Nuclear Fusion and Plasma Physics

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Lecture 7 - 7 November 2022

Magnetic confinement

- The tokamak: concept and its main features
- The stellarator

Waves in plasmas

- Importance of plasma waves
- Mathematical techniques (linearisation, Fourier transform)
- Group and phase velocities
- Ideal MHD waves
  - shear Alfvén
  - compressional Alfvén and magnetosonic waves

1 Magnetic confinement

1.1 The tokamak

Axi-symmetric torus, large toroidal magnetic field, small poloidal magnetic field, large pressure. Four main features:

1. TF coils.
2. OH transformer ($\rightarrow$ current for equilibrium, heating).
3. Vertical field system ($\rightarrow$ for toroidal force balance).
4. Shaping coils ($\rightarrow$ to improve MHD stability and alleviate plasma-wall interactions).

1.2 The stellarator

Rotational transform from external coils only.

- No need for plasma current.
- Steady-state.

$\Rightarrow$ See viewgraphs for a qualitative discussion.

2 Waves in plasmas

All plasma particles are "sources" for Maxwell's equations. Therefore most dynamical processes in plasmas are related to electromagnetic waves and oscillations. Waves are used to heat plasmas, and to drive current non–inductively. Another example of the importance of waves is the role that microscopic electromagnetic waves and instabilities play in producing transport of particles and energy in plasmas well above the levels due to collisional effects.

2.1 Mathematical technique

We will use normal mode (or plane wave) analysis. This corresponds to considering all quantities in Fourier space, using the Fourier transform defined for any quantity $\mathbf{g}$ as

$$\widetilde{\mathbf{g}}(\mathbf{k},\omega) = \frac{1}{(2\pi)^4}\int d^3x \int dt\, \mathbf{g}(\mathbf{x},t) e^{-i(\mathbf{k}\cdot\mathbf{x}-\omega t)}, \tag{2.1}$$

with the inverse transform given by

$$\mathbf{g}(\mathbf{x},t) = \int d^3k \int d\omega\, \widetilde{\mathbf{g}}(\mathbf{k},\omega) e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}. \tag{2.2}$$

This will lead to complex quantities. Naturally, all physical quantities are real, and we will need to consider the real part at the end of all calculations.

The Fourier transformation is a *linear* operation. Its use comes from the fact that by using it we can split a complicated problem into small pieces, solve it for these small pieces, and combine the pieces together to form the complete solution. This implies that the system of equations to be solved is *linear*.

When the system of equations to be solved is non-linear, we *linearise* it considering small perturbations to an existing equilibrium. Take for example the continuity equation (a differential equation) for the mass density ρ and the fluid velocity $\mathbf{u}$:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \tag{2.3}$$

where $\rho \equiv \rho(\mathbf{x}, t)$ and $\mathbf{u} \equiv \mathbf{u}(\mathbf{x}, t)$.

1. Choose an equilibrium $\rightarrow$ no time dependence $\rightarrow$ steady state:

$$\rho_0(\mathbf{x}) = \rho_0 \quad \text{(uniform equilibrium)}, \qquad \mathbf{u}_0(\mathbf{x}) = 0 \quad \text{(static equilibrium)} \tag{2.4}$$

2. Consider small perturbations to this equilibrium

$$\rho = \rho_0 + \rho_1(\mathbf{x}, t), \qquad \left|\frac{\rho_1}{\rho_0}\right| \ll 1 \quad \text{(expansion parameter)} \tag{2.5}$$

3. Linearise by retaining first order terms only to get the linearised continuity equation

$$\begin{aligned}
\frac{\partial(\rho_0 + \rho_1)}{\partial t} \quad &+ \quad \nabla \cdot \left((\rho_0 + \rho_1)(\underbrace{\mathbf{u}_0}_{=0} + \mathbf{u}_1) \right) = 0 \\
\underbrace{\frac{\partial \rho_0}{\partial t}}_{\text{Order 0; } = 0 \text{ by definition}} + \underbrace{\frac{\partial \rho_1}{\partial t}}_{\text{Order 1}} \quad &+ \quad \underbrace{\nabla \cdot (\rho_0 \mathbf{u}_1)}_{\text{Order 1 and } \rho_0 = \text{cte}} + \underbrace{\nabla \cdot (\rho_1 \mathbf{u}_1)}_{\text{Order 2; neglected}} = 0 \\
\frac{\partial \rho_1}{\partial t} \quad &+ \quad \rho_0 \nabla \cdot \mathbf{u}_1 = 0.
\end{aligned} \tag{2.6}$$

4. Now we consider normal modes, i.e. we consider the perturbed quantities as Fourier transforms:

$$\rho_1(\mathbf{x}, t) = \int d^3k \int d\omega \, \widetilde{\rho}_1(\mathbf{k}, \omega) e^{i(\mathrm{k}\cdot\mathrm{x} - \omega t)} \tag{2.7}$$

and the same for $\mathbf{u}_1$. Thus

$$\begin{aligned}
& \frac{\partial}{\partial t} \left\{ \int d^3k \int d\omega \; \widetilde{\rho}_1(\mathbf{k}, \omega) e^{i(\mathrm{k}\cdot\mathrm{x} - \omega t)} \right\} \\
+ \quad & \rho_0 \nabla \cdot \left\{ \int d^3k \int d\omega \; \widetilde{\mathbf{u}}_1(\mathbf{k}, \omega) e^{i(\mathrm{k}\cdot\mathrm{x} - \omega t)} \right\} = 0 \\
\Rightarrow \quad & \int d^3k \int d\omega \Big[-i\omega \widetilde{\rho}_1(\mathbf{k}, \omega) \Big] e^{i(\mathrm{k}\cdot\mathrm{x} - \omega t)} \\
+ \quad & \rho_0 \int d^3k \int d\omega \, \Big[i\mathbf{k} \cdot \widetilde{\mathbf{u}}_1(\mathbf{k}, \omega) \Big] e^{i(\mathrm{k}\cdot\mathrm{x} - \omega t)} = 0.
\end{aligned} \tag{2.8}$$

In general we can make the following formal substitutions:

$$\nabla \rightarrow i\mathbf{k} \qquad \text{and} \qquad \frac{\partial}{\partial t} \rightarrow -i\omega. \tag{2.9}$$

In our example the linearised continuity equation becomes in Fourier space an algebraic equation:

$$-i\omega\widetilde{\rho}_1 + i\rho_0\mathbf{k}\cdot\widetilde{\mathbf{u}}_1 = 0. \tag{2.10}$$

In the following we will drop the tilde symbol to simplify the notation.
Note that it is important to refer to the equilibrium, with respect to which the linearisation is done.

