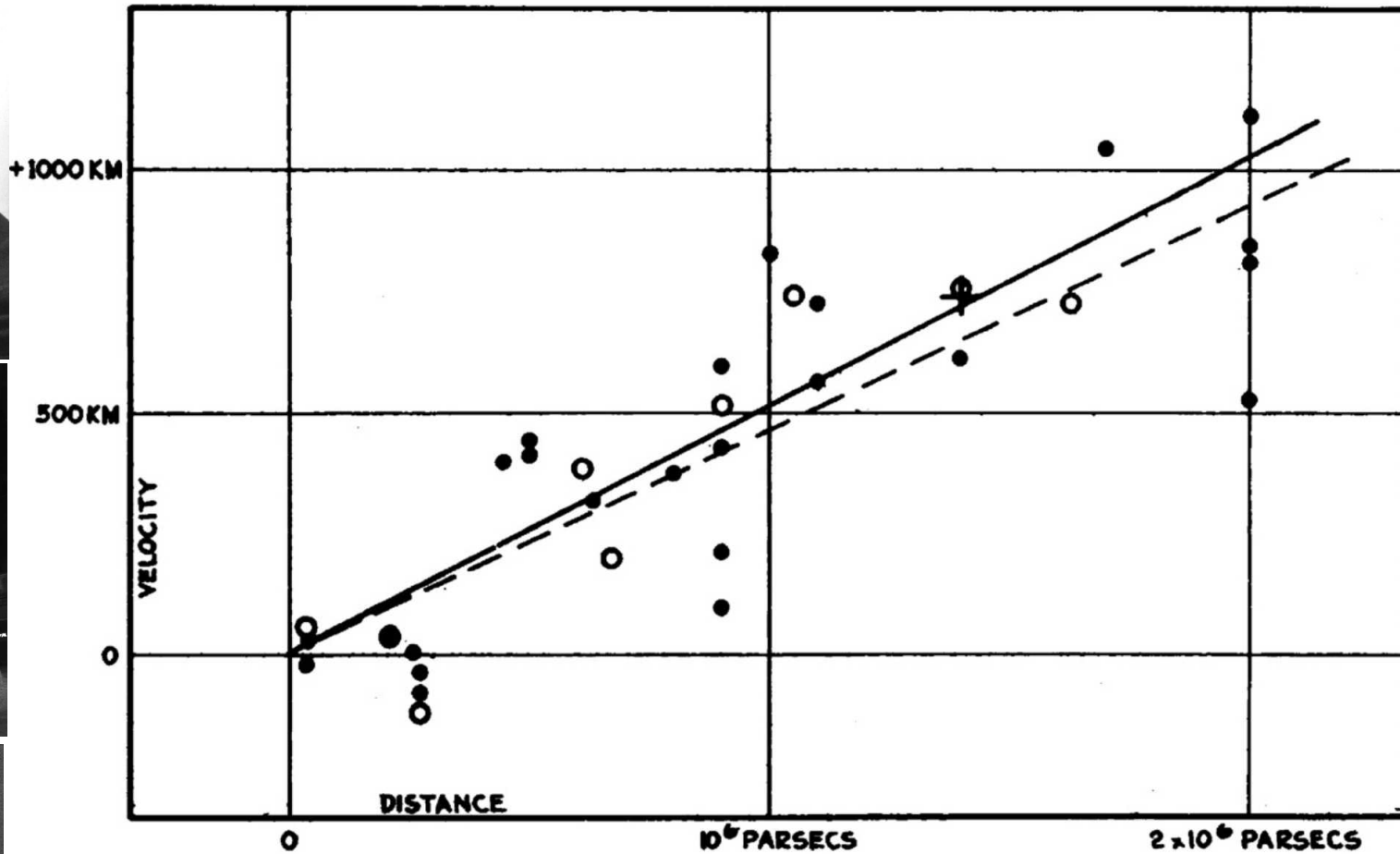


Credit : Hubble

Observation of Galaxies

Hubble Lemaître Law

A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae

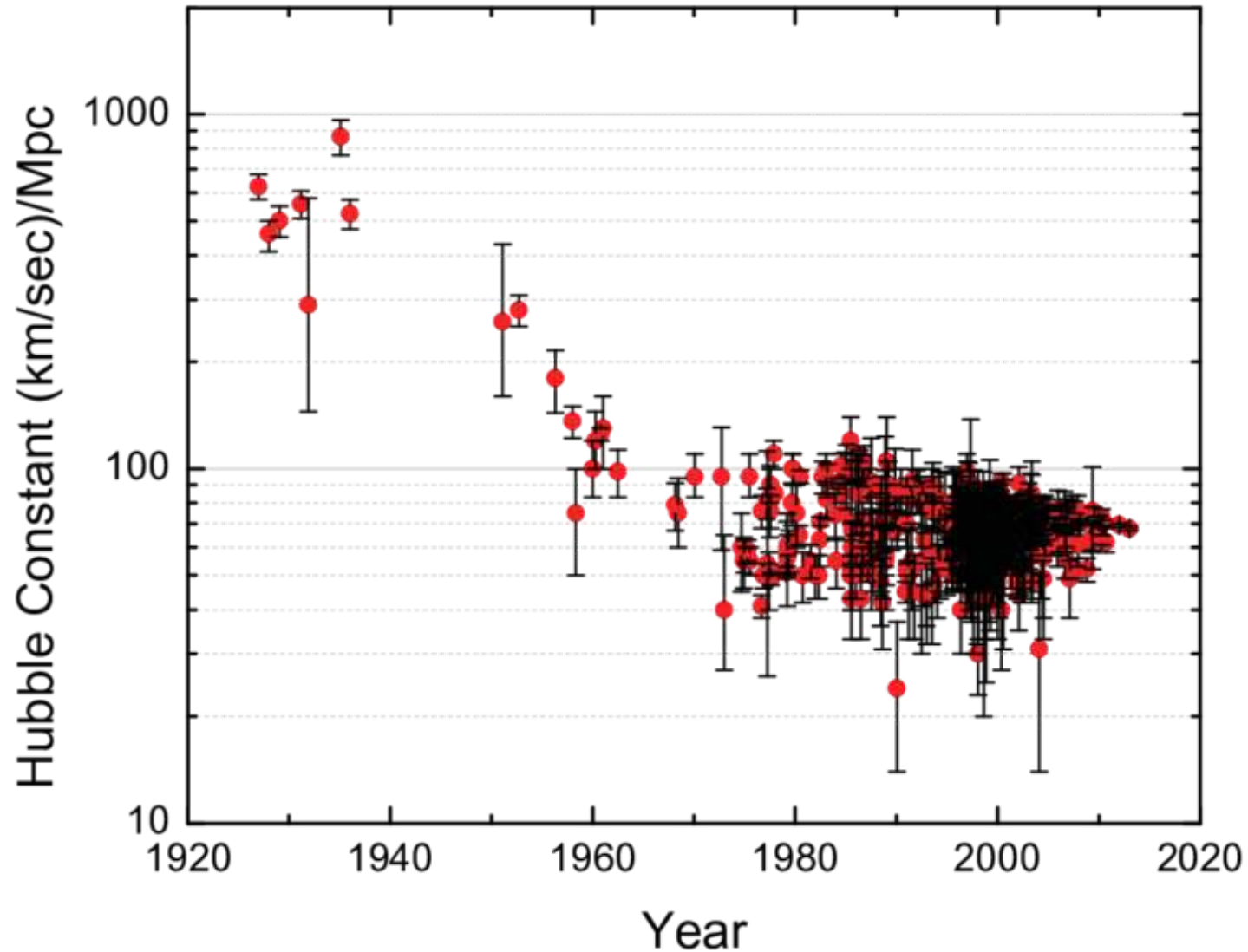


Hubble 1929

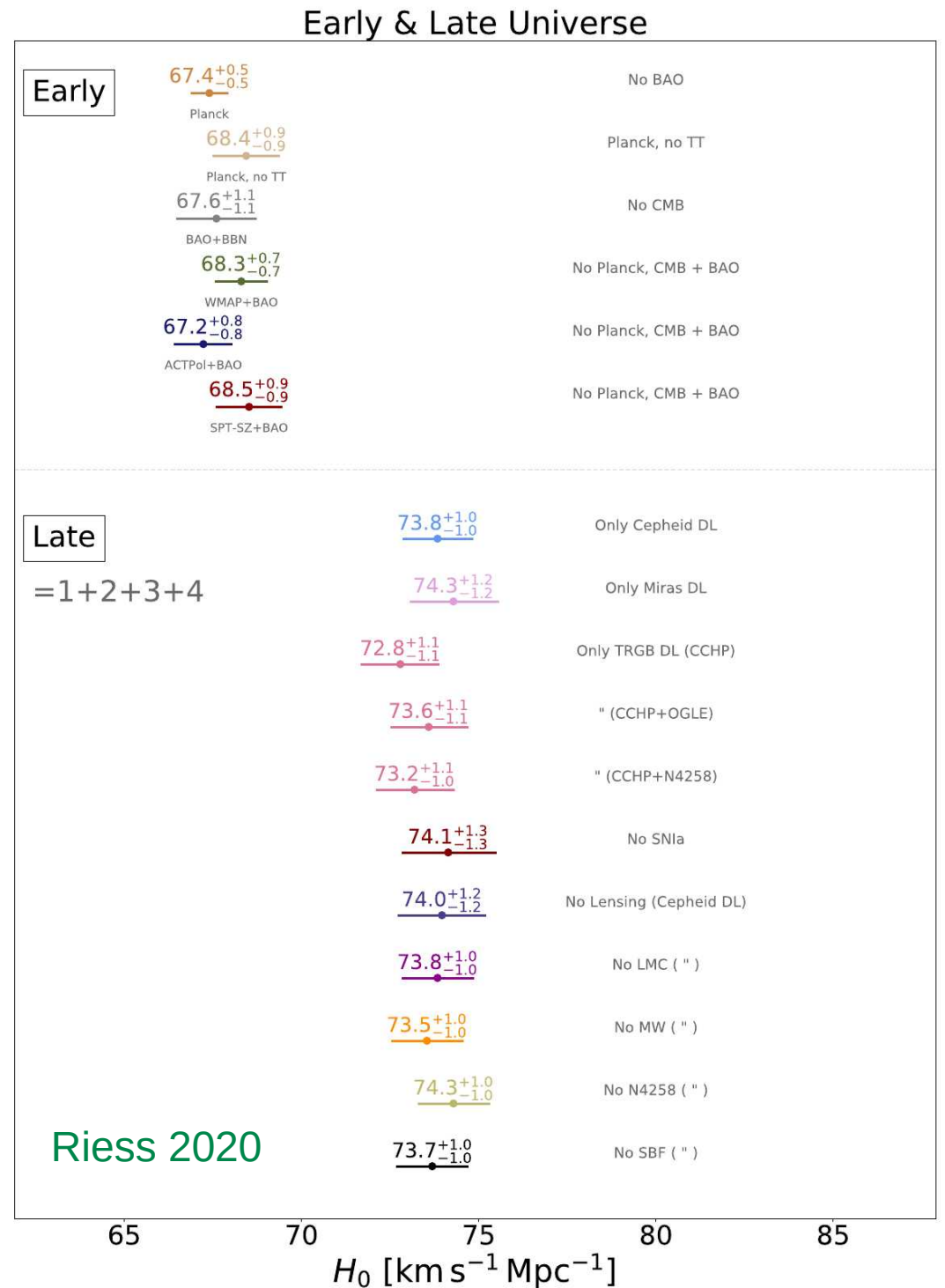
$$v = H_0 d$$

$$H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1} \cong 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

Historical evolution of the Hubble constant

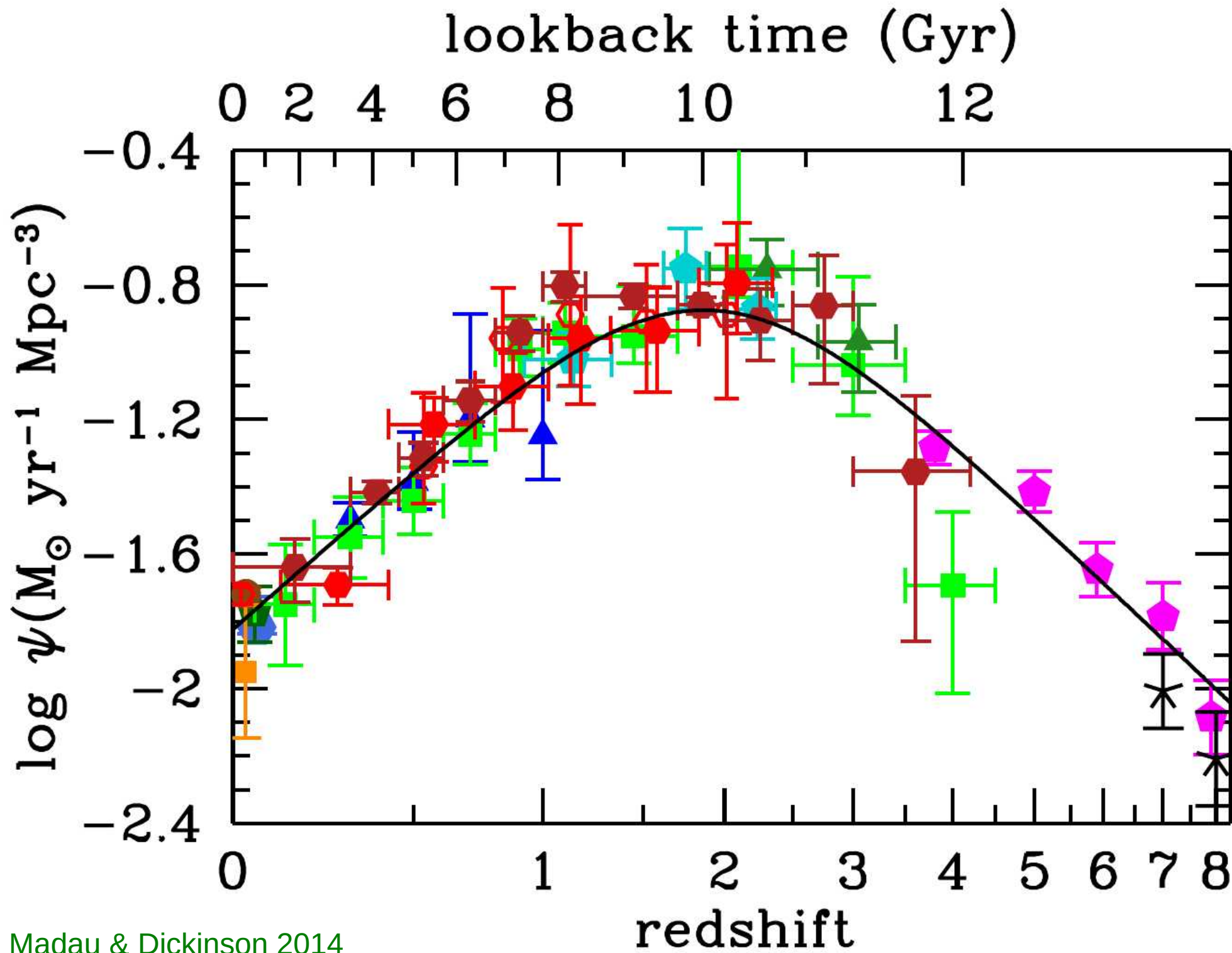


The Hubble Tension



Observation of Galaxies

The cosmic star formation history



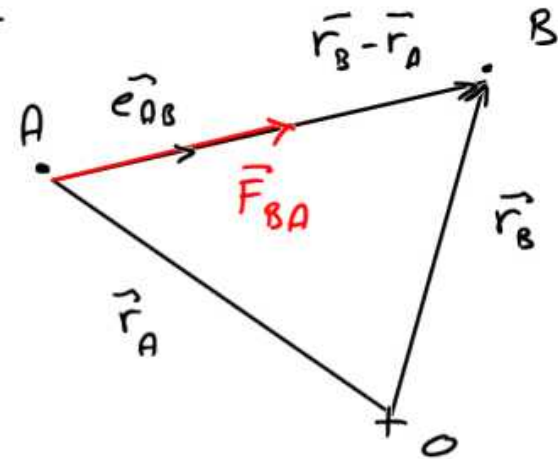
**The Gravity :
a long distance force**

The gravity : a long range force

$$\vec{F}_{BA} = \frac{G m_A m_B}{|\vec{r}_B - \vec{r}_A|^2} \vec{e}_{AB}$$

$$\vec{F}_{BA} = \frac{G m_A m_B}{|\vec{r}_B - \vec{r}_A|^3} \vec{r}_B - \vec{r}_A$$

$$|\vec{F}_{BA}| = \frac{G m_A m_B}{r_{AB}^2}$$



$$\vec{e}_{AB} = \frac{\vec{r}_B - \vec{r}_A}{|\vec{r}_B - \vec{r}_A|}$$

$$r_{AB} = |\vec{r}_B - \vec{r}_A|$$

$$[G] = \frac{\text{cm}^3}{\text{g s}^2} \equiv \frac{\text{erg cm}}{\text{g}^2}$$

$$[\text{erg}] = \frac{\text{cm}^2}{\text{s}^2} \text{ g}$$

Contrary to, for example, molecular forces, gravity is a long range force, i.e.: **we cannot neglect distant regions**

