Astrophysics III: Stellar and galactic dynamics

Exercises

Problem 1:

For the following potentials, derive the analytic expression for the acceleration $\vec{a}$ at an arbitrary point $\vec{x}$ in Cartesian coordinates. Also, derive the orbital circular period for a radius r.

a) Point mass:

$$\Phi(r) = -\frac{GM}{r}$$

b) Plummer-Schuster:

$$\Phi(r) = -\frac{GM}{\sqrt{e^2 + r^2}}$$

c) Miyamoto-Nagai:

$$\Phi(R, z) = -\frac{GM}{\sqrt{R^2 + (a + \sqrt{b^2 + z^2})^2}}$$

d) Harmonic potential:

$$\Phi(x, y, z) = \frac{1}{2}\omega_x^2 x^2 + \frac{1}{2}\omega_y^2 y^2 + \frac{1}{2}\omega_z^2 z^2$$

Problem 2:

Insert the results from Problem 1 into an orbital integrator provided (in `pm.py`, `plummer.py`, `miyamoto.py` and `harmonique.py`), which uses a Runge-Kutta solver. You should start with the point mass (`pm.py`). (Specifically the forces, circular velocities and periods in the relevant python routines.)

You can verify your results by using the script orbit.py. The following command print some information about the script.

```
./orbit.py --help
```

Problem 3:

Using the integrator for the orbits, attempt to find the appropriate initial conditions for each of the potentials proposed in problem 1 in order to generate:

a) circular orbits

b) quasi-periodic orbits

c) resonant orbits

For each case, examine the conserved integrals (energy, angular momentum, the projected angular momentum on z).