2.2 Phase and group velocities

Phase velocity

$$\mathbf{v}_{\mathrm{ph}} = \frac{\omega}{k}\frac{\mathbf{k}}{k}. \tag{2.11}$$

It *can* be $|\mathbf{v}_{\mathrm{ph}}| > c$, as $\mathbf{v}_{\mathrm{ph}}$ does not carry information.

Group velocity

$$\mathbf{v}_{\mathrm{g}} = \frac{\partial\omega}{\partial\mathbf{k}}. \tag{2.12}$$

It *cannot* be $|\mathbf{v}_{\mathrm{g}}| > c$, as $\mathbf{v}_{\mathrm{g}}$ does carry information.

2.3 Ideal MHD waves

The ideal MHD system can be reduced by combining its equations, obtaining

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0 \qquad \rho\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = -\nabla p + \frac{1}{\mu_0}(\nabla\times\mathbf{B})\times\mathbf{B} \tag{2.13}$$

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) \qquad \frac{\mathrm{d}}{\mathrm{d}t}(p\rho^{-\gamma}) = 0 \quad \text{(see footnote[1])} \tag{2.14}$$

This is a system of 8 equations with 8 unknowns: ρ, p, $\mathbf{u}$, $\mathbf{B}$. We now consider small perturbations to a uniform and static (no flow) equilibrium

$$\mathbf{B}(\mathbf{x},t) = \mathbf{B}_0 + \mathbf{B}_1(\mathbf{x},t) \qquad \mathbf{u}(\mathbf{x},t) = \mathbf{u}_1(\mathbf{x},t) \tag{2.15}$$

$$\rho(\mathbf{x},t) = \rho_0 + \rho_1(\mathbf{x},t) \qquad p(\mathbf{x},t) = p_0 + p_1(\mathbf{x},t) \tag{2.16}$$

and linearize the original system of equations with respect to the equilibrium

$$\frac{\partial\rho_1}{\partial t} + \rho_0\nabla\cdot\mathbf{u}_1 = 0 \qquad \rho_0\frac{\partial\mathbf{u}_1}{\partial t} = -\nabla p_1 + \frac{1}{\mu_0}(\nabla\times\mathbf{B}_1)\times\mathbf{B}_0 \tag{2.17}$$

$$\frac{\partial\mathbf{B}_1}{\partial t} = \nabla\times(\mathbf{u}_1\times\mathbf{B}_0) \qquad p_1 = \frac{\gamma p_0}{\rho_0}\rho_1 \equiv c_s^2\rho_1 \quad \text{(see footnote[2])} \tag{2.18}$$

[1] Rewriting the continuity equation as $\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho\nabla\cdot\mathbf{u}$, we have another form of the equation of state: $\frac{\mathrm{d}p}{\mathrm{d}t} + \gamma p\nabla\cdot\mathbf{u} = 0$.

Here $c_s \equiv \sqrt{\gamma p_0/\rho_0}$ is the *sound speed*. After elimination of p_1 and Fourier transformation this becomes

$$-\omega\rho_1 + \rho_0 \mathbf{k}\cdot\mathbf{u}_1 = 0 \tag{2.19}$$

$$-\omega\rho_0\mathbf{u}_1 = -\mathbf{k}\rho_1 c_s^2 + \frac{1}{\mu_0}(\mathbf{k}\times\mathbf{B}_1)\times\mathbf{B}_0 \tag{2.20}$$

$$-\omega\mathbf{B}_1 = \mathbf{k}\times(\mathbf{u}_1\times\mathbf{B}_0) \tag{2.21}$$

The shear Alfvén wave

Without loss of generality we can choose $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$ and $k_y = 0$ (see figure 1). Let us now consider the particular case of a transverse wave $u_{1x} = u_{1z} = 0$, i.e.

$$\mathbf{k} = (k_x, 0, k_z) \tag{2.22}$$

$$\mathbf{u}_1 = (0, u_{1y}, 0) \tag{2.23}$$

We will treat the case $u_{1x} \neq 0 \neq u_{1z}$ later.

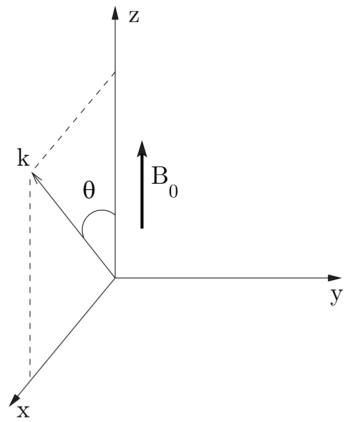

Figure 1: *Notation for the study of MHD waves.*

Eq.(2.19) gives $\begin{pmatrix} k_x \\ 0 \\ k_z \end{pmatrix} \cdot \begin{pmatrix} 0 \\ u_{1y} \\ 0 \end{pmatrix} = 0$

Therefore $\rho_1 = 0$, i.e. there is no variation of the mass density and we can conclude that the wave is of non–compressional type.

