Astrophysics III, Dr. Yves Revaz | EPFL

4th year physics | Exercises week 5
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Astrophysics III: Stellar and galactic dynamics

Exercises

Problem 1:

In practice, is often useful to derive, for example the density $\rho(r)$ knowing the gravitational field $g(r) = -\frac{\Phi}{r}$, or the potential $\Phi(r)$ knowing the cumulative mass $M(r)$. Using the relations presented during the lectures, express successively $\rho(r)$, $\Phi(r)$, $M(r)$ and $\frac{\Phi}{r}$ as a function of respectively $\rho(r)$, $\Phi(r)$, $M(r)$ and $\frac{\Phi}{r}$ as given in the following table :

	$\rho(r)$	$\Phi(r)$	$M(r)$	$\frac{\Phi}{r}$
$\rho(r)$	$\rho(r)$	$\frac{1}{4\,\pi\,G}\frac{1}{r^2}\frac{}{r}\left(r^2\frac{\Phi}{r}\right)$	$\frac{1}{4\pi r^2}\frac{M(r)}{r}$	$\frac{1}{4\,\pi\,G}\frac{1}{r^2}\frac{}{r}\left(r^2\frac{\Phi}{r}\right)$
$\Phi(r)$	$-\frac{G\,M(r)}{r} - 4\pi G\int_r^\infty r'\,r'\,\rho(r')$	$\Phi(r)$	$-G\int_r^\infty r'\,\frac{M(r')}{r'^2}$	$-\int_r^\infty r'\,\frac{\Phi}{r}$
$M(r)$	$4\pi\int_0^r r'\,r'^2\,\rho(r')$	$\frac{r^2}{G}\frac{\Phi}{r}$	$M(r)$	$\frac{r^2}{G}\frac{\Phi}{r}$
$\frac{\Phi}{r}$	$\frac{4\pi G}{r^2}\int_0^r r'\,r'^2\,\rho(r')$	$\frac{\Phi}{r}$	$\frac{GM(r)}{r^2}$	$\frac{\Phi}{r}$

Problem 2:

Derive the potential of a razor-thin infinite slab of constant surface density Σ_0.

Problem 3:

Derive the potential of an infinite wire of constant linear density λ_0.

Problem 4:

Derive the density and circular velocity corresponding to the NFW potential

$$\Phi(r) = v_s^2\left[1 - \frac{\ln(1 + r/r_s)}{r/r_s}\right]$$

Problem 5:

The isochrone potential is given by

$$\Phi(r) = -\frac{GM}{b + \sqrt{b^2 + r^2}}$$

What is the density profile that gives this potential? What is the circular velocity?

Problem 6:

Using Gauss' theorem, derive the surface density for the Kuzmin disk potential at z=0

$$\Phi_K(R, z) = -\frac{GM}{\sqrt{R^2 + (a + |z|)^2}}$$

Problem 7:

The surface density of a Mestel disk is defined as:

$$\Sigma(R) = \begin{cases} \frac{v_0^2}{2\pi G R} & (R < R_{\max}) \\ 0 & (R \geq R_{\max}) \end{cases} \tag{1}$$

Show that in the limit $R_{\max} \to \infty$, the circular velocity is the constant v_0.