Nuclear Fusion and Plasma Physics - Exercises

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Problem Set 3 - 10 October 2022

Exercise 1 - Plasma production

Semiconductor manufacturers use plasmas for surface treatment of materials. In a vacuum chamber of dimensions $0.5\,\mathrm{m} \times 0.5\,\mathrm{m} \times 0.5\,\mathrm{m}$ an inert gas is partially ionized by radio waves. Consider the case where the gas used is Argon ($p = 10^{-4}$ torr, $n_e = 10^{16}\,\mathrm{m}^{-3}$, $T_e = 3\,\mathrm{eV}$, and $T_i = 0.1\,\mathrm{eV}$ - first ionisation), and the temperature of the neutral gas is assumed to be 25 °C .

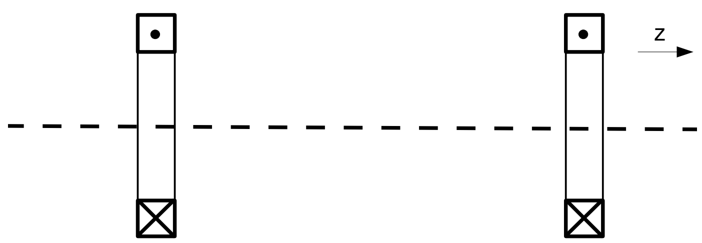
a) Calculate the relative ionization degree of the gas used, α. (def. $\alpha = \frac{n_e}{n_e + n_{gas}}$)

b) Estimate the electron-neutral collision frequency (ν_{en}) assuming a cross section $\sigma = 1000\,\pi\,a_0^2$, where $a_0 = 5.29 \times 10^{-11}$ m is the Bohr radius.

c) Can we consider this gas to be a plasma? Why?

Exercise 2 - Mirror effect

Consider the following configuration of two cylindrical current carrying coils:



a) Draw a sketch of the magnetic field lines.

b) Draw a sketch of the magnetic field intensity B_z (along the axis) as a function of z.

c) What is the trajectory of a particle that is initially traveling along the axis with velocity $\mathbf{v} = v_z \mathbf{e_z}$ (i.e. having *no* velocity component orthogonal to the magnetic field)?

d) Consider a particle on the z-axis in between the two magnets, having a velocity component $v_\perp$ perpendicular to the magnetic field, as well as a parallel component $v_{||}$.

   Use the adiabatic invariant

$$\frac{mv_\perp^2}{B} = \text{constant}$$

   and the conservation of kinetic energy to show that such a particle can be "reflected" by the magnetic field.

e) For a reflected particle, the parallel velocity at the reflection point is $v_{||} = 0$. Use this to derive a condition on the initial velocity of the particle on the midplane in order for it to be reflected. This defines the so-called loss cone , i.e. the portion of the velocity space that corresponds to particles that are lost from the mirror.

Exercise 3 - Confinement by a toroidal field

Consider the magnetic field generated by a long current carrying wire.

We know that in a non-uniform magnetic field the particles experience a drift, called the $\vec{\nabla} B$ drift, given by

$$\mathbf{v}_{\nabla B} = \mp \frac{v_\perp^2}{2\omega_c} \frac{\mathbf{B} \times \vec{\nabla} B}{B^2}$$

where the first sign corresponds to negative charges. Additionally, a particle in a curved magnetic field with radius of curvature $\mathbf{R_c}$ will have a *curvature* drift (due to the centrifugal force) given by

$$\mathbf{v}_{R_c} = \mp \frac{v_{||}^2}{\omega_c} \frac{\mathbf{R_c} \times \mathbf{B}}{B R_c^2}$$

where ω_c is the particle's cyclotron frequency $\omega_c = \frac{qB}{m}$

a) Find $\mathbf{B}$ for an infinitely-long current carrying wire (hint: use Ampère's law in differential or integral form). Use the result to find an expression for $\vec{\nabla} B$ (notice that B is the *magnitude* of $\mathbf{B}$).

b) Explain why you cannot confine a plasma with a simple toroidal field.