The component along the y-axis of eq.(2.20) becomes

$$\omega\rho_0 u_{1y} = -\frac{1}{\mu_0}\Big[(\mathbf{k}\times\mathbf{B}_1)\times\mathbf{B}_0\Big]_y = -\frac{1}{\mu_0}\left|\begin{array}{ccc} \hat{x} & \hat{y} & \hat{z} \\ (\mathbf{k}\times\mathbf{B}_1)_x & (\mathbf{k}\times\mathbf{B}_1)_y & (\mathbf{k}\times\mathbf{B}_1)_z \\ 0 & 0 & B_0 \end{array}\right|_y =$$

$$= \frac{B_0}{\mu_0}(\mathbf{k}\times\mathbf{B}_1)_x = \frac{B_0}{\mu_0}\left|\begin{array}{ccc} \hat{x} & \hat{y} & \hat{z} \\ k_x & 0 & k_z \\ B_{1x} & B_{1y} & B_{1z} \end{array}\right|_x = -\frac{B_0}{\mu_0}\, k_z B_{1y}$$

Eq.(2.21) gives

$$-\,\omega B_{1y} = \Big[\mathbf{k}\times(\mathbf{u}_1\times\mathbf{B}_0)\Big]_y = \Big[\mathbf{k}\times\hat{x}B_o u_{1y}\Big]_y = B_0 k_z u_{1y} \tag{2.24}$$

[2]From eq.(2.14) and eq.(2.16) we have $(p_0+p_1)(\rho_0+\rho_1)^{-\gamma} = p_0\rho_0^{-\gamma} \Rightarrow (p_0+p_1)(1-\gamma\frac{\rho_1}{\rho_0}) = p_0$. At the 'zero' order (i.e. neglecting all the perturbation terms labelled as '1') we simply have $p_0 \equiv p_0$, while at the first order we obtain $p_1 = \gamma p_0\frac{\rho_1}{\rho_0}$.

Then the system of eq.(2.19), eq.(2.20) and eq.(2.21) can be written as:

$$\rho_1 = 0, \tag{2.25}$$
$$\omega\rho_0 u_{1y} + \frac{k_z B_0}{\mu_0} B_{1y} = 0, \tag{2.26}$$
$$k_z B_0 u_{1y} + \omega B_{1y} = 0, \tag{2.27}$$

where eq.(2.26) and eq.(2.27) can be written as a homogenous linear system

$$\mathbf{A} \cdot \begin{pmatrix} u_{1y} \\ B_{1y} \end{pmatrix} = 0, \qquad \text{where} \qquad \mathbf{A} = \begin{pmatrix} \omega\rho_0 & \frac{k_z B_0}{\mu_0} \\ k_z B_0 & \omega \end{pmatrix}. \tag{2.28}$$

To have a non–trivial solution ($u_{1y} \neq 0 \neq B_{1y}$), we must have $\det \mathbf{A} = 0$. Thus, we obtain the following dispersion relation

$$\omega^2 = \frac{B_0^2}{\rho_0\mu_0} k_z^2 \equiv c_A^2 k_z^2 = c_A^2 k^2 \cos^2\theta, \tag{2.29}$$

where $c_A \equiv B_0/\sqrt{\mu_0\rho_0}$ is the *Alfvén speed*. Typical values are

Magnetosphere:

$$\left.\begin{array}{r} B \sim 5 \times 10^{-8}\ \mathrm{T} \\ n \sim 10^6\ \mathrm{m}^{-3} \end{array}\right\} \Rightarrow c_A \sim \frac{5 \times 10^{-8}}{\sqrt{1.7 \times 10^{-27} \cdot 10^6 \cdot 4\pi \cdot 10^{-7}}} \sim 10^6\ \mathrm{m/s}.$$

Tokamak:

$$\left.\begin{array}{r} B \sim 3\ \mathrm{T} \\ n \sim 10^{20}\ \mathrm{m}^{-3} \end{array}\right\} \Rightarrow c_A \sim \frac{3}{\sqrt{1.7 \times 10^{-27} \cdot 10^{20} \cdot 4\pi \cdot 10^{-7}}} \sim 6 \times 10^6\ \mathrm{m/s}.$$

The solution given by eq.(2.29) is called *shear Alfvén wave* or *non–compressional Alfvén wave*, as there is no density perturbation:

$$\rho_1 = \frac{\mathbf{k} \cdot \mathbf{u}_1}{\omega} = 0, \tag{2.30}$$

This is different from sound waves, for example. Note that

- The velocity of α particles born with energies 3.5 MeV is $> c_A$, so the α's become *resonant*[3] with Alfvén waves during slowing down in a fusion reactor.
- Alfvén waves are equivalent to waves on a string with tension S and mass per unit length M

$$M\frac{\partial^2 y}{\partial t^2} = S\frac{\partial^2 y}{\partial z^2} \qquad \Longrightarrow \qquad \omega^2 = \frac{S}{M}k_z^2 \tag{2.31}$$

  In the exercise you will show the analogy between a wave travelling along a magnetic field line and a chord.

[3] As we will see later in the kinetic model, the condition $v_{\text{particle}} \sim v_{\text{ph}}$ makes it possible that a strong interaction between waves and particles with exchange of energy may occur. This may lead to instabilities, and the particle motion may be affected by the wave.

The compressional Alfvén waves (fast waves) and the magneto–sonic waves

Now we consider the other case $u_{1x} \neq 0$, $u_{1y} = 0$, $u_{1z} \neq 0$, where the perturbation has a longitudinal component. Choosing $B_{1y} = 0$ we get with our previous choices $\mathbf{B}_0 = B_0 \mathbf{e}_z$ and $k_y = 0$ from eq.(2.21)

$$\mathbf{B}_1 = \frac{u_{1x} B_0}{\omega} (\mathbf{k} \times \hat{\mathbf{y}}). \tag{2.32}$$

By inserting ρ_1 from eq.(2.19) and $\mathbf{B}_1$ from eq.(2.32) in eq.(2.20), we get a linear system for u_{1x} and u_{1z}, which again has a non–trivial solution only if the determinant of the coefficient matrix vanishes. After some algebra one finds the dispersion relation

$$\omega^4 - \omega^2 k^2 (c_A^2 + c_s^2) + k_z^2 k^2 c_A^2 c_s^2 = 0, \tag{2.33}$$

which has the solutions

$$\omega^2 = \frac{1}{2} \big(c_A^2 + c_s^2 \big) k^2 \pm \sqrt{\frac{1}{4} \big(c_A^2 + c_s^2 \big)^2 k^4 - c_A^2 c_s^2 k^2 k_z^2}. \tag{2.34}$$

Note that

$$\left(\frac{c_s}{c_A} \right)^2 = \frac{\gamma p_0}{\rho_0} \frac{\mu_0 \rho_0}{B_0^2} = \frac{\gamma}{2} \frac{p_0}{\frac{B_0^2}{2\mu_0}} = \frac{\gamma}{2} \beta, \tag{2.35}$$

The pressure ratio β is an important parameter to characterize a plasma[4]. For many plasmas of interest we have $\beta \ll 1$, so $c_s \ll c_A$. In this limit the "+" branch of eq.(2.34) becomes

$$\omega^2 \simeq k^2 c_A^2. \tag{2.36}$$

This solution is called *fast wave* or *compressional Alfvén wave*[5]. For the "−" branch we find the so-called *slow wave* or *magneto–sonic wave*

$$\omega^2 \simeq c_s^2 k_z^2 = k^2 c_s^2 \cos^2 \theta. \tag{2.37}$$

These are *all* possible modes of oscillation that an (unbounded) "MHD plasma" can sustain. As we relax the assumptions that lead to the MHD model many other modes appear, for example separating ions and electrons in their oscillatory motion. To describe these modes we need a more detailed plasma model, as the multi–fluid or the kinetic models.

