Astrophysics III: Stellar and galactic dynamics

Solutions

Problem 1:

From Poisson's equation in spherical coordinates we get:

$$\nabla^2\Phi = 4\pi G\rho$$

$\nabla^2\Phi$ written in spherical coordinates, and considering a spherical potential we get:

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right)$$

one then obtains

$$\nabla^2\Phi = \frac{3GMb^2}{(r^2+b^2)^{5/2}}$$

and finally:

$$\rho = \frac{3M}{4\pi b^3}\left(1+\frac{r^2}{b^2}\right)^{-5/2}$$

Problem 2:

a) Point mass:

$$V_\mathrm{c}^2(r) = \frac{GM}{r}$$

b) Homogeneous sphere of radius a:

$$V_\mathrm{c}^2(r) = \begin{cases} \frac{GMr^2}{a^3} & \text{if } r < a \\ \frac{GM}{r} & \text{if } r \geq a \end{cases}$$

c) Plummer-Schuster potential:

$$V_\mathrm{c}^2(r) = \frac{GMr^2}{(r^2+a^2)^{3/2}}$$

d) Miyamoto-Nagai potential:

$$V_\mathrm{c}^2(R) = \frac{GMR^2}{\left[R^2+(a+b)^2\right]^{3/2}}$$

Problem 3:

We are still in the plane $z = 0$ (where the rotation curves are defined.) With the parametrization:

$$h_R = a + b$$
$$h_z = b$$

the circular velocity of the Miyamotio-Nagai potential can be written:

$$V_c^2(R) = \frac{GMR^2}{(R^2 + h_R^2)^{3/2}}$$

which is obviously independent of the scale height h_z. This parametrization is more telling than the a, b one: it shows how a Miyamoto-Nagai system has a circular velocity independent of the flattening of the potential. The two extremes are:

- spherical symmetry: $a = 0 \implies h_R = h_z = b$,
- thin disk: $b = 0 \implies h_R = a,\ h_z = 0$.

The rotation in the plane $z = 0$ is the same for these two extreme cases since V_c^2 is independent of h_z.

Problem 4:

In vc_plummer.py, the following line was required.

```
vc2_th = r**2/(r**2+b**2)**1.5
```

Problem 5:

In vc_miyamoto.py:

- ```
  vc2_Mr = Mr/r
  ```
- ```
  vc2_phi =  r *dphi
  ```
- ```
  vc2_th = r**2/(r**2+ (a+b)**2 )**1.5
  ```

In vc_homosphere.py:

- ```
  vc2_Mr = Mr/r
  ```
- ```
  vc2_phi =  r *dphi
  ```
- ```
  vc2_th_in  = r**2/a**3
  vc2_th_out = 1/r
  vc2_th = where(r<a,vc2_th_in,vc2_th_out)
  ```

In vc_pm.py:

- `vc2_Mr = Mr/r`
- `vc2_phi = r *dphi`
- `vc2_th = 1/r`