Nuclear Fusion and Plasma Physics - Exercises

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Exercise 1 - Perfect Plasma Power Reactor

Adapted from Freidberg, ex 5.4
In this exercise we examine the economics of an "ideal" fusion reactor. By "ideal" we mean that it has ideal physics performance, so any $\beta < 1$, n and τ_E are achievable. Also, we assume that components of the reactor have no cost, so any set of coils, auxiliary heating systems, etcetera, are available. Essentially, the reactor consists of only a toroidal blanket-and-shield surrounding the first wall. Neutronics show that the required blanket-and-shield thickness is $b = 1.5\,\mathrm{m}$. The first wall material limits the maximum wall loading to[1] $L_{Wmax} = 4\,\mathrm{MW/m^2}$. The electric power output is assumed to be $P_E = 1\,\mathrm{GW}$ at a thermal conversion efficiency of $\eta_t = 0.35$. Assume that each fusion reaction ultimately produces 22.4 MeV of thermal energy (3.5 MeV from the α particles, 14.1 MeV from the neutrons and 4.8 MeV from each fusion neutron when breeding Tritium in the Lithium blanket). Now, define the mass utilization factor as F= reactor mass/electric power. For a fission reactor a typical value of F is 1 ton/MW.

- For a toroidal fusion reactor, determine the values of the major and minor radius (R_0 and a, respectively) that minimize F (i.e., a more economic reactor). Assume that all the produced neutrons pass through the first wall releasing 30% of their energy and are then absorbed by the surrounding blanket-and-shield. The average density of the blanket-and-shield is about $\rho_{blanket} = 3 \times 10^3 \mathrm{kg/m^3}$.
- Compare the result with the value for a fission reactor.

Exercise 2 - Plasma β and diamagnetism

The plasma β (ratio between plasma and magnetic field pressure, $\beta = \frac{nT}{B_0^2/2\mu_0}$) is a fundamental quantity for the design and optimization of a magnetic fusion reactor. β is also a

[1]Note that L_W, the wall power loading in [MW/m^2], is referred to as P_W in [Freidberg]. The notation chosen here is to avoid confusion with P_E which is the electric power in [W]

measure of the plasma diamagnetism, i.e. the fact that the plasma reduces the value of an externally applied magnetic field.

a) Show the origin of plasma diamagnetism, considering the individual motion of electrons and ions in the simplest possible magnetic configuration, that of an infinitely long cylinder, whose axis coincides with the direction of the field lines.

b) Demonstrate that the diamagnetic field induced by the plasma is proportional to the plasma pressure (nT), i.e. to β.

c) Qualitatively, discuss why β cannot be increased indefinitely.

Reminder: Motion of an electron around a field line: cyclotron frequency $\Omega_j = \frac{q_j B}{m_j}$; Larmor radius $\rho_{L,j} = \frac{v_\perp}{\Omega_j}$, where $v_\perp$ is the velocity perpendicular to B.

Exercise 3 - Violating quasi-neutrality

In the definition of a plasma it is stated that, although it is an ionized gas made of separate positive and negative charges, it is globally neutral.

Considering a typical fusion plasma with density $n_e = 10^{20}\,\mathrm{m}^{-3}$, $T = 10\,\mathrm{keV}$, 100% deuterium, and characteristic size = 1 m,

a) Evaluate the order of magnitude of the density of force (force per unit volume) that would be established in a plasma if a small violation of neutrality is introduced, i.e. if the density of positive charges exceeds the density of negative charges by 1% everywhere.

b) Compare the result in (a) to other forces that can be present in the plasma volume (gravitation, pressure, ...)

Hint: use a 1-D model for the plasma, considering only one direction, x, and assume that the densities and temperature are constant and uniform.