Illustration: an homogeneous medium ($\rho(\vec{r}) = \rho_0$)

Force on point O due to a thin shell of mass Δm at distance r

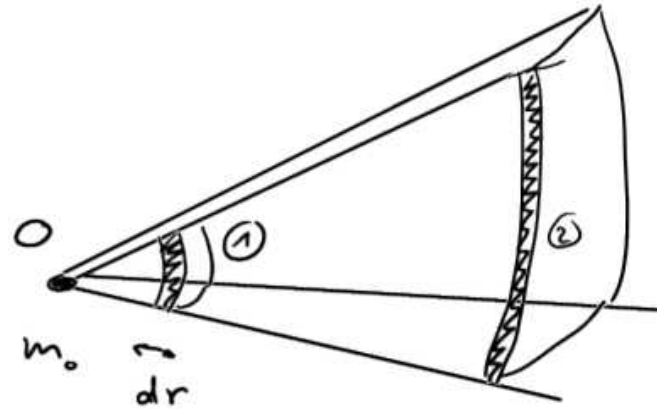
$$\Delta F = \frac{G m_0 \Delta m}{r^2}$$

but $\Delta m = \rho r^2 \Delta \Omega dr$

thus

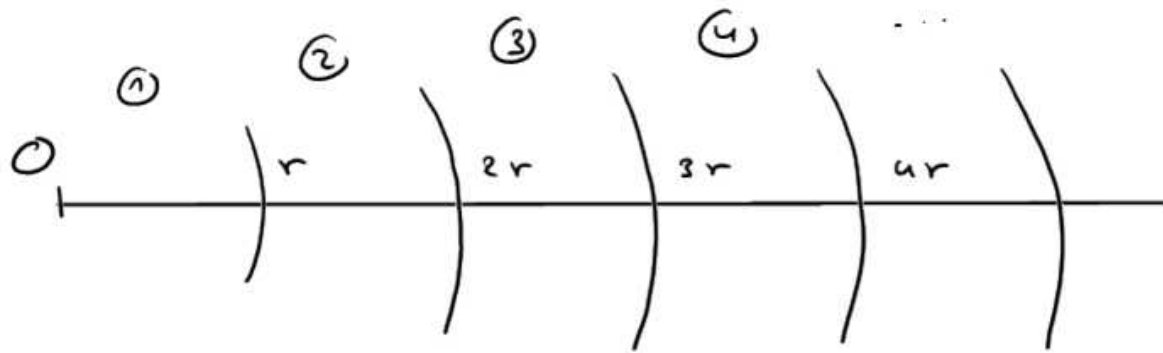
$$\Delta F = G m_0 \rho \Delta \Omega dr$$

etc indep. of r



solid angle $\Delta \Omega$

Split the space in shells of thickness r



$$\Delta F_1 = \int_0^r \Delta F = \int_0^r dr G m_0 \rho \Delta R = G m_0 \rho \Delta R r$$

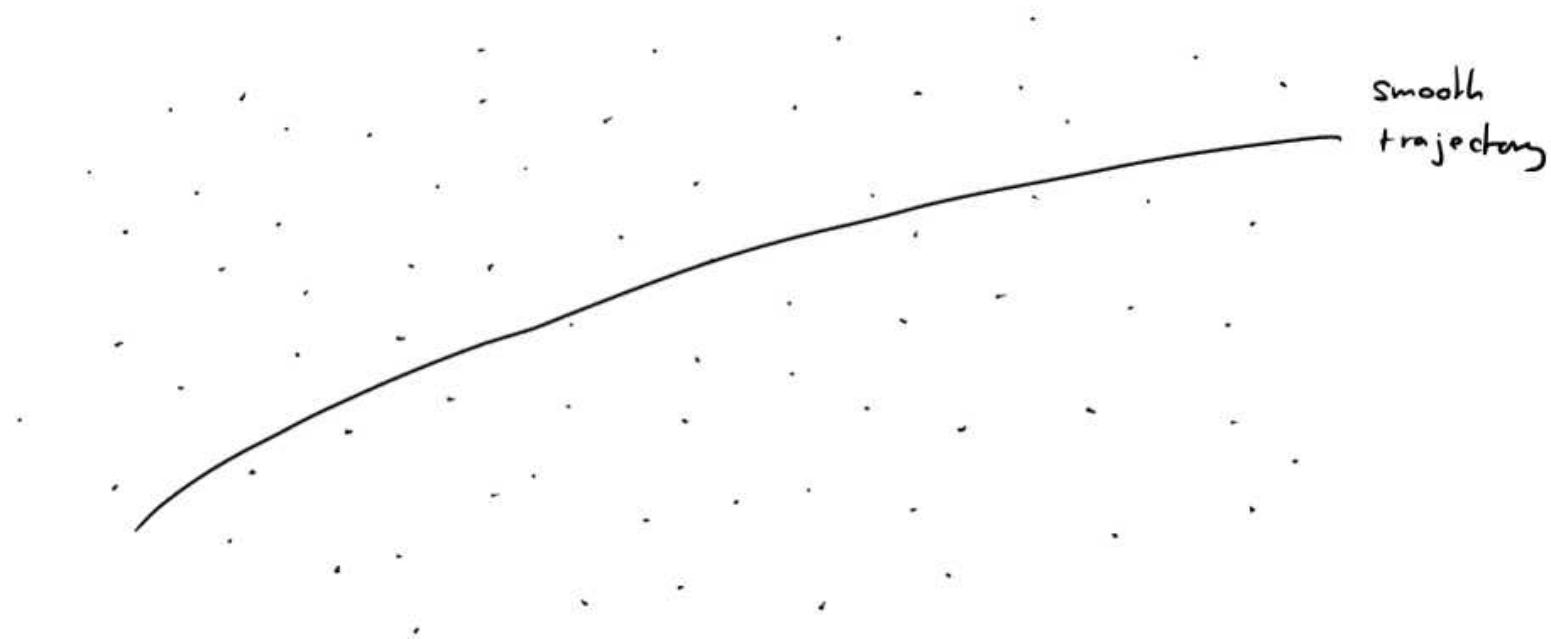
$$\Delta F_2 = \int_r^{2r} \Delta F = \dots = G m_0 \rho \Delta R r$$

As the contribution of all shell is the same, the contribution of the stars with $r' > r$ will dominate over the ones with $r' < r$

We cannot neglect regions at large distances !

Corollary : As the force is dominated by the mass at large scale, the force varies smoothly along the trajectory of a particle (star).

Stellar systems can be modeled by smooth mass distributions



Notes

① 2D case

$$\delta F = - \frac{G m_0}{r^2} \delta m = - \frac{G m_0}{r^2} \Sigma \delta \theta r dr = - G m_0 \rho \delta \theta dr \frac{1}{r}$$

$$F_r = - G m_0 \Sigma \delta \theta$$

Σ : surface density

1D case

$$\delta F = - \frac{G m_0}{r^2} \delta m = - \frac{G m_0}{r^2} \lambda dr$$

$$F_r = - G m_0 \lambda \frac{1}{r}$$

λ : linear density

② Molecular dynamics

"long distance" attractive force
between two molecules,

Van der Waals Force $\sim r^{-7}$

\Rightarrow local molecules dominates

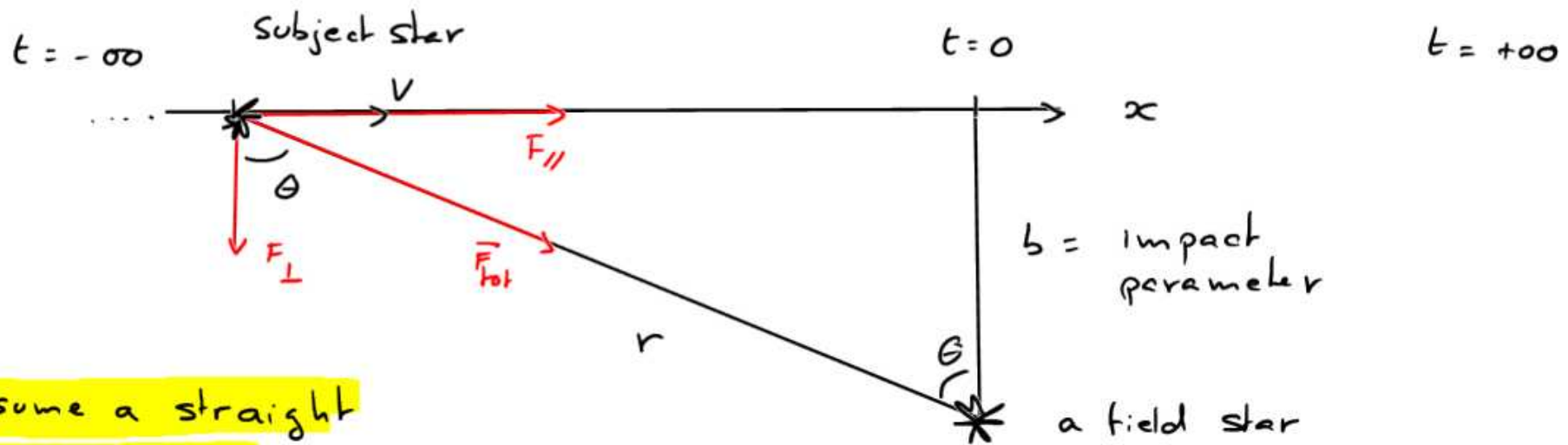
Relaxation Time

Question :

How accurate is the assumption that a galaxy may be modelled as a smooth distribution ?

- 1) Effect of one star on the orbit of a peculiar star.
- 2) Effect of all stars of a stellar system on a peculiar star.
- 3) Under which conditions the orbit of a peculiar star is strongly influenced by the discrete nature of the stellar system (importance of “collision” with other stars).

- ① Estimate the effect of one star on the trajectory of a peculiar star
-



We assume a straight line trajectory

- 1) acceleration along x (F_{\parallel}) does not matter, as it is symmetric (the star decelerate after passing the field star)
- 2) acceleration perpendicular to x (F_{\perp})

$$\begin{aligned}
 |F_{\perp}| &= |F_{\text{tot}}| \cos \Theta = \frac{Gmm}{r^2} \cos \Theta && \text{but } \cos \Theta = \frac{b}{r} \\
 &= \frac{Gmm}{x^2 + b^2} \frac{b}{\sqrt{x^2 + b^2}} = \frac{Gmm}{(x^2 + b^2)^{3/2}} b \\
 &= \frac{Gmm}{\left(1 + \frac{x^2}{b^2}\right)^{3/2}} \frac{1}{b^2} = \frac{Gmm}{b^2} \left(1 + \frac{x^2}{b^2}\right)^{-3/2}
 \end{aligned}$$

with $x = vt$

$$F_{\perp} = |F_{\perp}| = \frac{Gmm}{b^2} \left(1 + \left(\frac{vt}{b}\right)^2\right)^{-3/2}$$

Newton 2nd law

$$F_{\perp} = m a_{\perp} = m \frac{dV_{\perp}}{dt}$$

Integrating over time from $t = -\infty$ to $t = \infty$

$$\int_{-\infty}^{\infty} F_{\perp} dt = \int_{-\infty}^{\infty} m \frac{dV_{\perp}}{dt} dt = m \Delta V_{\perp} \quad \text{net velocity increase}$$

$$= \int_{-\infty}^{\infty} \frac{GmM}{b^2} \left(1 + \left(\frac{vt}{b}\right)^2\right)^{-\frac{3}{2}} dt$$

$$\Delta V_{\perp} = \frac{GmM}{b^2} \int_{-\infty}^{\infty} \left(1 + \left(\frac{vt}{b}\right)^2\right)^{-\frac{3}{2}} dt$$

$$\text{with } s = \frac{vt}{b} \quad ds = \frac{v}{b} dt$$

$$\delta v_{\perp} = \frac{Gm}{b^2} \int_{-\infty}^{\infty} (1 + s^2)^{-\frac{3}{2}} dt = 2 \frac{Gm}{bv}$$

$$\delta v_{\perp} = \frac{Gm}{b^2} \cdot \frac{2b}{v}$$

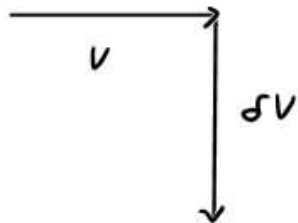
$\underbrace{\hspace{10em}}$
 acceleration
 at the closest
 approach

$\underbrace{\hspace{10em}}$
 "duration"
 of the closest
 approach

Note : our hypothesis of a straight line is ok if $\frac{\delta v}{v} \ll 1$

$$\Rightarrow b \gg \frac{2Gm}{v^2} = b_{g0}$$

b_{g0} define
 as $v = \delta v$



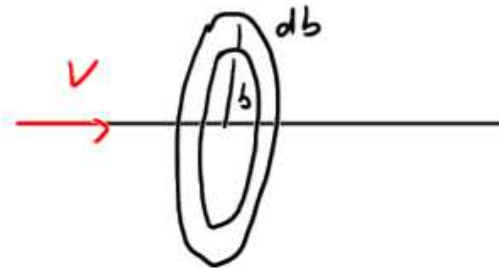
② Effect of all stars of a stellar system on the trajectory of a peculiar star

N : total number of stars

R typical size of the system

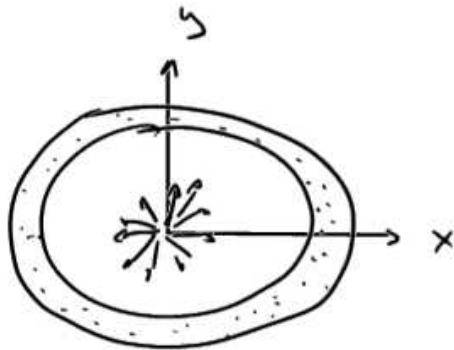
- number density of stars per unit of surface $n = \frac{N}{\pi R^2}$
- number of stars met by the star with $[b, b + db]$
which induces a $\delta v_{\perp} = \frac{e G m}{b v}$ change of v_{\perp}

$$\begin{aligned} \delta N &= 2\pi b db \cdot n \\ &= 2\pi b db \frac{N}{\pi R^2} \end{aligned}$$

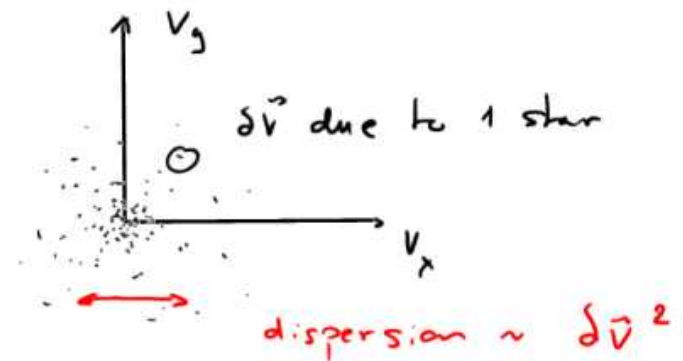
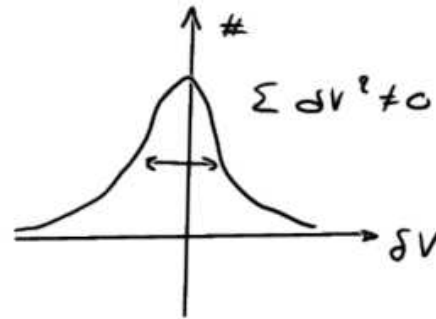