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[4] $B_0^2/2\mu_0$ is often referred to as "magnetic pressure".

[5] $\rho_1 \neq 0 \longleftrightarrow \nabla \cdot \mathbf{u}_1 \neq 0$

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Nuclear Fusion and Plasma Physics

Lecture 7

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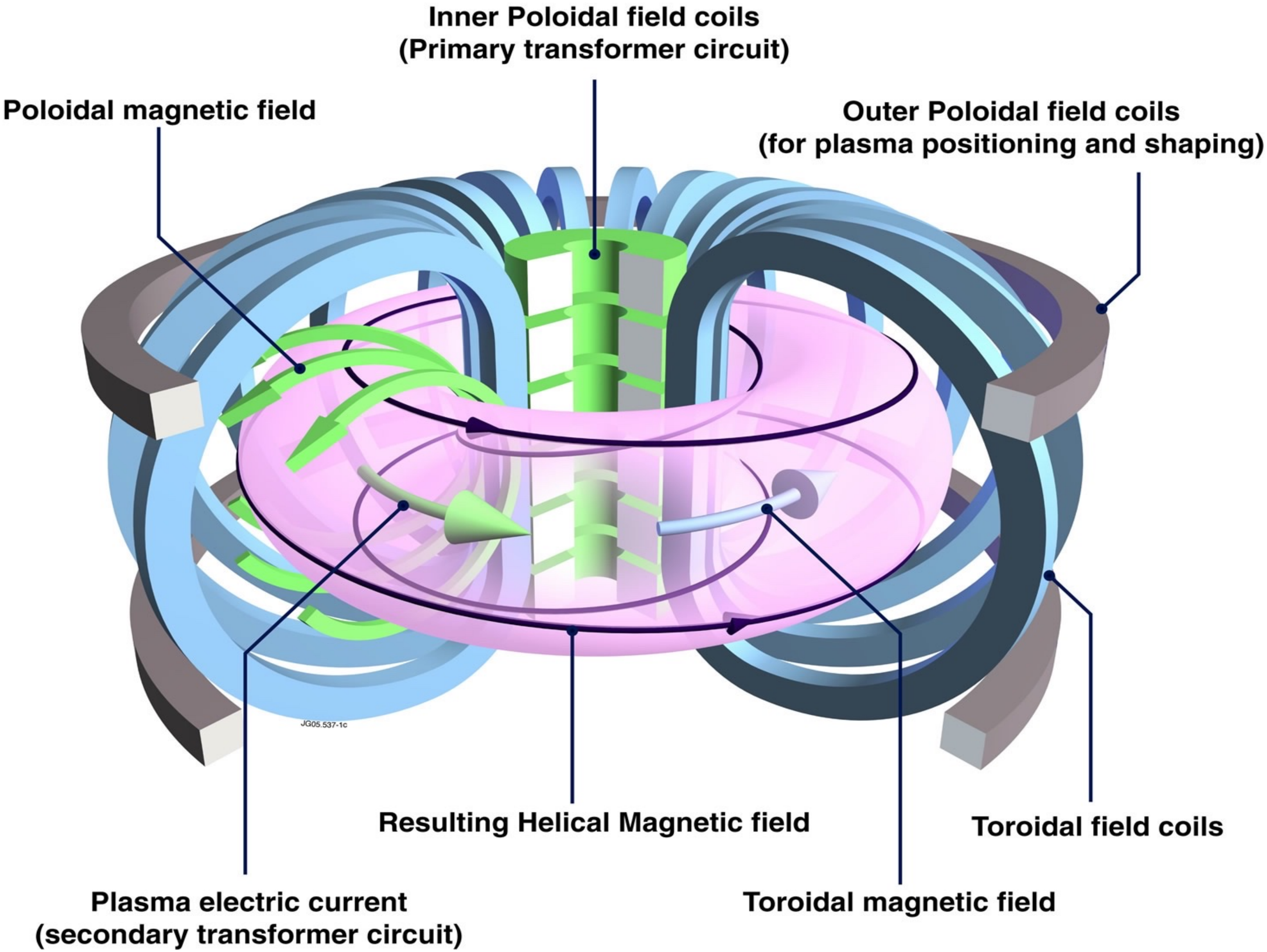
The Tokamak



Inner Poloidal field coils
(Primary transformer circuit)
Poloidal magnetic field
Outer Poloidal field coils
(for plasma positioning and shaping)
JG05.537-1c
Resulting Helical Magnetic field
Toroidal field coils
Plasma electric current
(secondary transformer circuit)
Toroidal magnetic field

