Astrophysics III, Dr. Yves Revaz

 $\begin{array}{l} \text{4th year physics} \\ 19.10.2022 \end{array}$

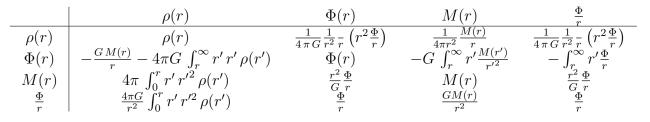
<u>Exercises week 5</u> Autumn semester 2022

EPFL

Astrophysics III: Stellar and galactic dynamics <u>Exercises</u>

Problem 1:

In practice, is often useful to derive, for example the density $\rho(r)$ knowing the gravitational field $g(r) = -\frac{\Phi}{r}$, or the potential $\Phi(r)$ knowing the cumulative mass M(r). Using the relations presented during the lectures, express successively $\rho(r)$, $\Phi(r)$, M(r) and $\frac{\Phi}{r}$ as a function of respectively $\rho(r)$, $\Phi(r)$, M(r) and $\frac{\Phi}{r}$ as given in the following table :



Problem 2:

Derive the potential of a razor-thin infinite slab of constant surface density Σ_0 .

Problem 3:

Derive the potential of an infinite wire of constant linear density λ_0 . **Problem 4:**

Derive the density and circular velocity corresponding to the NFW potential

$$\Phi(r) = v_s^2 \left[1 - \frac{\ln(1 + r/r_s)}{r/r_s} \right]$$

Problem 5:

The isochrone potential is given by

$$\Phi(r) = -\frac{GM}{b + \sqrt{b^2 + r^2}}$$

What is the density profile that gives this potential? What is the circular velocity?

Problem 6:

Using Gauss' theorem, derive the surface density for the Kuzmin disk potential at $z{=}0$

$$\Phi_K(R,z) = -\frac{GM}{\sqrt{R^2 + (a+|z|)^2}}$$

Problem 7:

The surface density of a Mestel disk is defined as:

$$\Sigma(R) = \begin{cases} \frac{v_0^2}{2\pi GR} & (R < R_{\max}) \\ 0 & (R \ge R_{\max}) \end{cases}$$
(1)

Show that in the limit $R_{\max} \to \infty$, the circular velocity is the constant v_0 .