Each star in the ring induces a $\delta\vec{V}$ of the same amplitude, such that in average $\sum \delta\vec{V} \cong 0$

however $\sum \delta\vec{V}^2 \neq 0$



configuration space



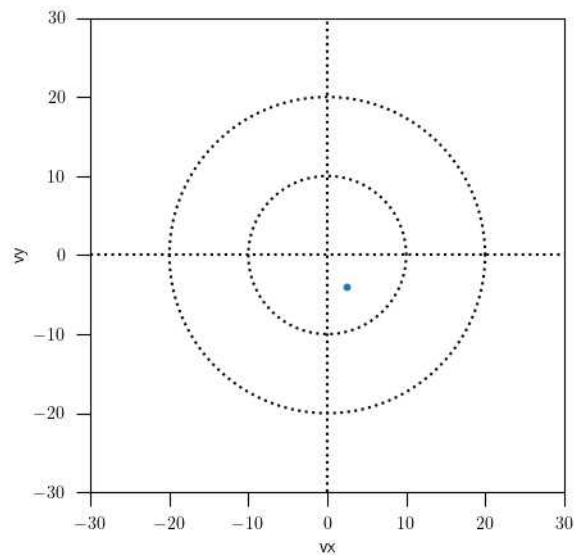
velocity space

Estimation of $\sum \delta\vec{V}^2$

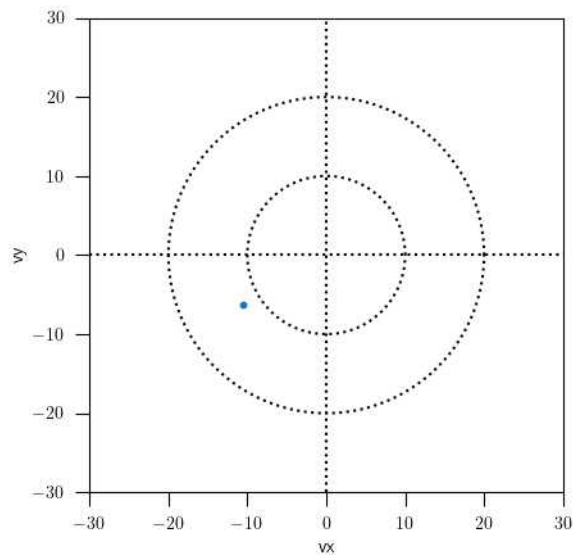
$$\sum \delta\vec{V}^2 = \delta N \cdot \delta\vec{V}^2 = \left(2\pi b db \frac{N}{\pi R^2} \right) \left(\frac{2 G m}{b v} \right)^2 = \frac{8 N G^2 m^2}{v^2 R^2} \frac{db}{b}$$

Spread in the velocity space due to two-body encounters

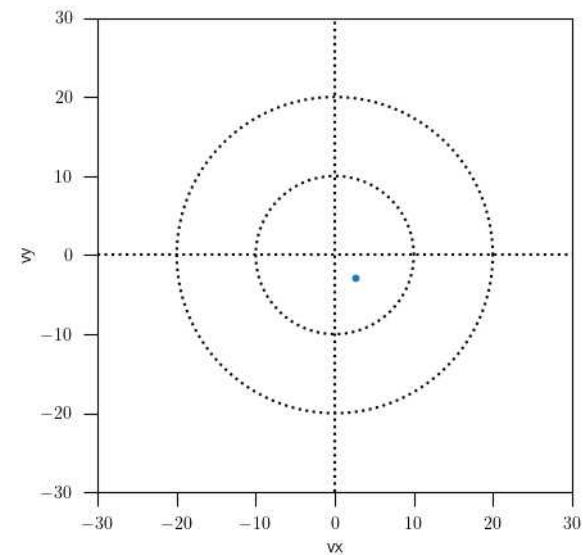
N=1



N=1

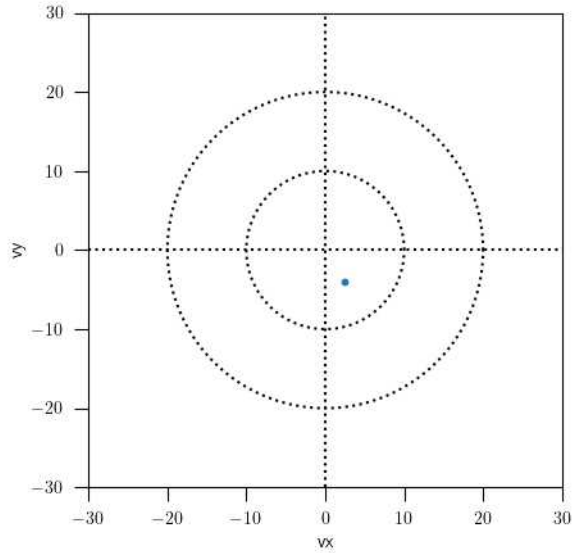


N=1

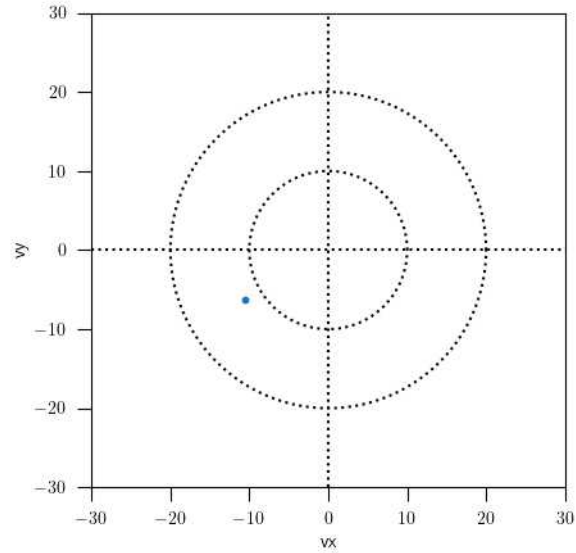


Spread in the velocity space due to two-body encounters

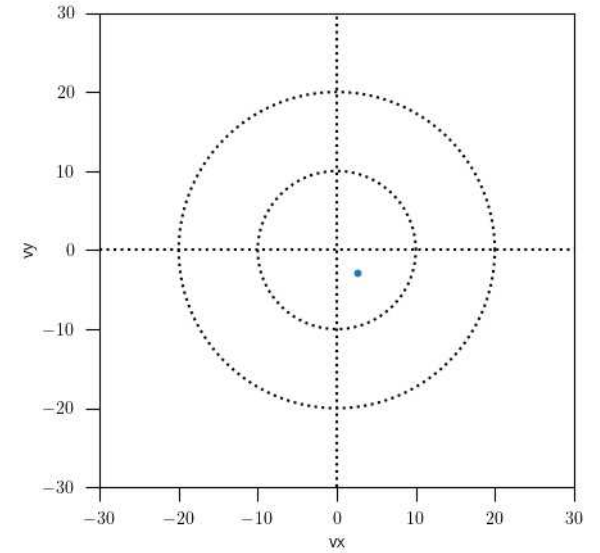
N=1



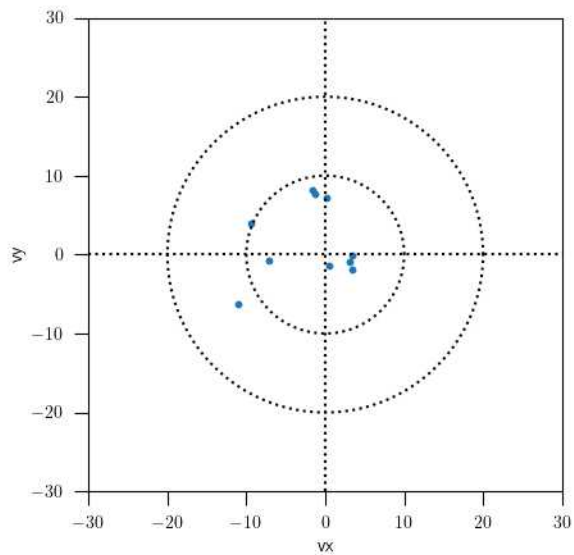
N=1



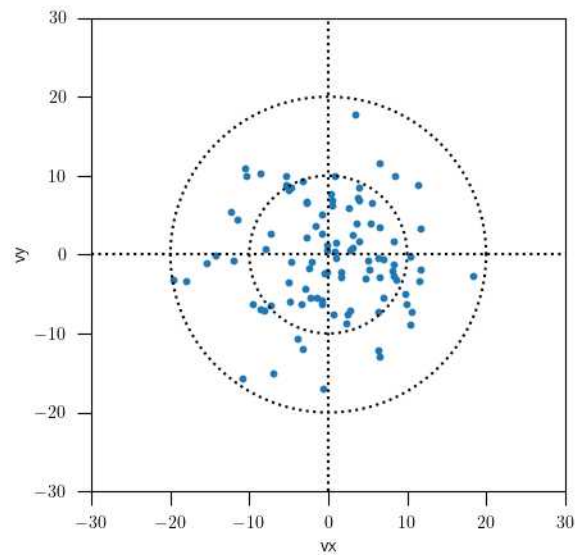
N=1



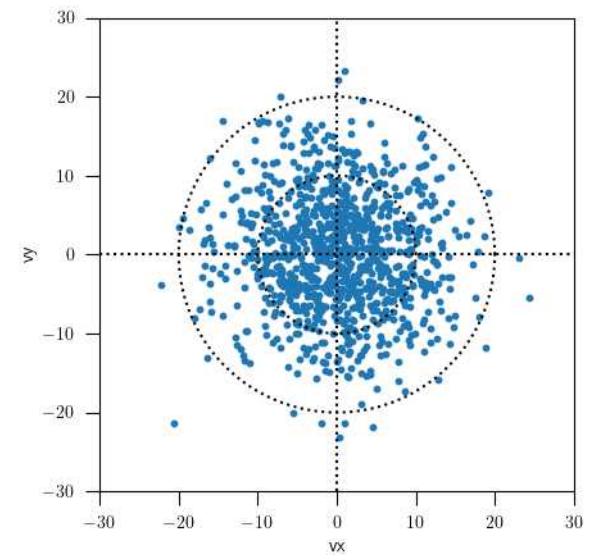
N=10



N=100



N=1000



For all encounters, we integrate over b from b_{\min} to b_{\max}

$$b_{\min} := \beta_1 b_{s0} \quad \text{if } b < b_{s0} \quad \Delta V \sim v \quad \beta_1 \approx 1$$

$$b_{\max} := \beta_2 R \quad \text{if } b > R \quad \text{the density is no longer constant} \quad \beta_2 \approx 1$$

we get

ln : Coulomb logarithm

$$\Delta V^2 = 8N \left(\frac{Gm}{vR} \right)^2 \int_{b_{\min}}^{b_{\max}} \frac{db}{b} = 8N \left(\frac{Gm}{vR} \right)^2 \ln \left(\frac{b_{\max}}{b_{\min}} \right)$$

$$\Delta V^2 = 8N \left(\frac{Gm}{vR} \right)^2 \left[\ln \left(\frac{R}{b_{s0}} \right) + \ln \left(\frac{\beta_2}{\beta_1} \right) \right]$$

- variation due to one crossing.

≈ 0

Crossing time / relaxation time

Typical velocity of one star (circular orbit)

$$v^2 \sim \frac{GMm}{R}$$

$$R = \frac{GMm}{v^2}$$

$$\Delta v^2 = 8N \left(\frac{Gm}{vR} \right)^2 \ln \Lambda = 8N \left(\frac{v}{N} \right)^2 \ln \Lambda = 8 \frac{v^2}{N} \ln \Lambda$$

$$\Delta v^2 = 8 \frac{v^2}{N} \ln \Lambda$$

- Each time a star will cross the system, the square of its velocity will change by an amount ΔV^2

$$n_{\text{cross}} \Rightarrow \Delta V_{n_{\text{cross}}}^2 = n_{\text{cross}} \cdot V^2 \frac{8}{N} \ln \Lambda$$

- n_{relax} : number of crossing time to have $\Delta V^2 \cong V^2$

$$n_{\text{relax}} \cdot V^2 \frac{8}{N} \ln \Lambda = V^2$$

$$n_{\text{relax}} = \frac{N}{8 \ln \Lambda}$$

Crossing time t_{cross}

Time for the star to cross the system

$$t_{\text{cross}} \cong \frac{R}{V} \quad \left(V^2 \sim \frac{GM_{\text{in}}}{R} \right)$$

Relaxation time t_{relax}

Time for n_{relax} crossing.

$$t_{\text{relax}} = \frac{N}{8 \ln 2} \cdot t_{\text{cross}}$$

\equiv Time after which a star change significantly its orbit with respect to a smooth density field

If $t < t_{\text{relax}}$: the perturbations of nearby stars does not matter \Rightarrow
collisionless system

Estimations for stellar systems

N, m, R

using

$$b_{50} = \frac{2Gm}{v^2} \quad \text{circular orbit: } a = \frac{v^2}{r} = \frac{GNm}{r^2} \Rightarrow v^2 = \frac{GNm}{r}$$

we get

$$b_{50} = \frac{2R}{N}$$

$$\frac{R}{b_{50}} = \frac{N}{2}$$

$$\ln \mathcal{L} \approx \ln N$$

$$t_{\text{rdax}} = \frac{N}{8 \ln \mathcal{L}} \cdot t_{\text{cross}} \Rightarrow$$

$$t_{\text{rdax}} \approx \frac{0.1 N}{\ln N} t_{\text{cross}}$$

$$\ln \mathcal{L} \approx \ln N$$

$$\frac{1}{8} \approx 0.1$$

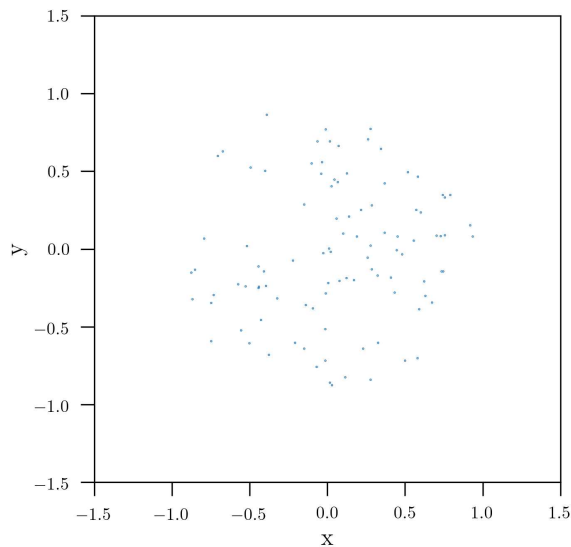
Numerical application / Relaxation Time

	N	R	b_{90}	$\ln(R/b_{90})$	t_{relax}
Globular Cluster	10^5	10 pc	2×10^{-4} pc	~ 10	~ 1 Gyr
Dwarf Galaxy	10^6	1 kpc	2×10^{-3} pc	~ 13	~ 4 Gyr
Spiral Galaxy	10^{10}	15 kpc	3×10^{-6} pc	~ 20	$\gg t_{\text{Hubble}}$
Galaxy Cluster	10^{13}	1 Mpc	2×10^{-7} pc	~ 30	$\gg t_{\text{Hubble}}$

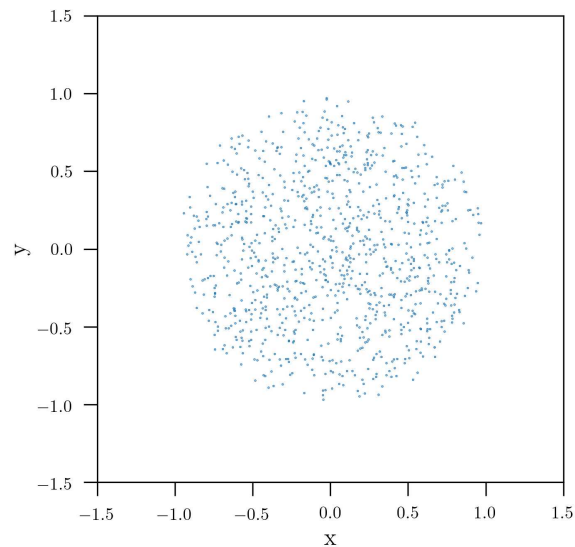
Numerical illustration

Orbit of a point mass in an homogeneous sphere
sampled with a discrete number of stars.