Main features of Tokamaks

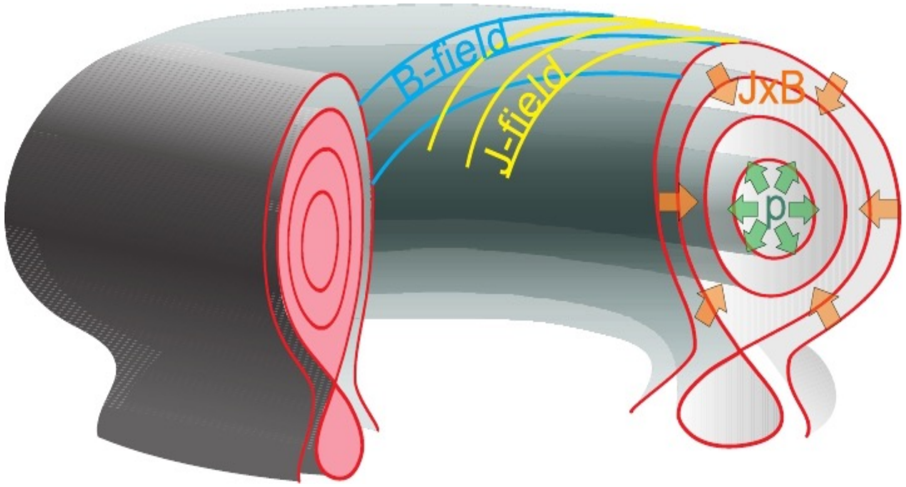
Axi-symmetric torus, large toroidal magnetic field, small poloidal magnetic field, large pressure

Four main features/components

- Toroidal field coils
  - main confinement field
- 'Ohmic' transformer
  - current for equilibrium, heating
- Vertical field system
  - toroidal force balance, contrasts hoop force
- Shaping coils
  - to improve MHD stability and alleviate plasma-wall interactions

Plasma equilibrium $\mathbf{j}\times\mathbf{B} = \nabla p$

Nested magnetic surfaces on which p is constant and current lies



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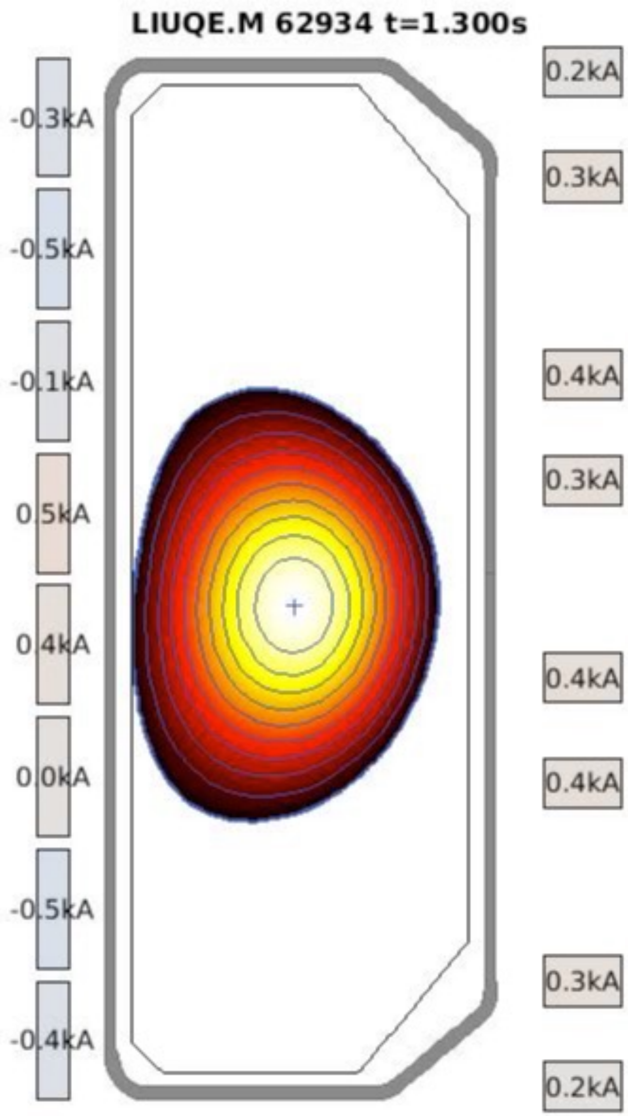
Plasma equilibrium $\mathbf{j}\times\mathbf{B} = \nabla p$

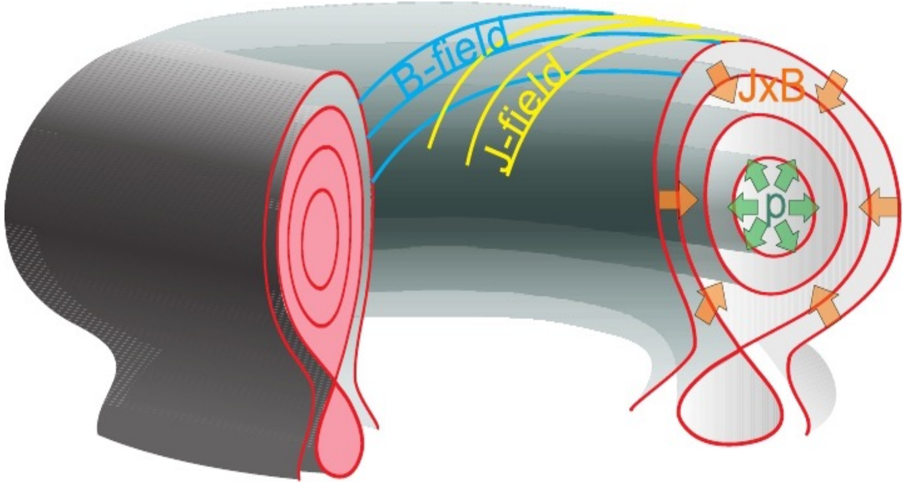
Nested magnetic surfaces on which p is constant and current lies

