Nuclear Fusion and Plasma Physics - Exercises

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Exercise 1 - Two derivations of the continuity equation

Derive the (mass) continuity equation for a plasma fluid without sources or sinks, in the two approaches, Eulerian and Lagrangian.

Hint:

- In the Eulerian approach, one needs to consider a fixed point in space, with a fixed volume, through which the fluid moves. What remains constant is the observed volume element.
- In the Lagrangian approach, one follows the evolution of a set of particles that initially occupy a volume element. What remains constant is the number of particles in the volume, not the volume itself. In this case one needs to consider the total (or "convective") derivative.
- In both cases, the starting point is the conservation of mass.

Exercise 2 - Magnetic field diffusion in a resistive plasma

a) Starting from the resistive MHD equations, find the diffusion equation for the magnetic field in a plasma.

b) Estimate the diffusion time of the magnetic field in the ITER plasma (characteristic length L = 3 m, electron temperature $T_e = 10$ keV and density $n_e = 10^{20}\,\mathrm{m}^{-3}$).

Hint:

- The resistive MHD equations are:

$$\begin{aligned}
\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) &= 0\\
\rho\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} &= \mathbf{J}\times\mathbf{B} - \nabla p\\
\mathbf{E} + \mathbf{u}\times\mathbf{B} &= \eta\mathbf{J}\\
\frac{\mathrm{d}}{\mathrm{d}t}(p\rho^{-\gamma}) &= 0\\
\nabla\times\mathbf{B} &= \mu_0\mathbf{J}\\
\nabla\times\mathbf{E} &= -\frac{\partial\mathbf{B}}{\partial t}\\
\nabla\cdot\mathbf{B} &= 0\\
\nabla\cdot\mathbf{J} &= 0
\end{aligned}$$

- A diffusion equation is an equation which contains terms of the form $\frac{\partial}{\partial t}()$ and $\nabla^2()$.

- You may find the following vector identities useful:

$$\begin{aligned}
&\nabla\times(A\times B) = A(\nabla\cdot B) - B(\nabla\cdot A) + (B\cdot\nabla)A - (A\cdot\nabla)B\\
&\nabla\times(\nabla\times A) = \nabla(\nabla\cdot A) - \nabla^2 A
\end{aligned}$$