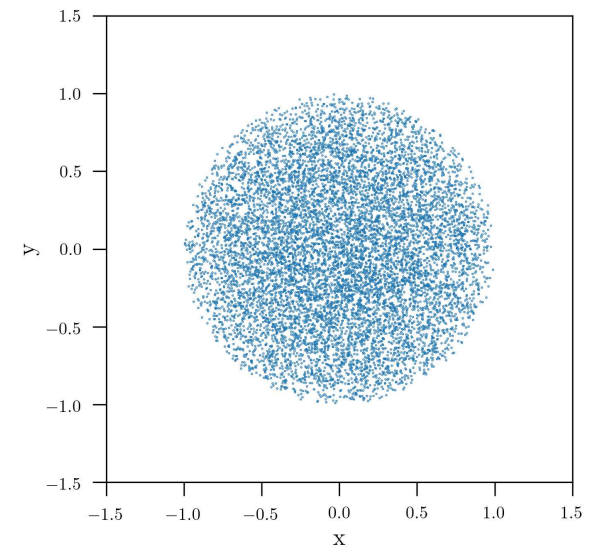
$N = 100$



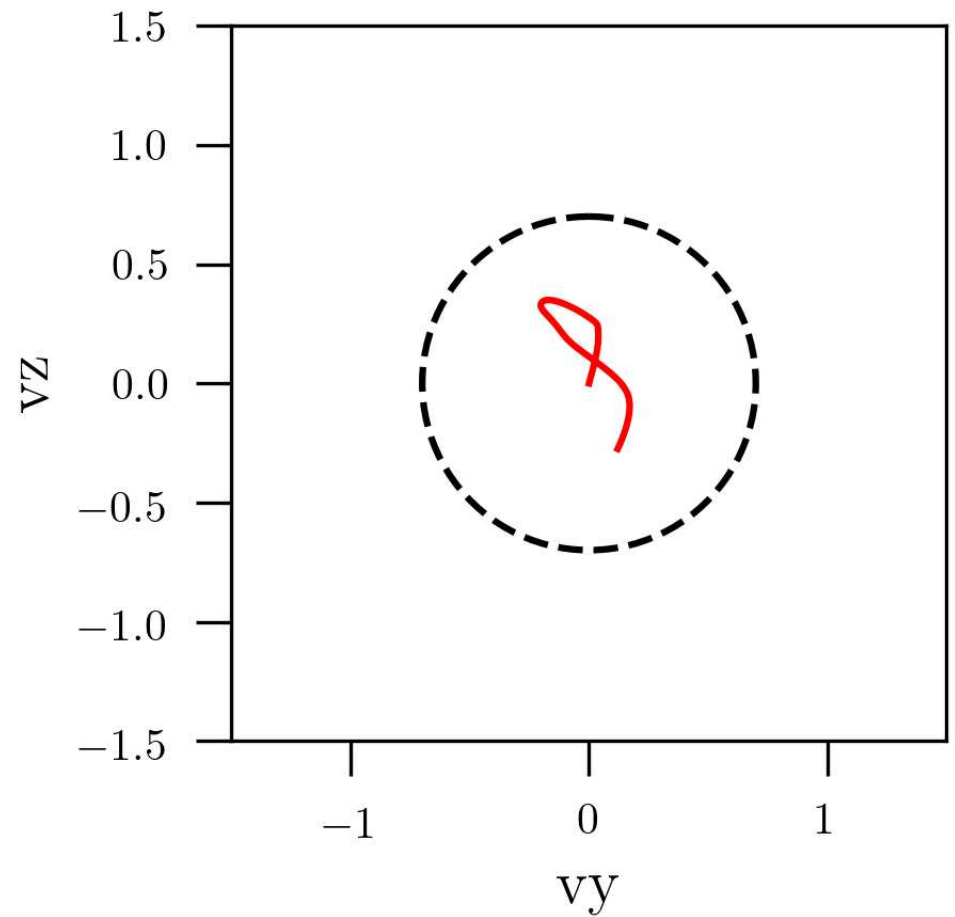
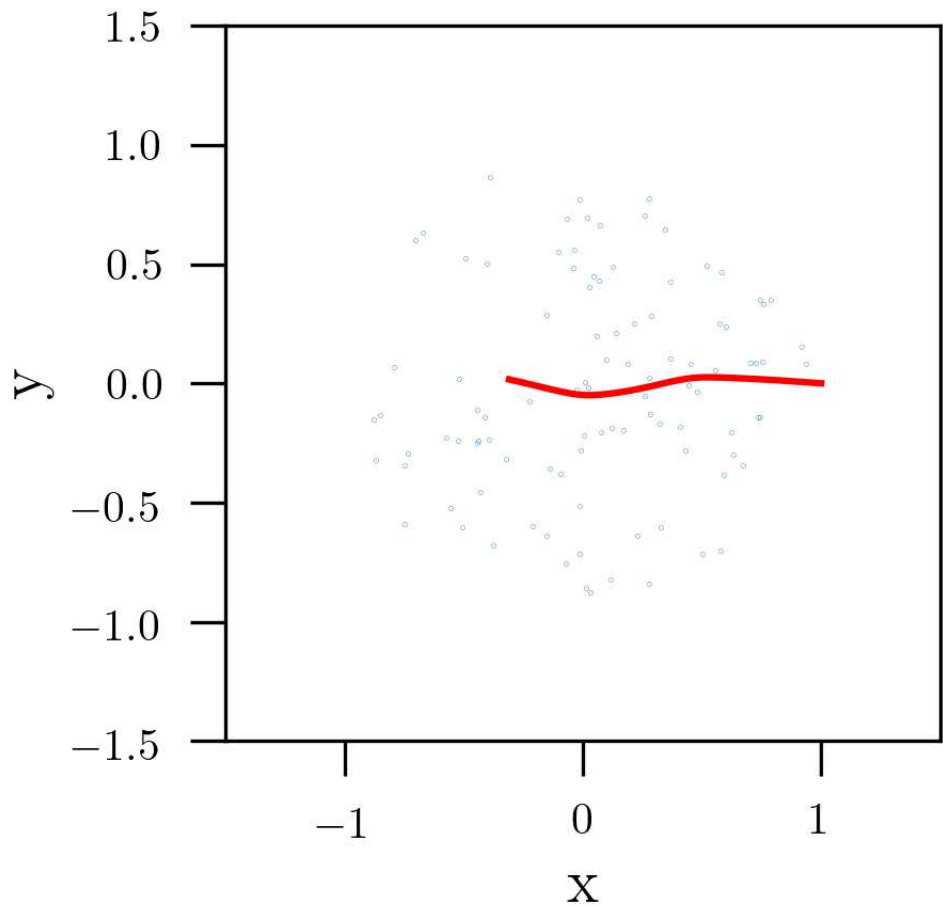
$N = 1000$



$N = 10000$

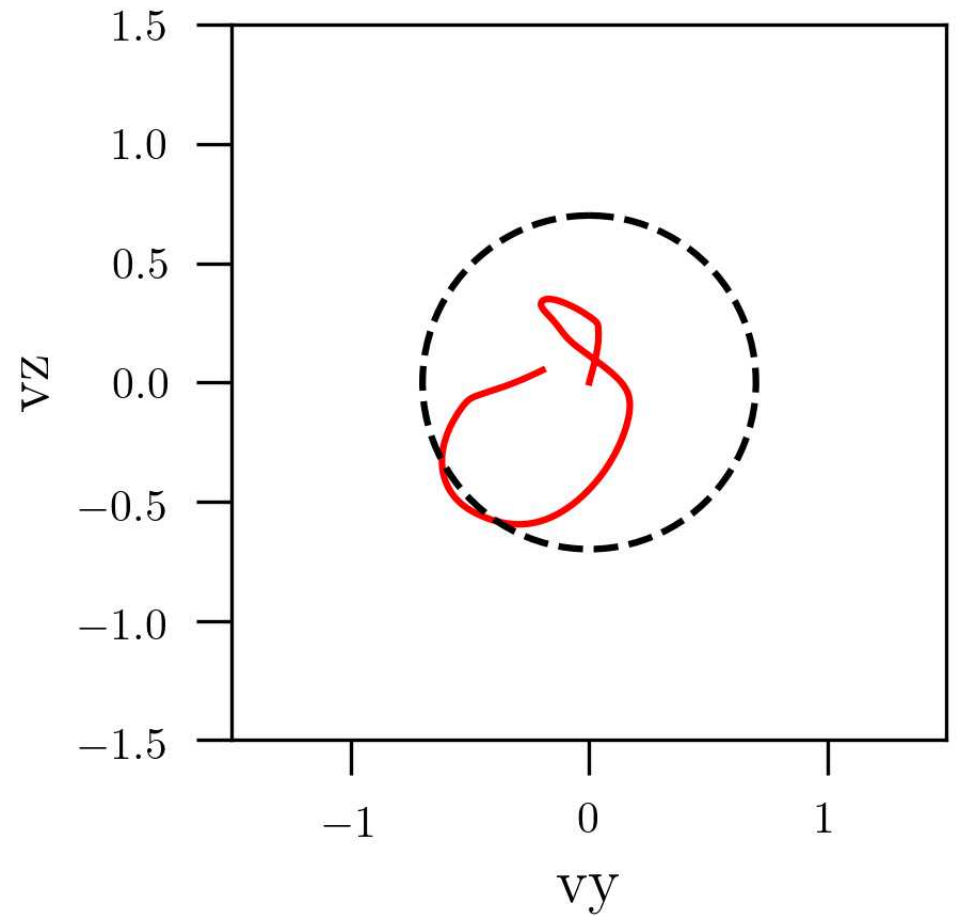
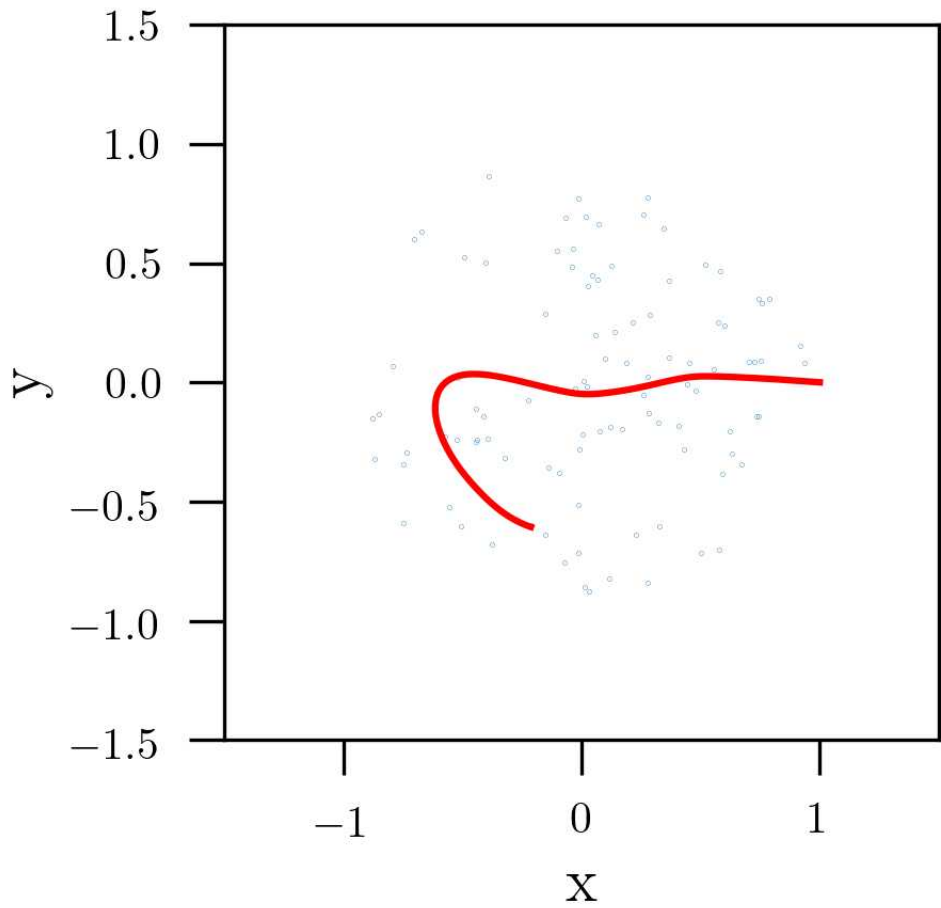


$N = 100$ Time = 2.00



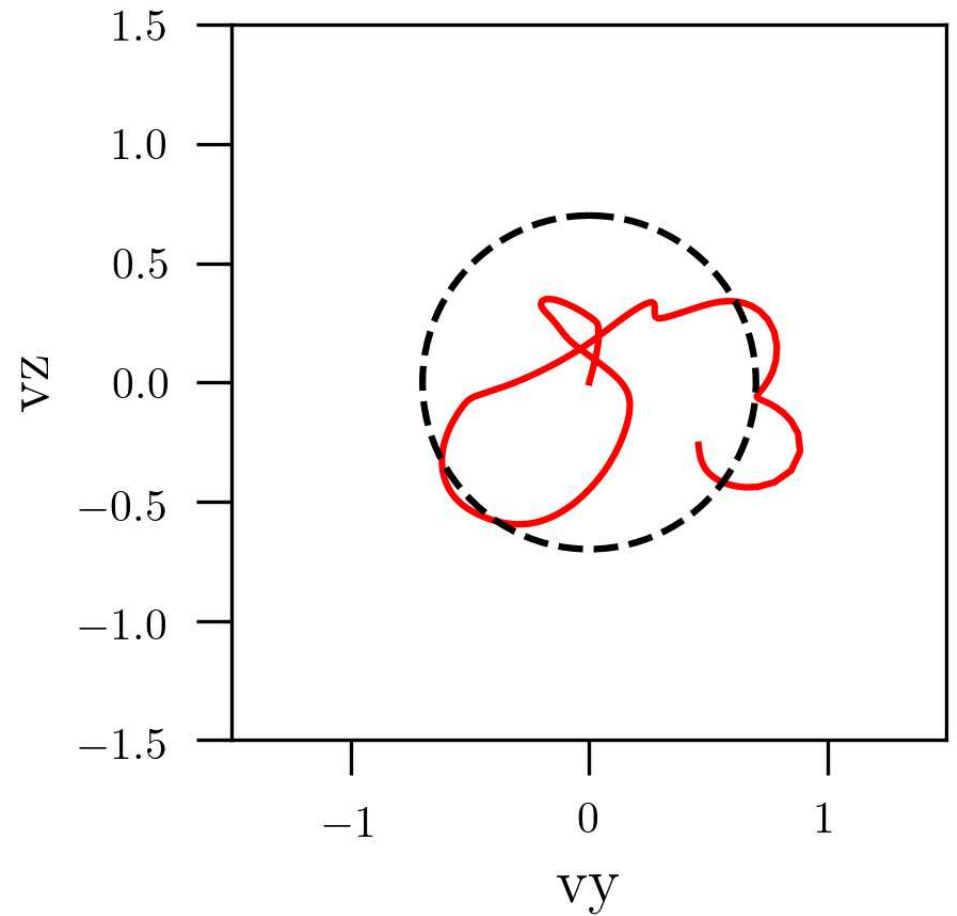
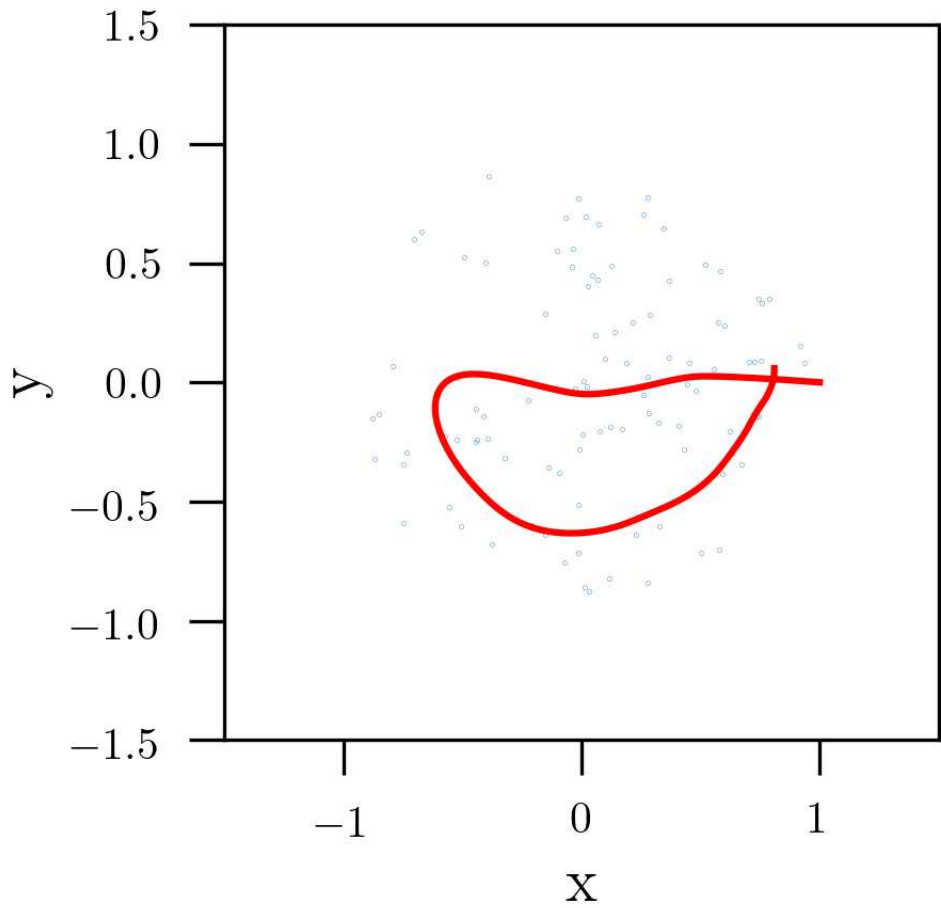
$t_{\text{relax}} = 3$