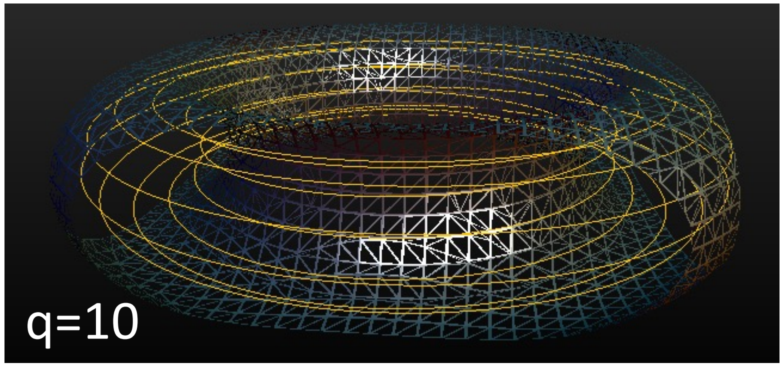
Ex. of TCV plasma evolution

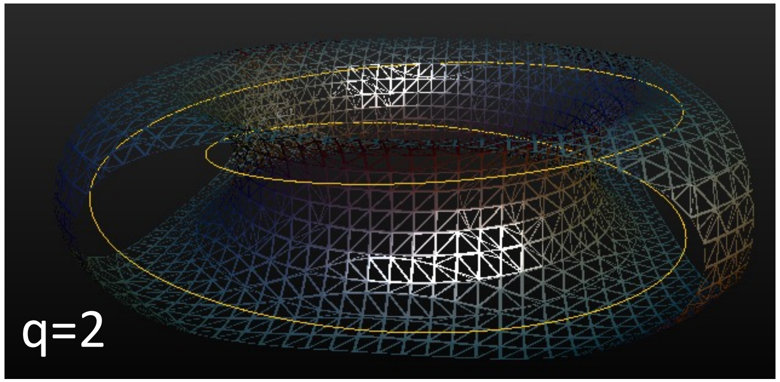


Plasma equilibrium $\mathbf{j}\times\mathbf{B} = \nabla p$

Nested magnetic surfaces on which p is constant and current lies







Tokamak equilibrium characterised by

- Safety factor q = toroidal turns / poloidal turns (pitch of field lines)
- Normalised pressure $\beta = nT/(B^2/2\mu_0)$

Plasma stability

Stability

Destabilising: gradients of current and pressure

Stabilising: B-field line bending and compression

Instabilities

Ideal ($\eta=0$): fast, no change in B-field topology

Resistive ($\eta\neq0$): slower, possibility of feedback control, change in B-field topology (magnetic islands)

MHD stability imposes limits on optimisation of fusion parameters

Current limit

- Limits energy confinement time
  - $\tau_E \propto 1/q \sim I_p$ for fixed B-field
- Can be improved by shaping the plasma

Limit in normalised pressure $\beta \propto nT/B^2$

- Limits fusion power for given B (cost!)
  - $P_{fus} \propto \beta^2 B^4$
- Can be improved by shaping the plasma

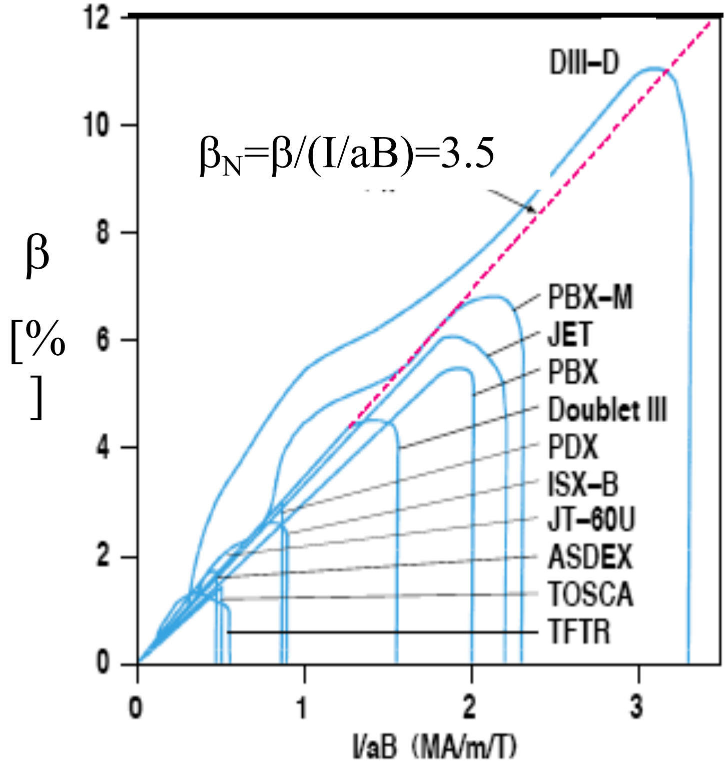
Density limit

- Limits fusion power
  - $P_{fus} \propto n^2 \langle \sigma v \rangle$
- Can be improved by peaked radial profiles

Ideal plasma stability

Ideal limit in β and current is generally understood

- The need to optimise fusion power ($P_{fus} \propto \beta^2$) pushes operation close to limits



Violation of linear stability results in rapid loss of plasma: disruptions

- Toroidal E-field can lead to *runaway* electrons, damaging wall
- Plasma currents intercepted by conducting surfaces and fast variation of flux lead to large thermal loads and e.m. forces