$N = 100$ Time = 4.00



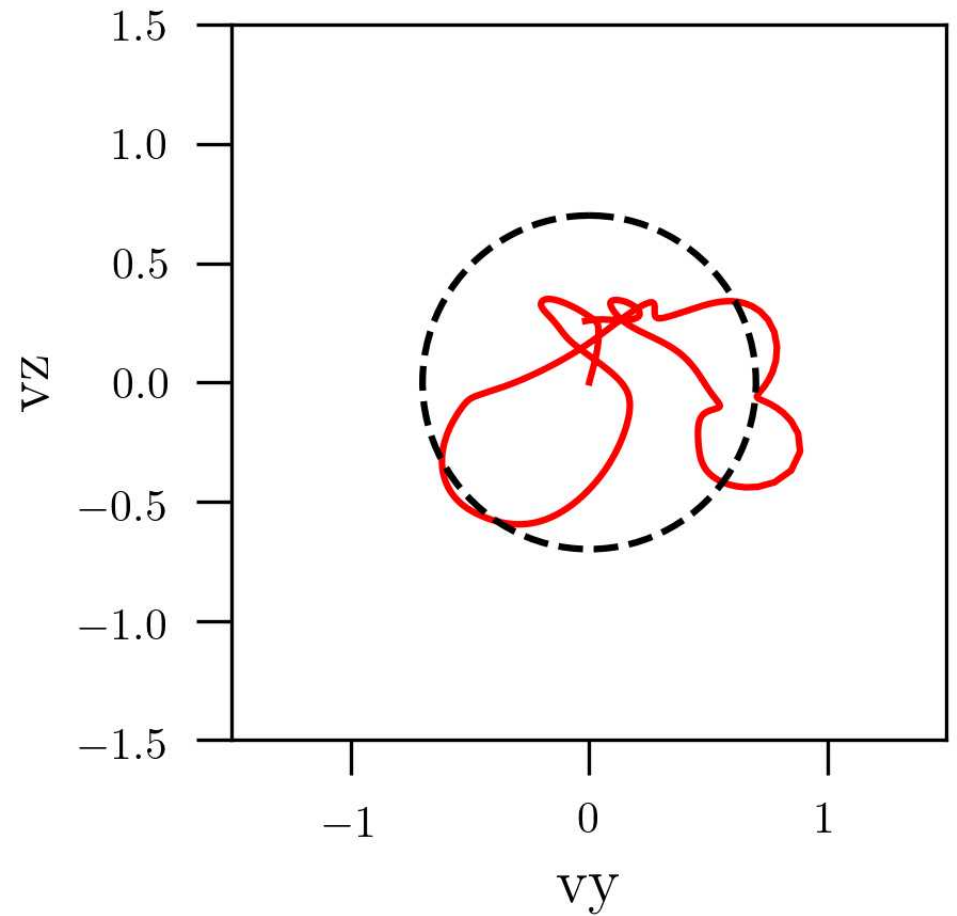
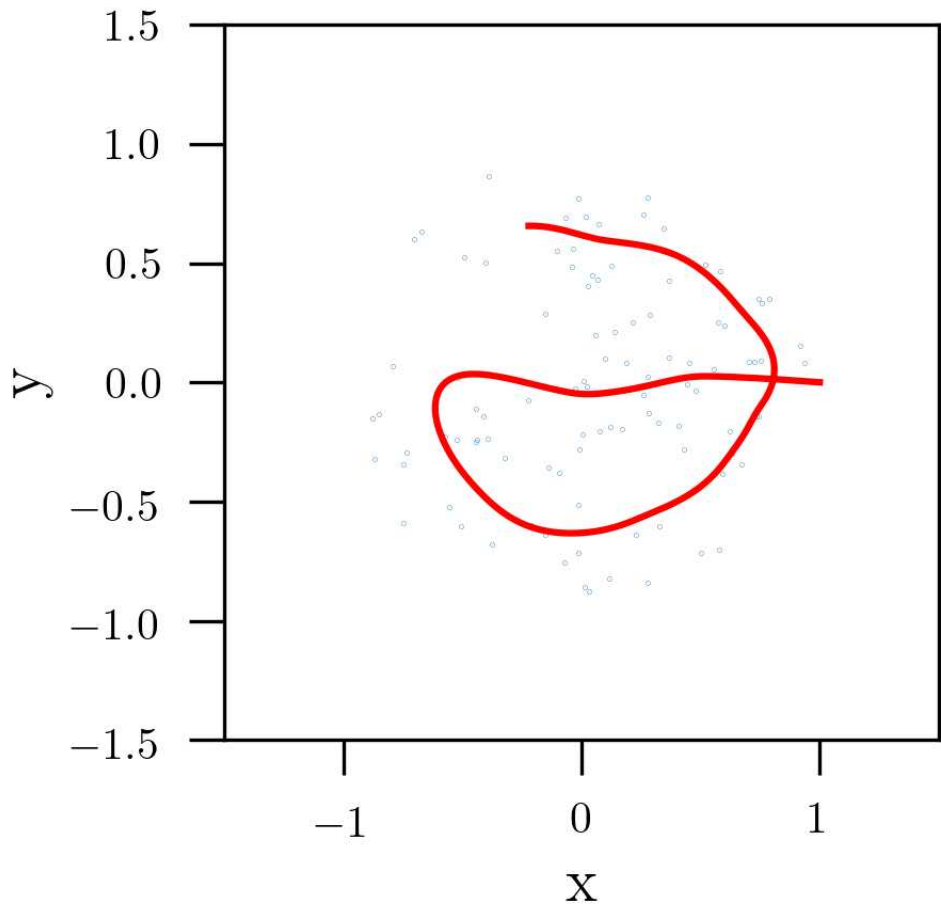
$t_{\text{relax}} = 3$

$N = 100$ Time = 6.00



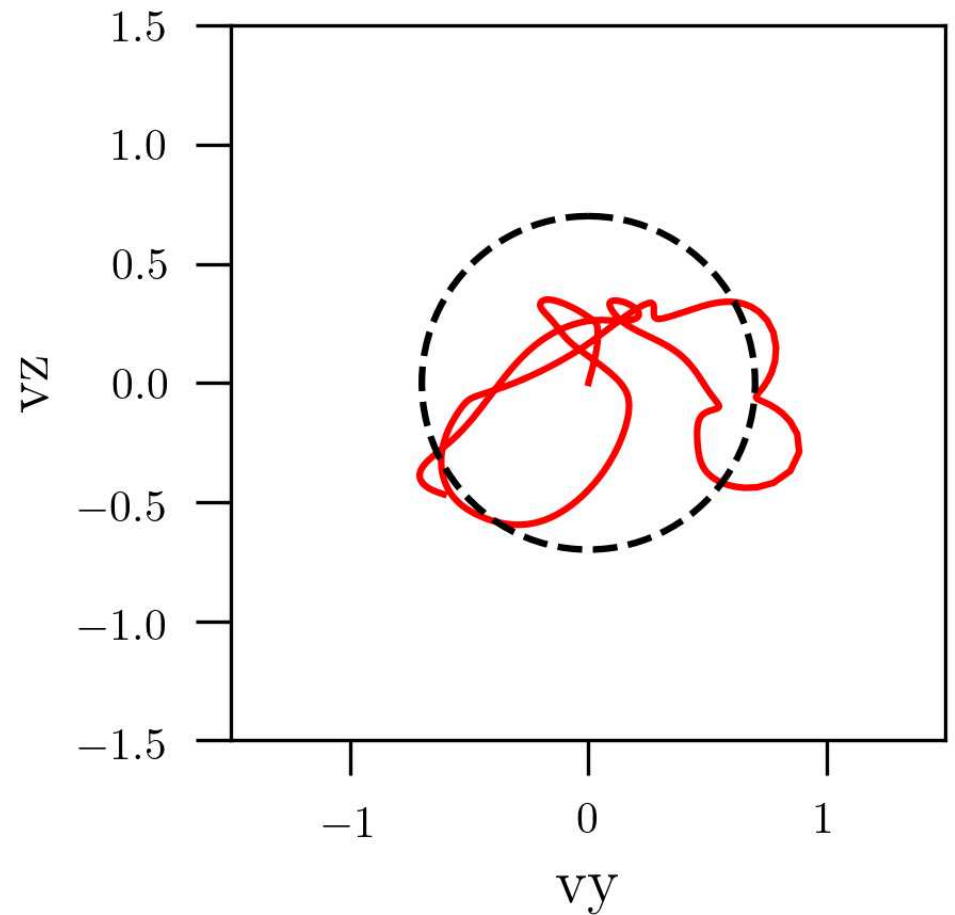
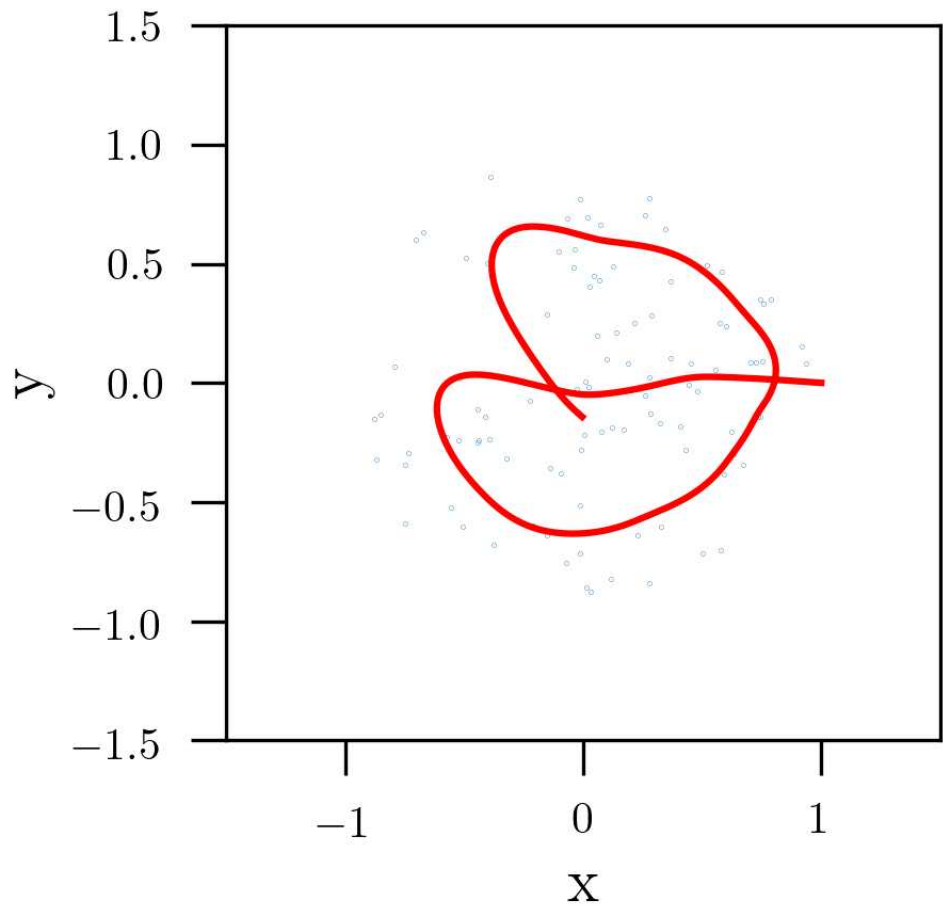
$t_{\text{relax}} = 3$

$N = 100$ Time = 8.00



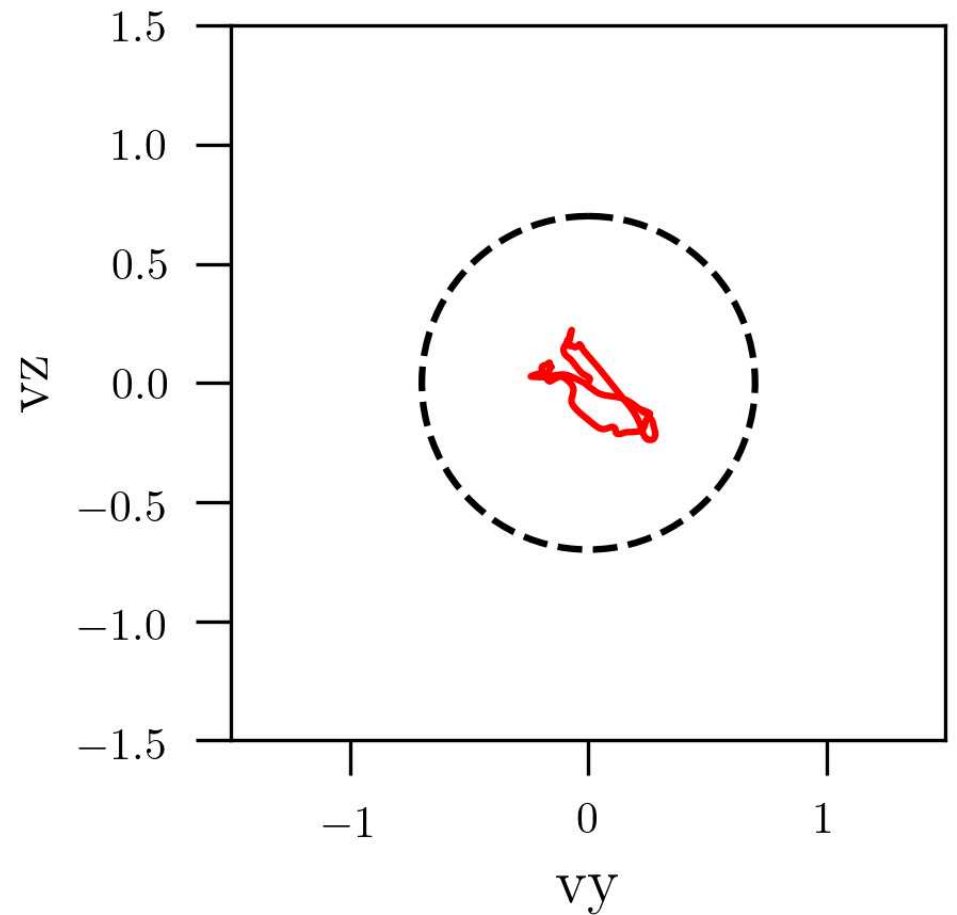
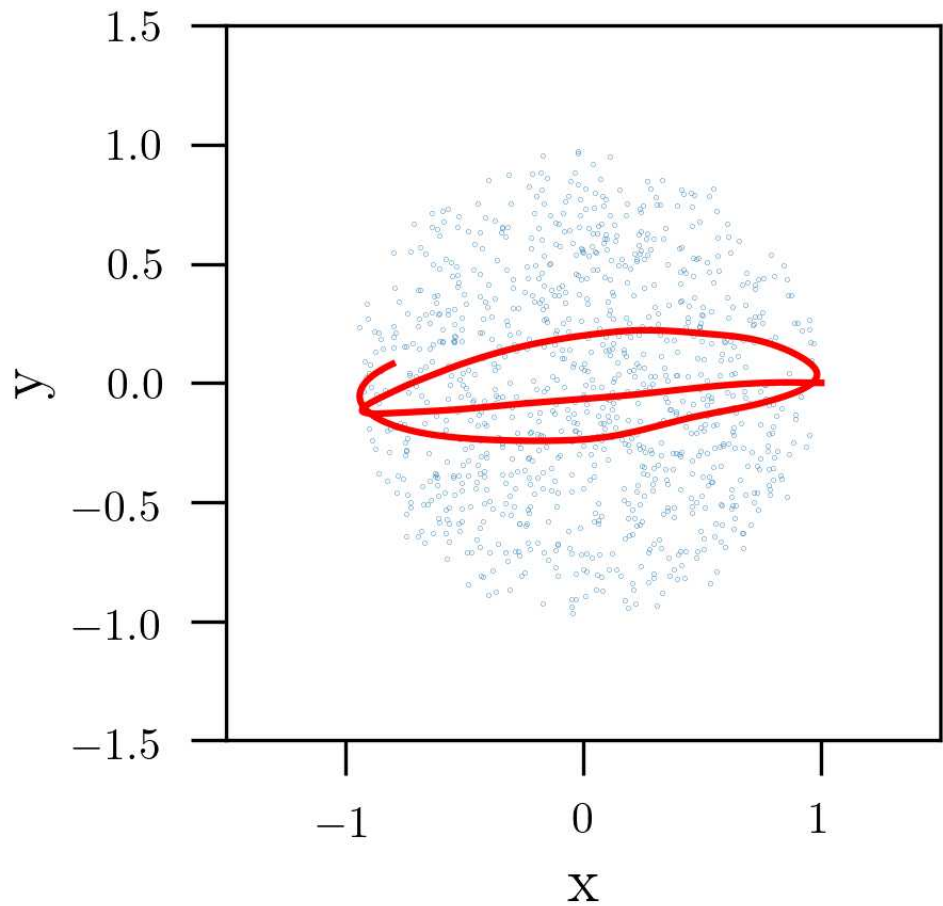
$t_{\text{relax}} = 3$

$N = 100$ Time = 10.00



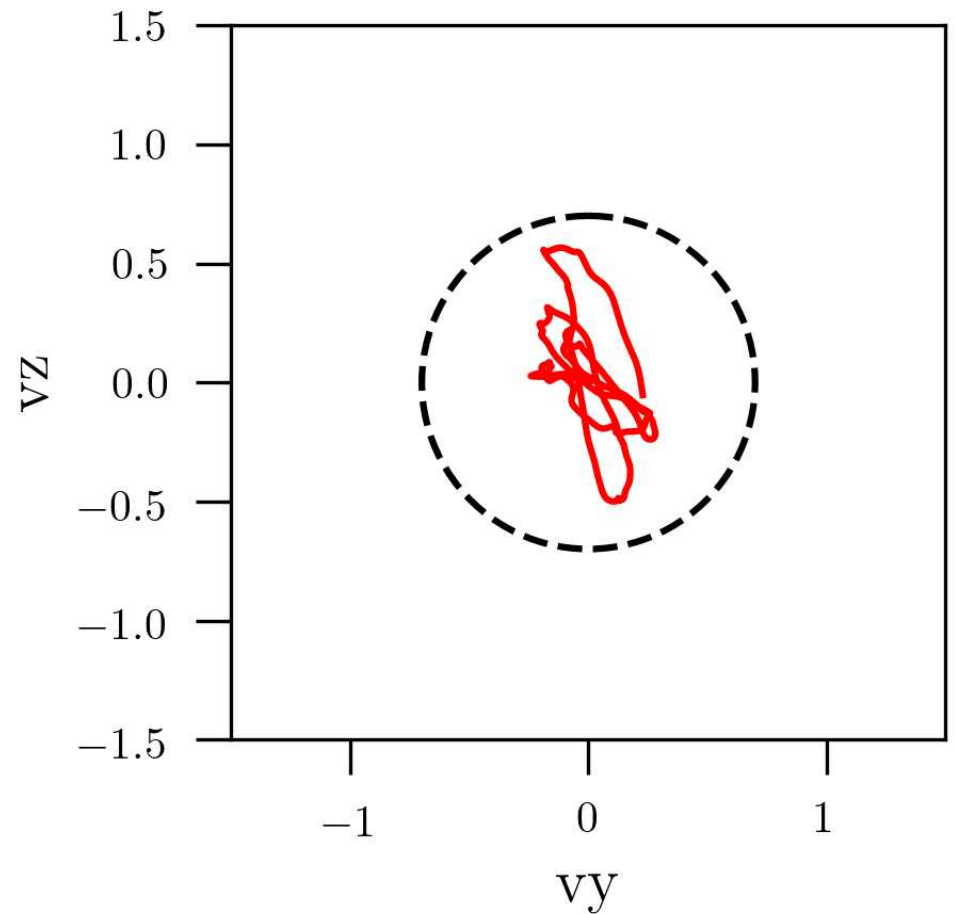
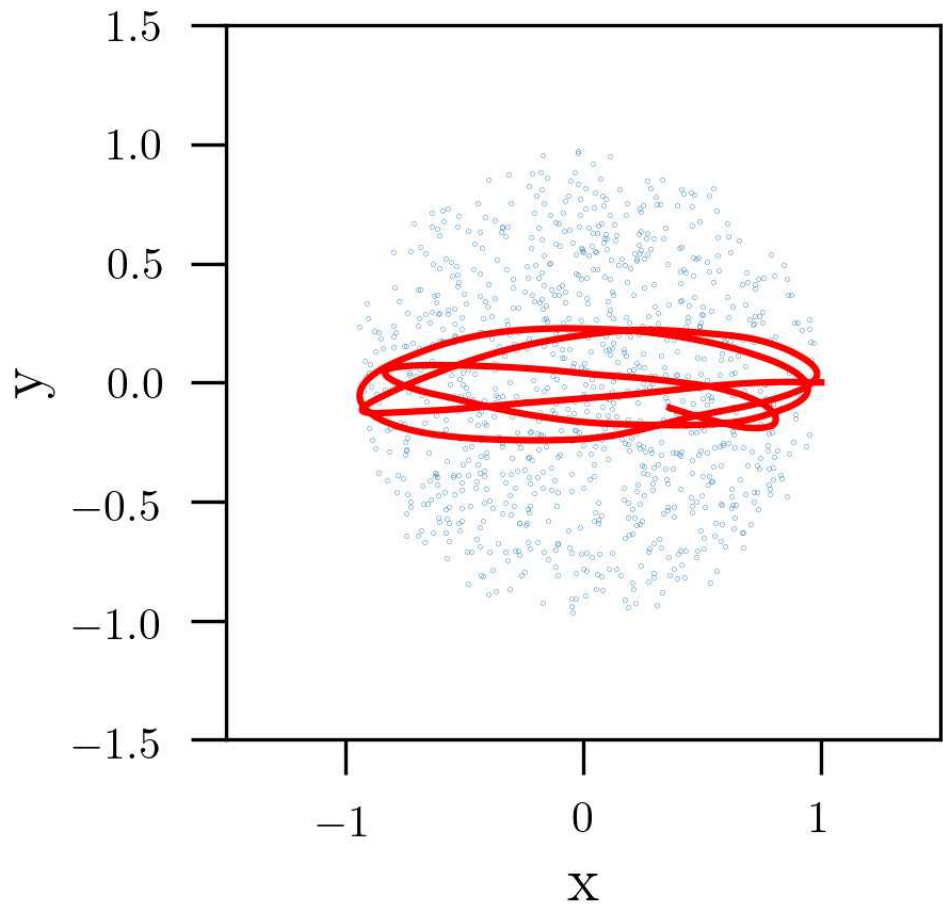
$t_{\text{relax}} = 3$

$N = 1000$ Time = 10.00



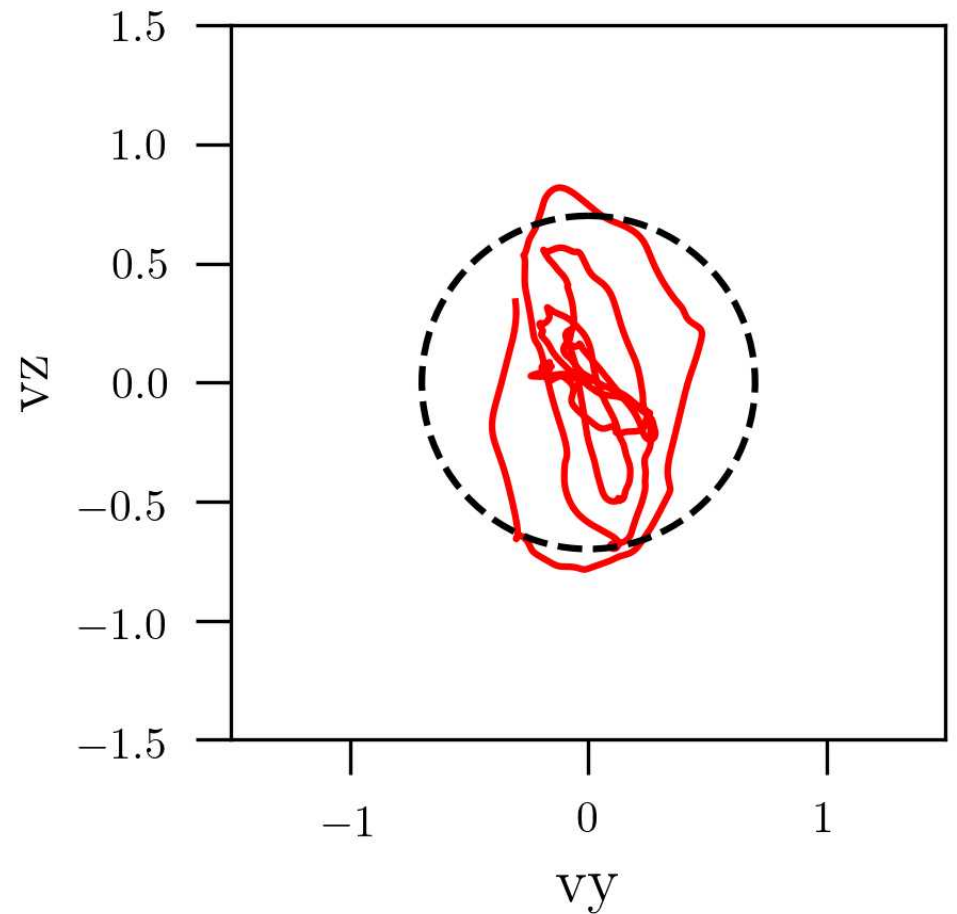
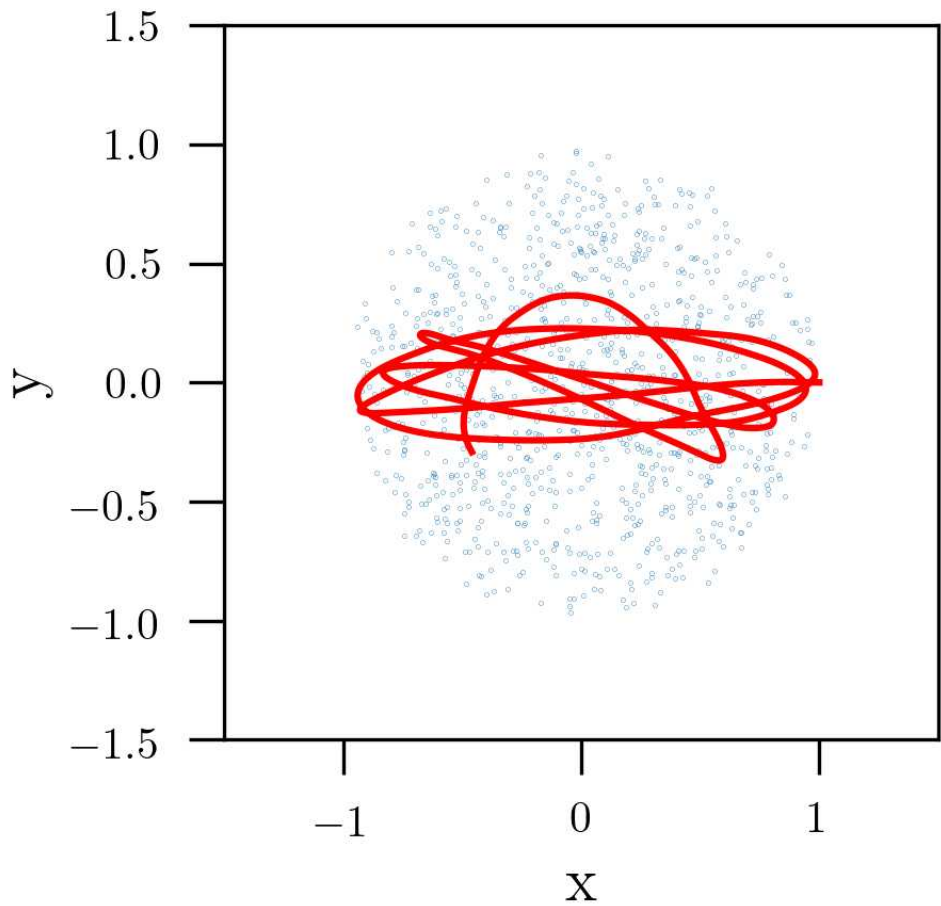
$t_{\text{relax}} = 20$