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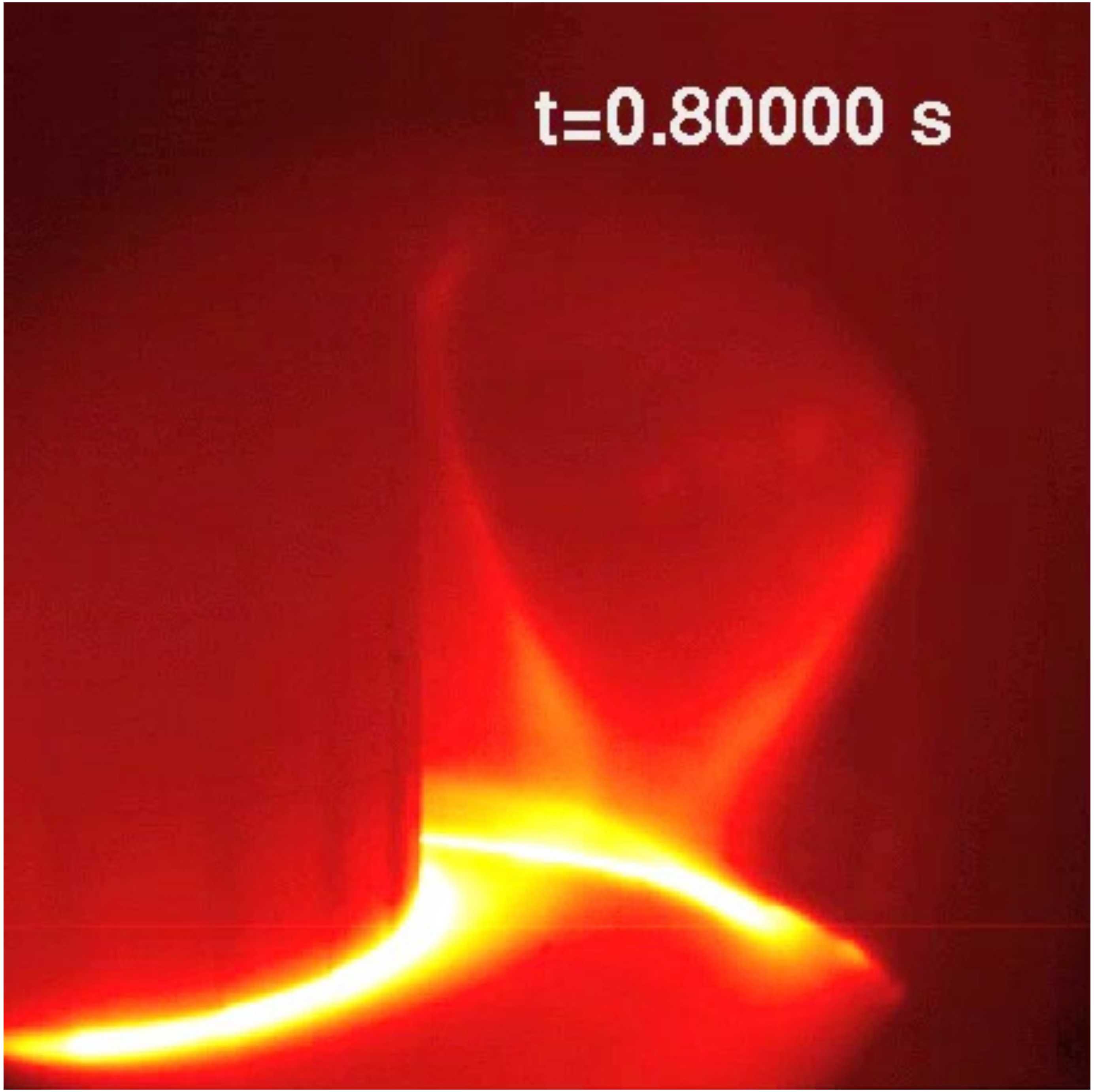
A TCV tokamak discharge



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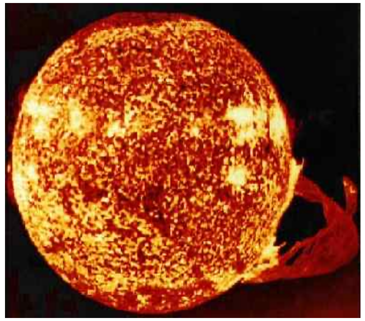
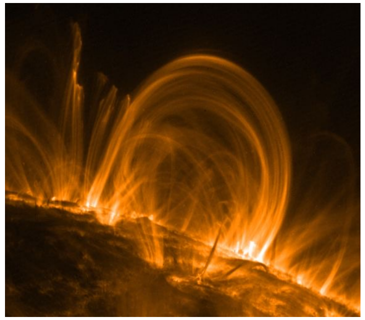
Tokamak *physics* challenges



Large power density and gradients (10MW/m^3), anisotropy, no thermal equilibrium

Macro-instabilities and relaxation processes

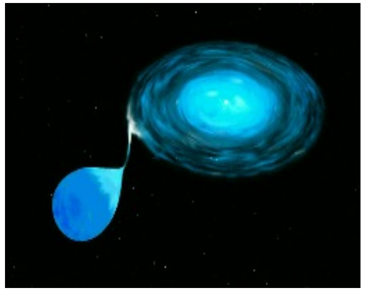
solar flares, substorms



Dual fluid/particle nature

Wave-particle interaction (resonant, nonlinear)

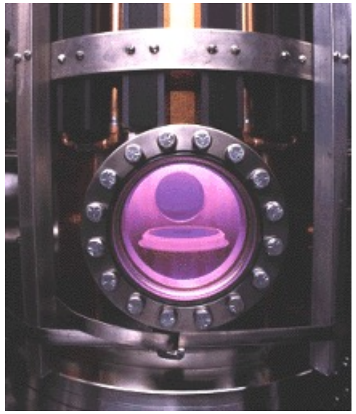
coronal heating



Turbulent medium

Non-collisional transport and losses

accretion disks



Plasma-neutral transition, wall interaction

plasma manufacturing

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Huge range in temporal (10^{-10}→10^5 s) and spatial scales (10^{-6}→10^4 m)