$N = 1000$ Time = 20.00



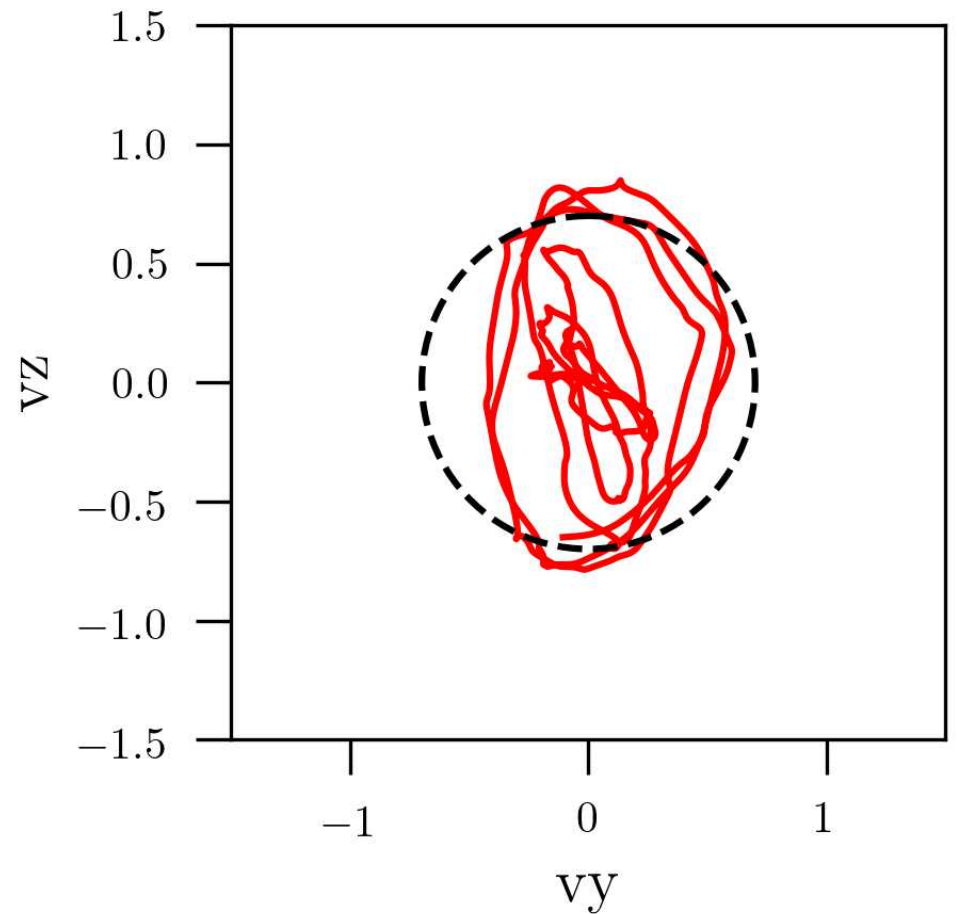
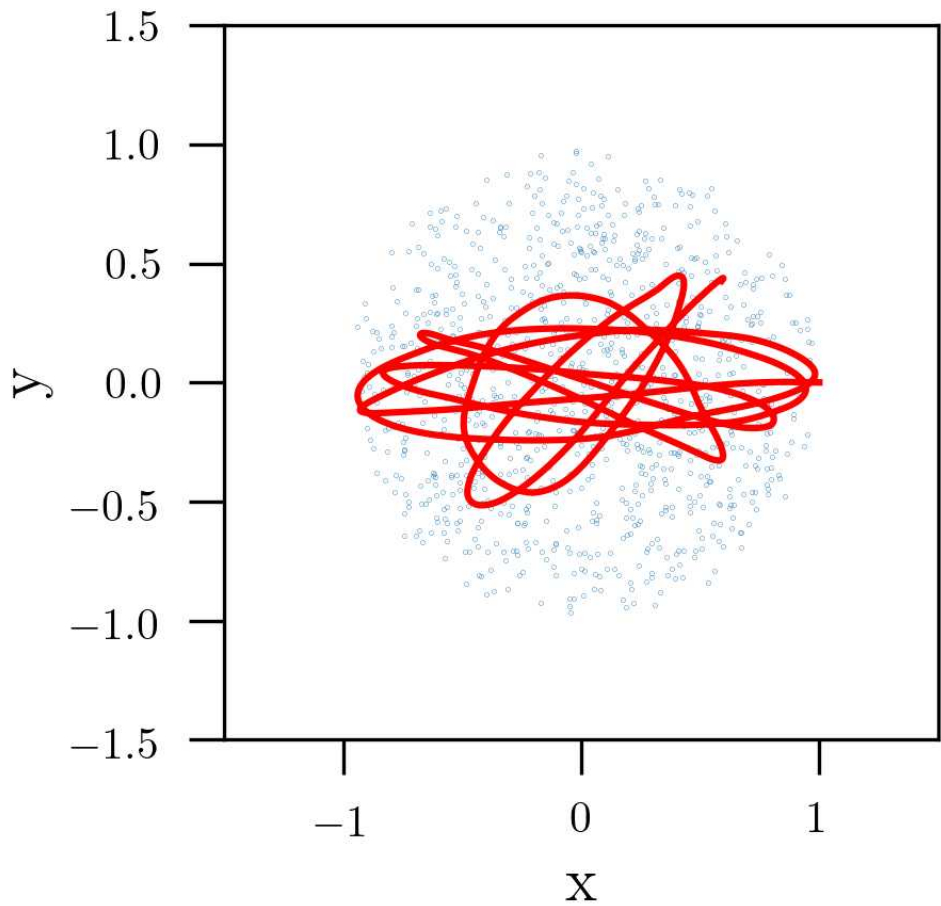
$t_{\text{relax}} = 20$

$N = 1000$ Time = 30.00



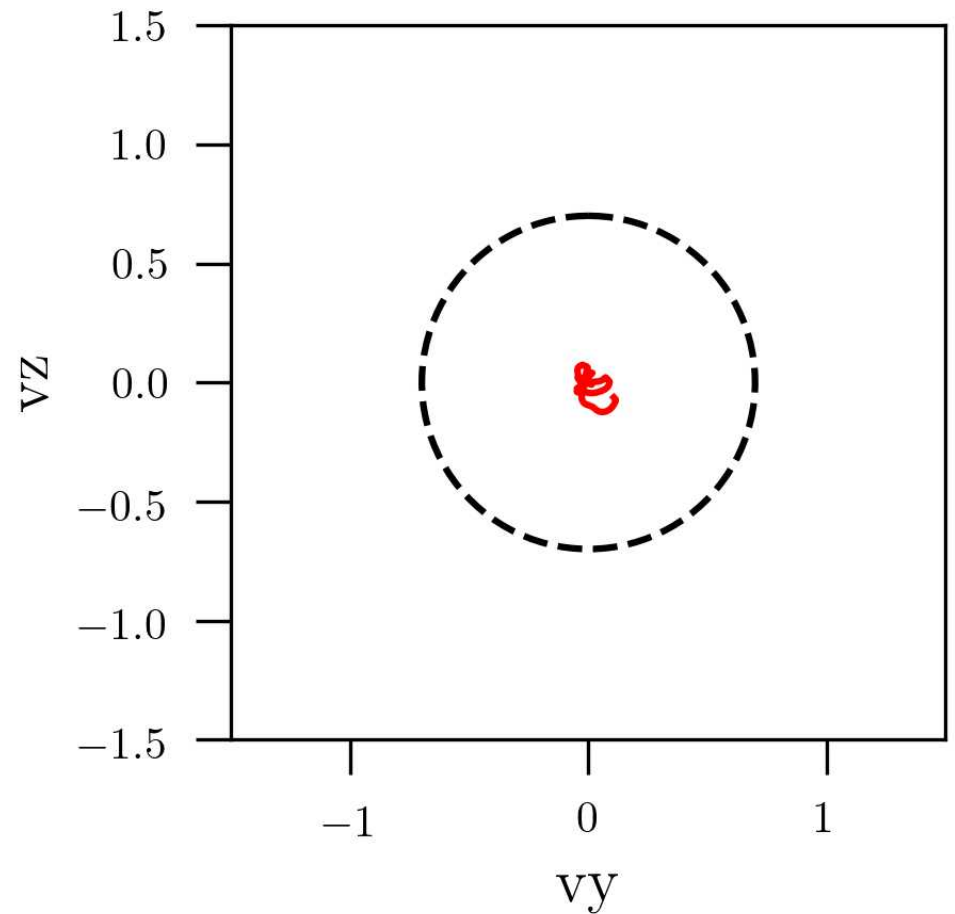
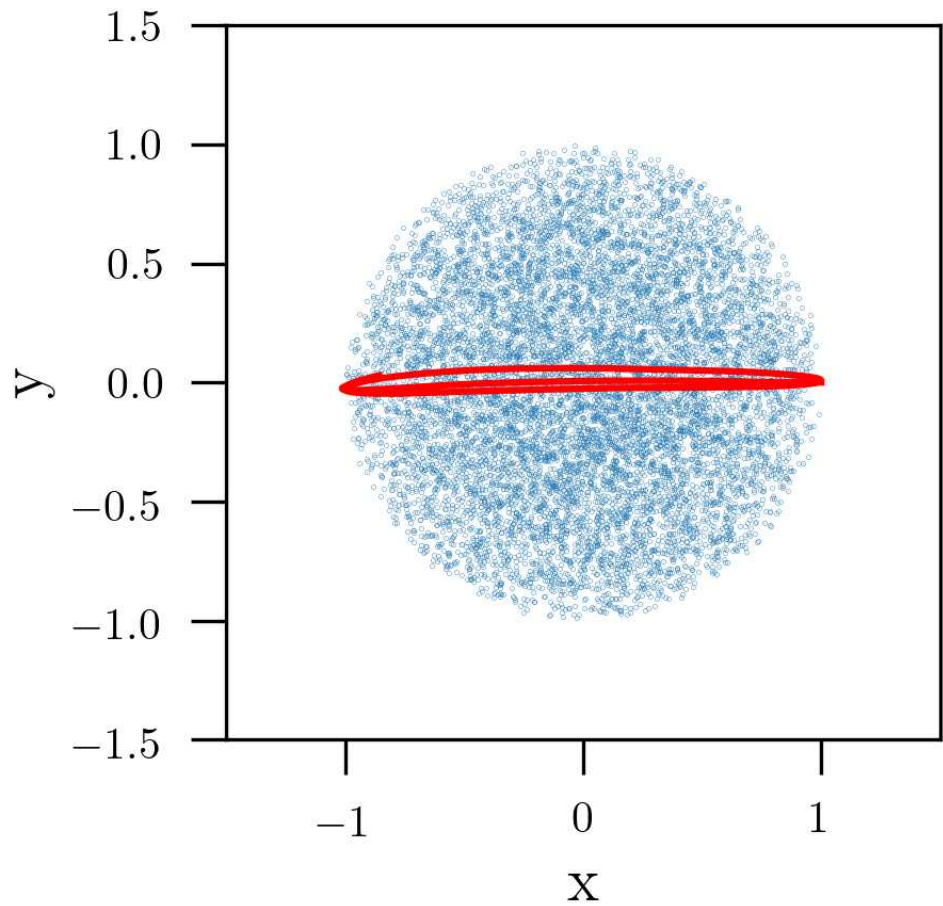
$t_{\text{relax}} = 20$

$N = 1000$ Time = 40.00



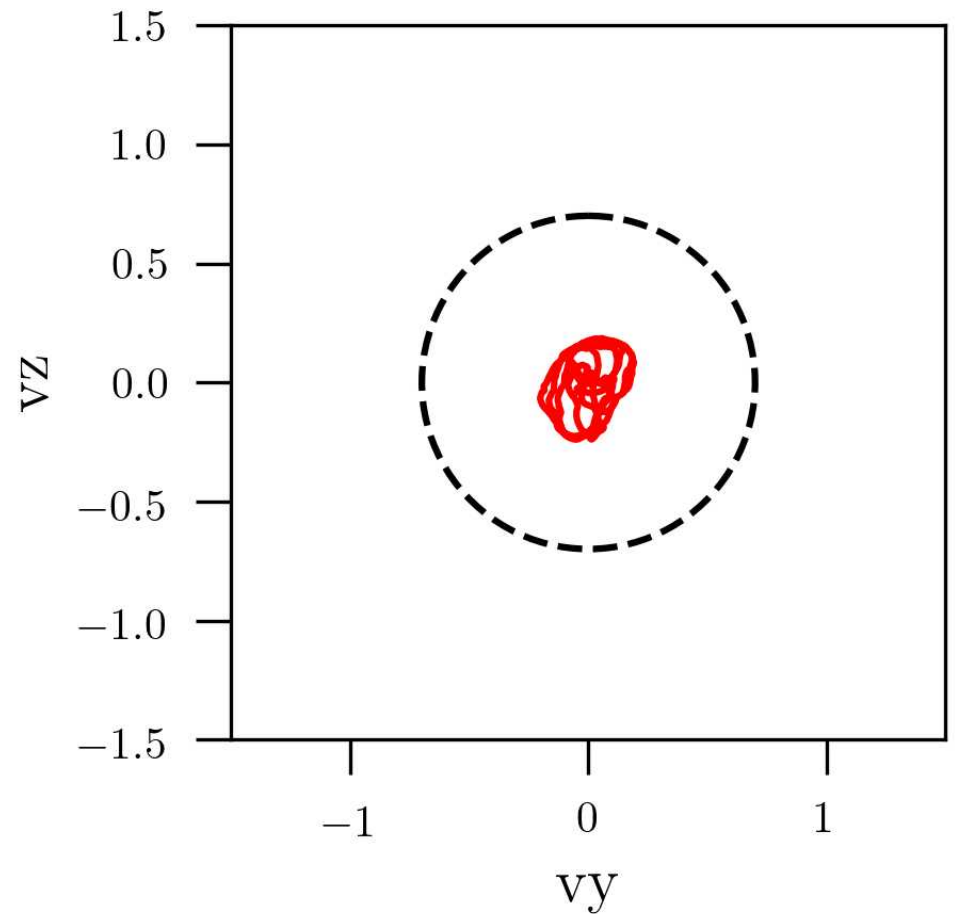
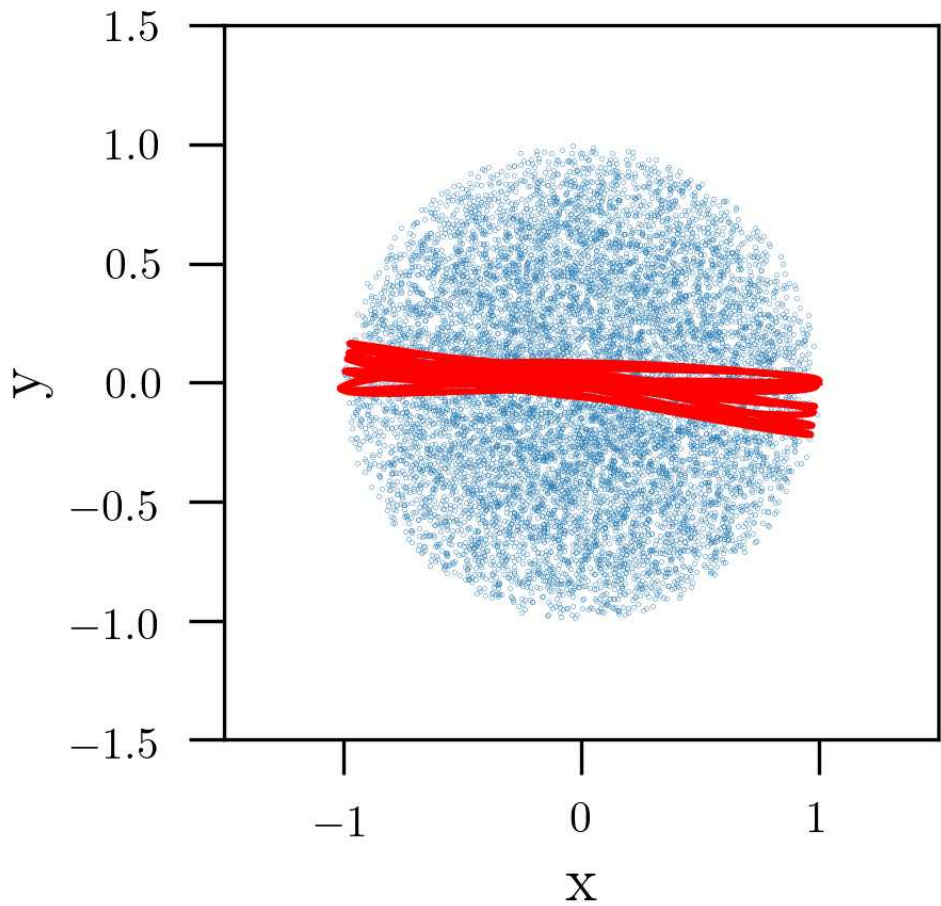
$t_{\text{relax}} = 20$

$N = 10000$ Time = 10.00



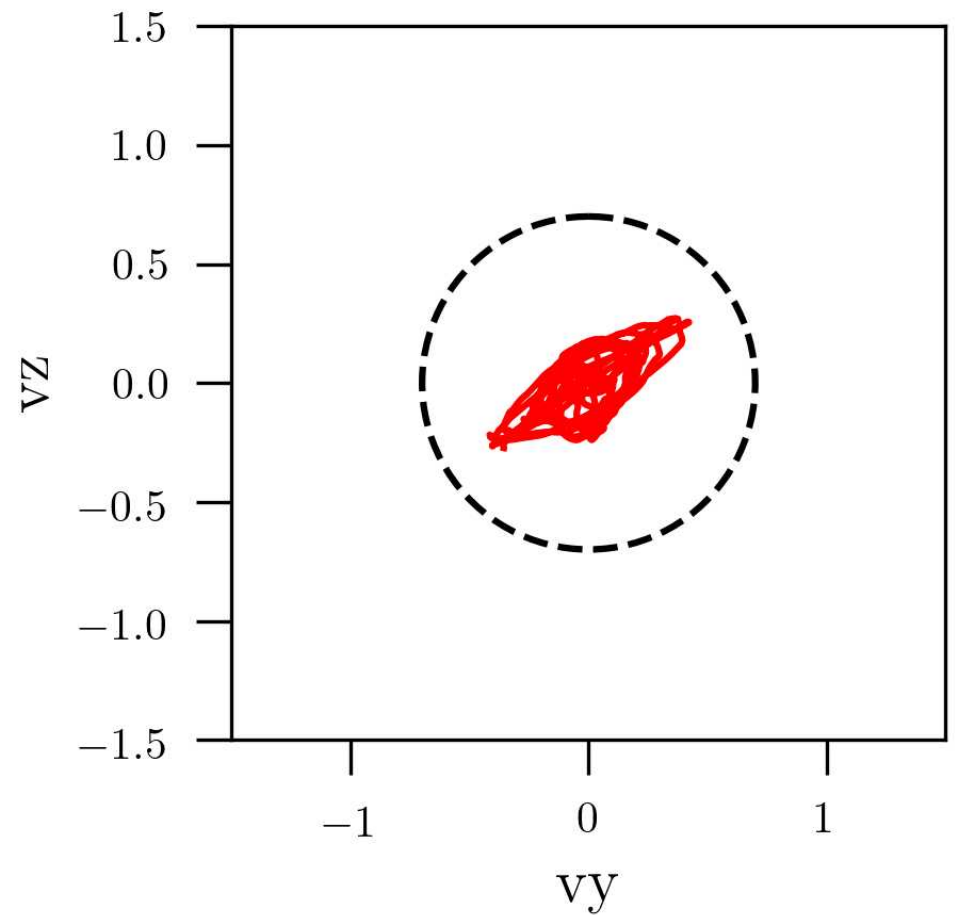
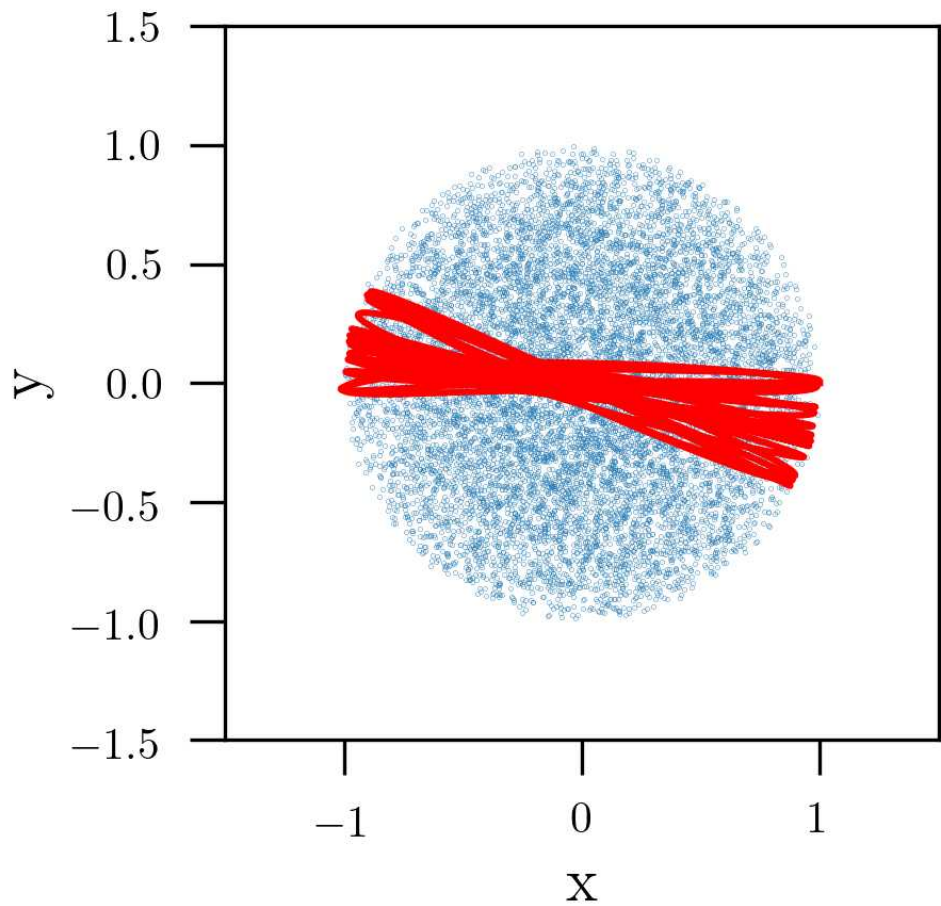
$t_{\text{relax}} = 150$

$N = 10000$ Time = 40.00



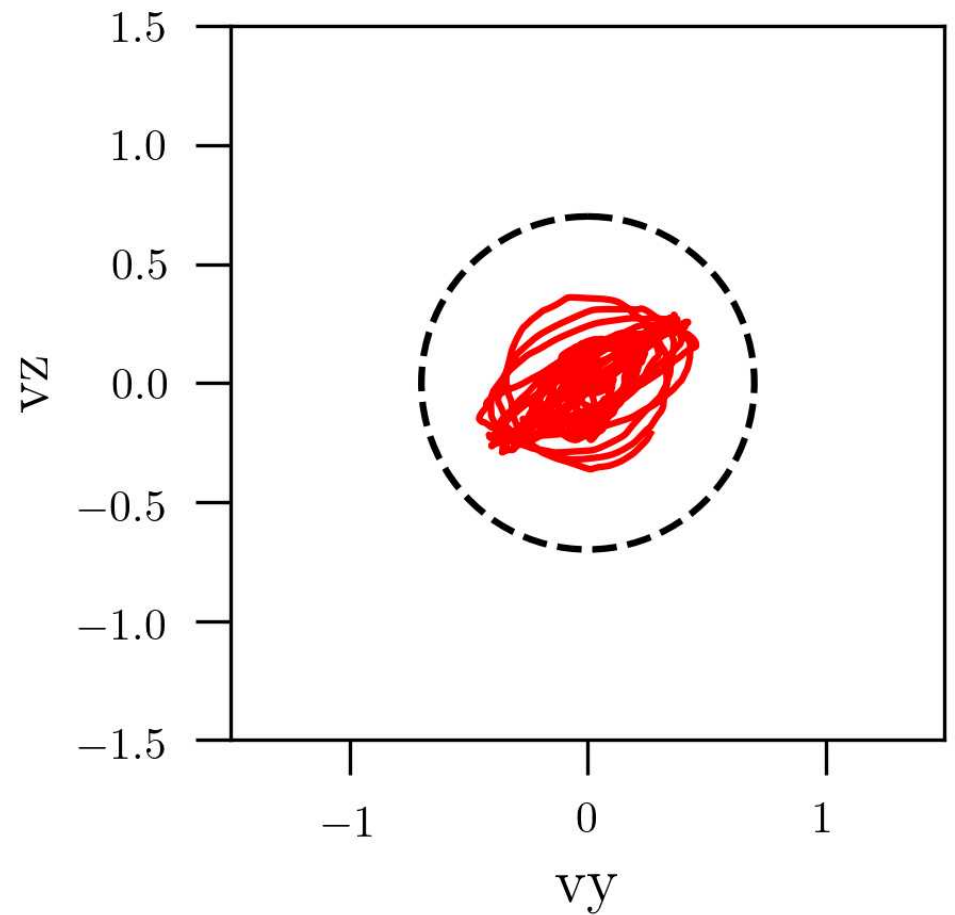
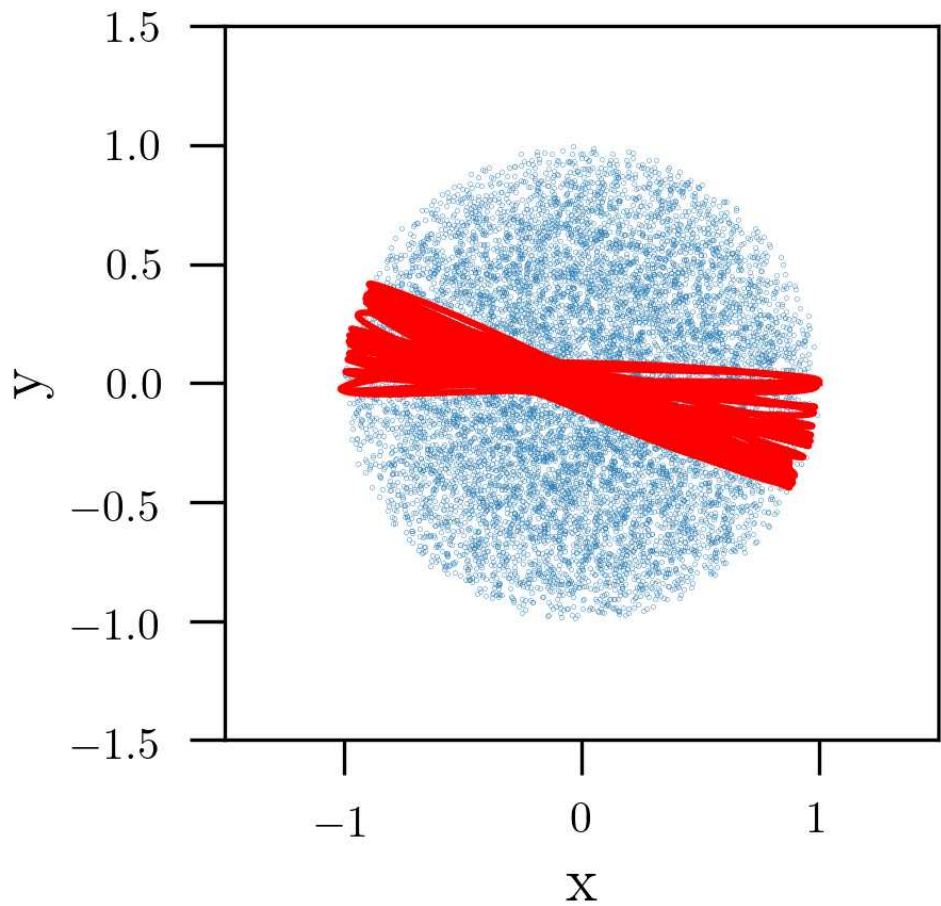
$t_{\text{relax}} = 150$

$N = 10000$ Time = 80.00



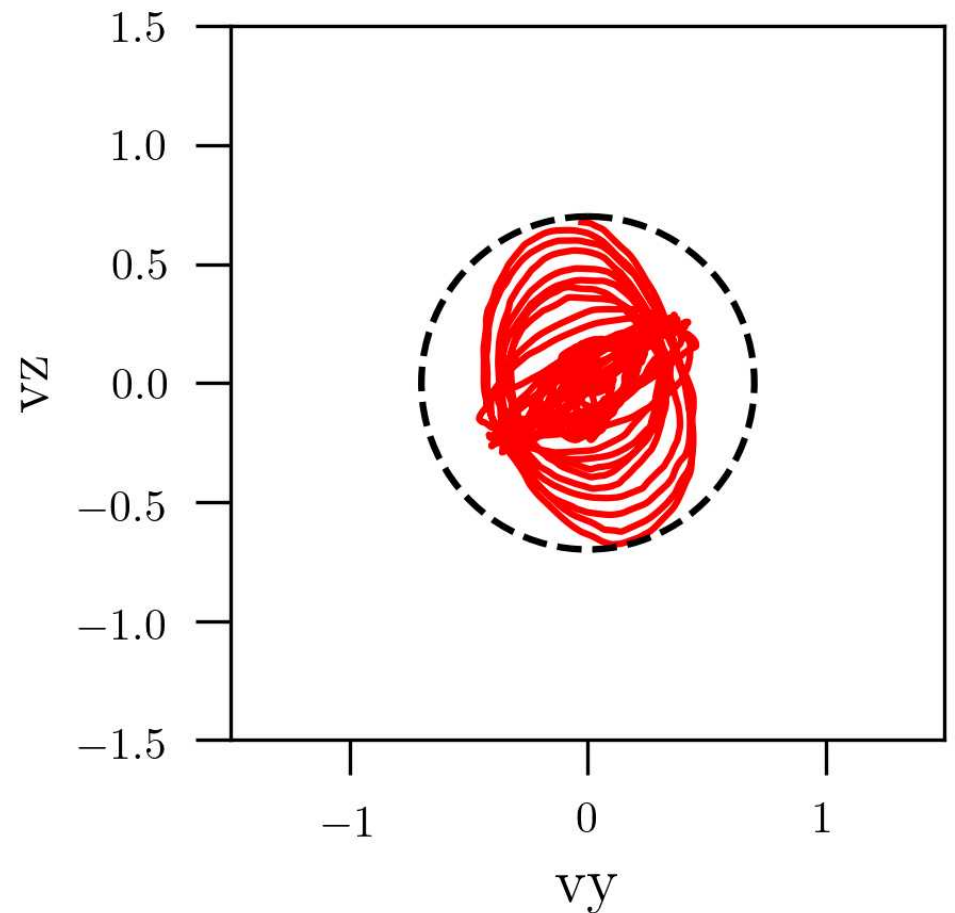
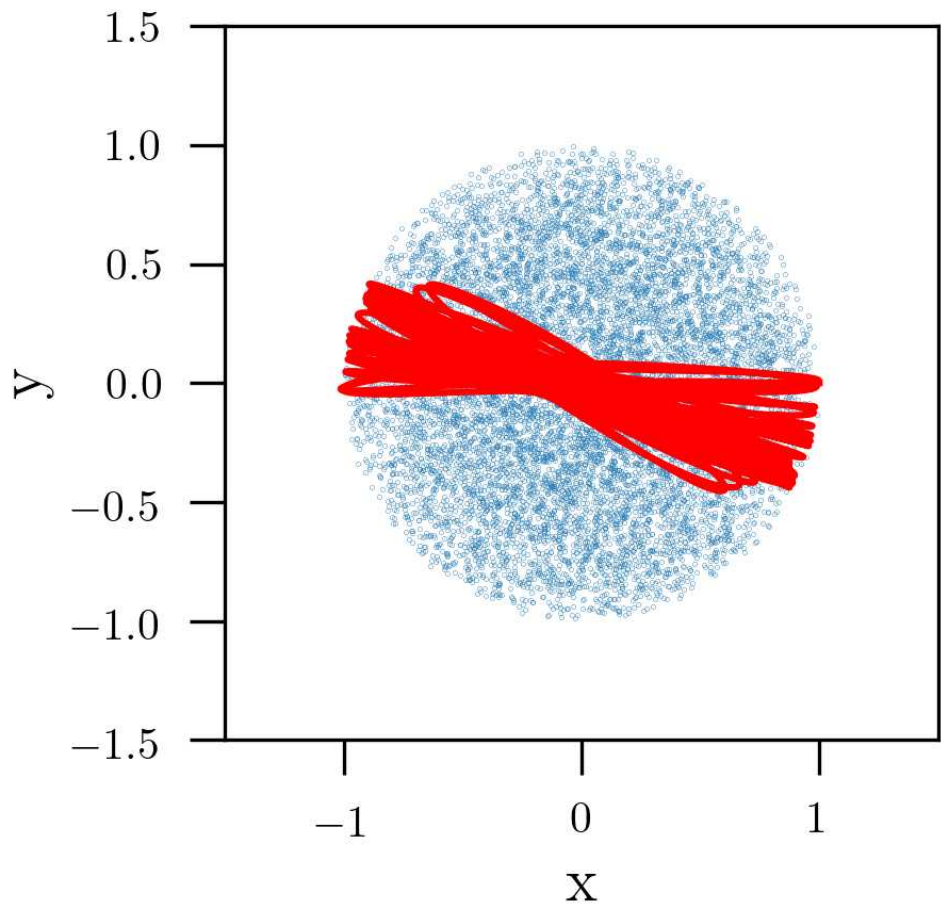
$t_{\text{relax}} = 150$

$N = 10000$ Time = 120.00



$t_{\text{relax}} = 150$

$N = 10000$ Time = 160.00



$t_{\text{relax}} = 150$

The End