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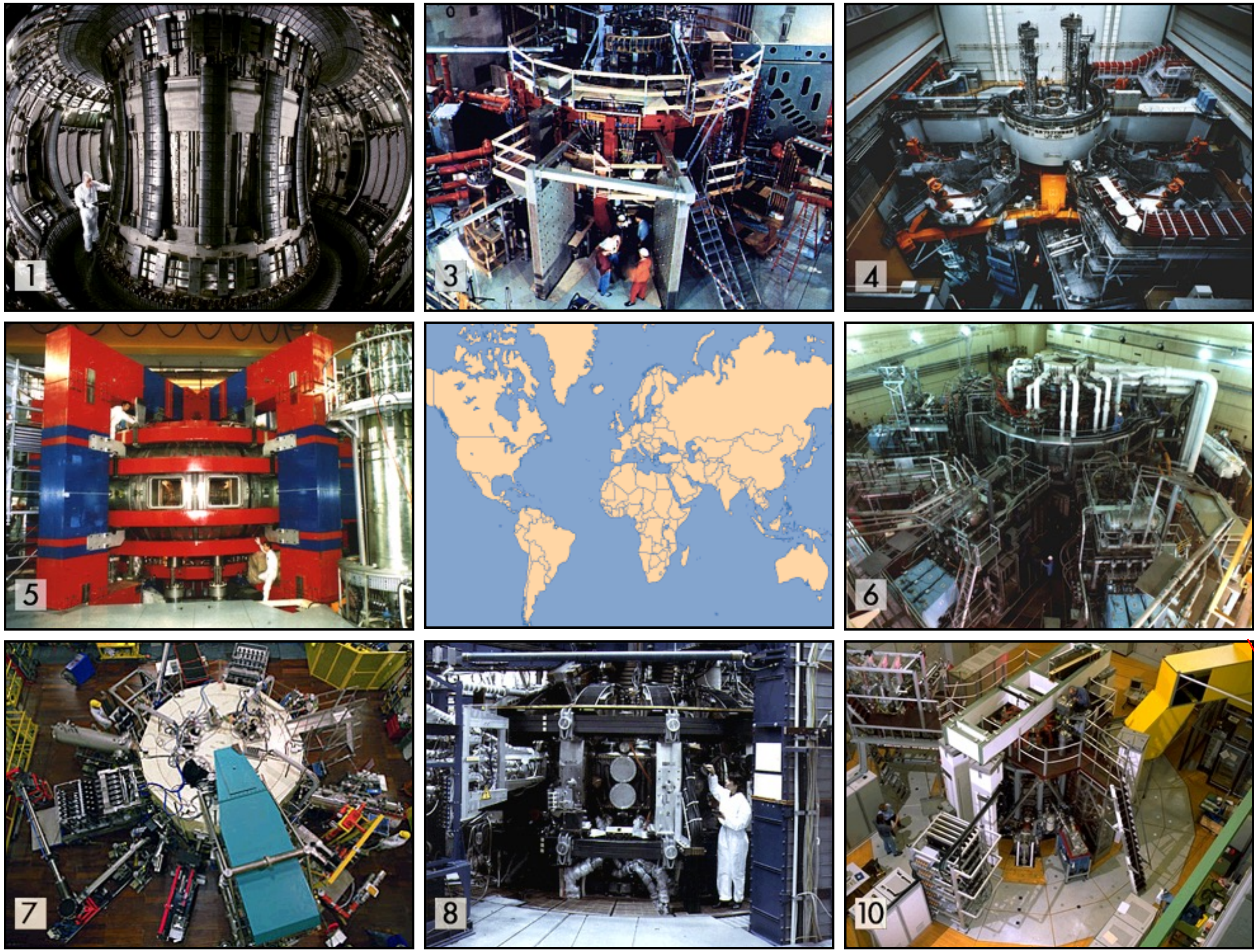
Tokamaks around the world



Europe: ~10

Japan: ~5

Russia: ~5

USA: 3

Others: 10

~40 tokamaks in operation or under construction (India, Korea, China)

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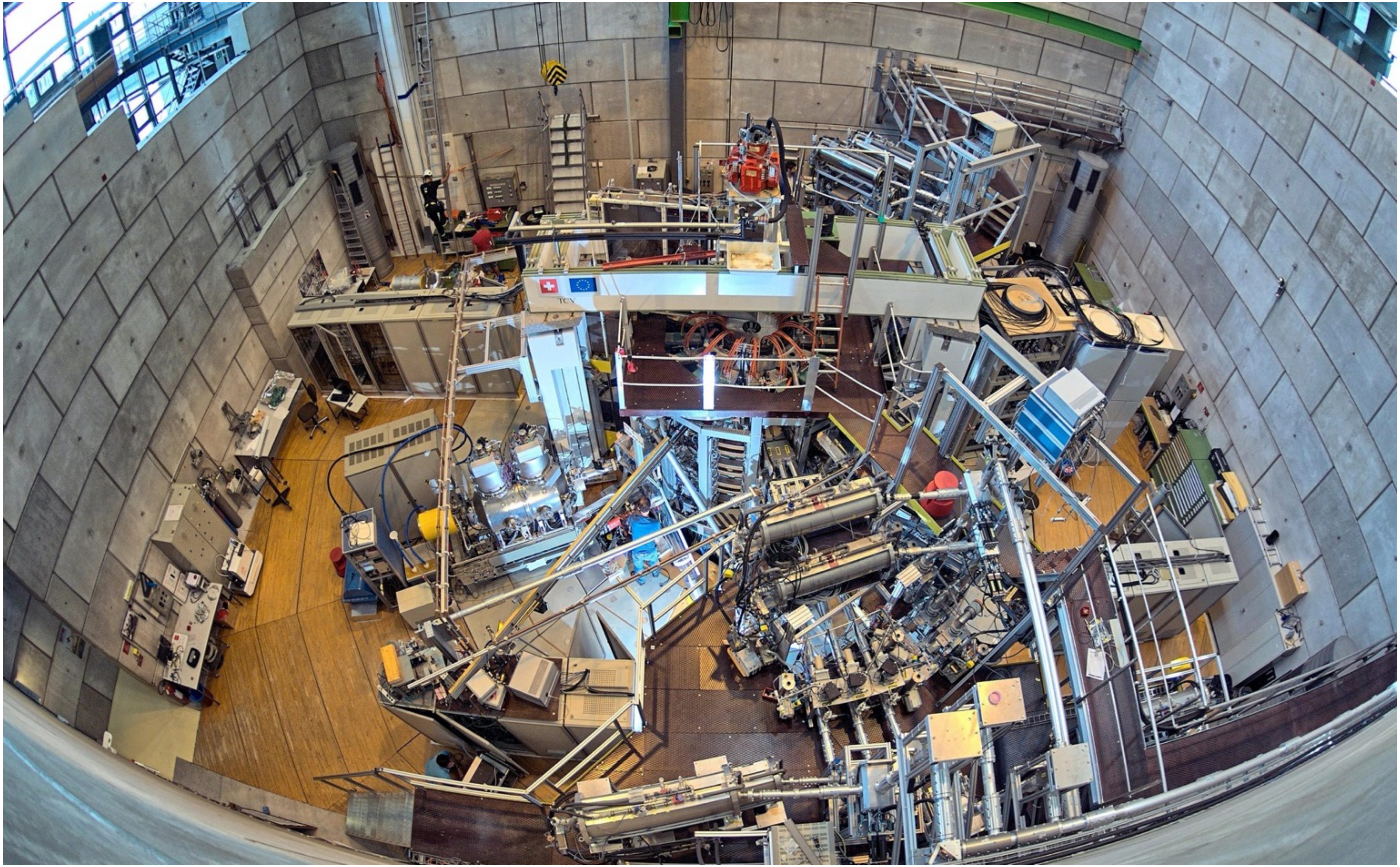
Tokamak à Configuration Variable



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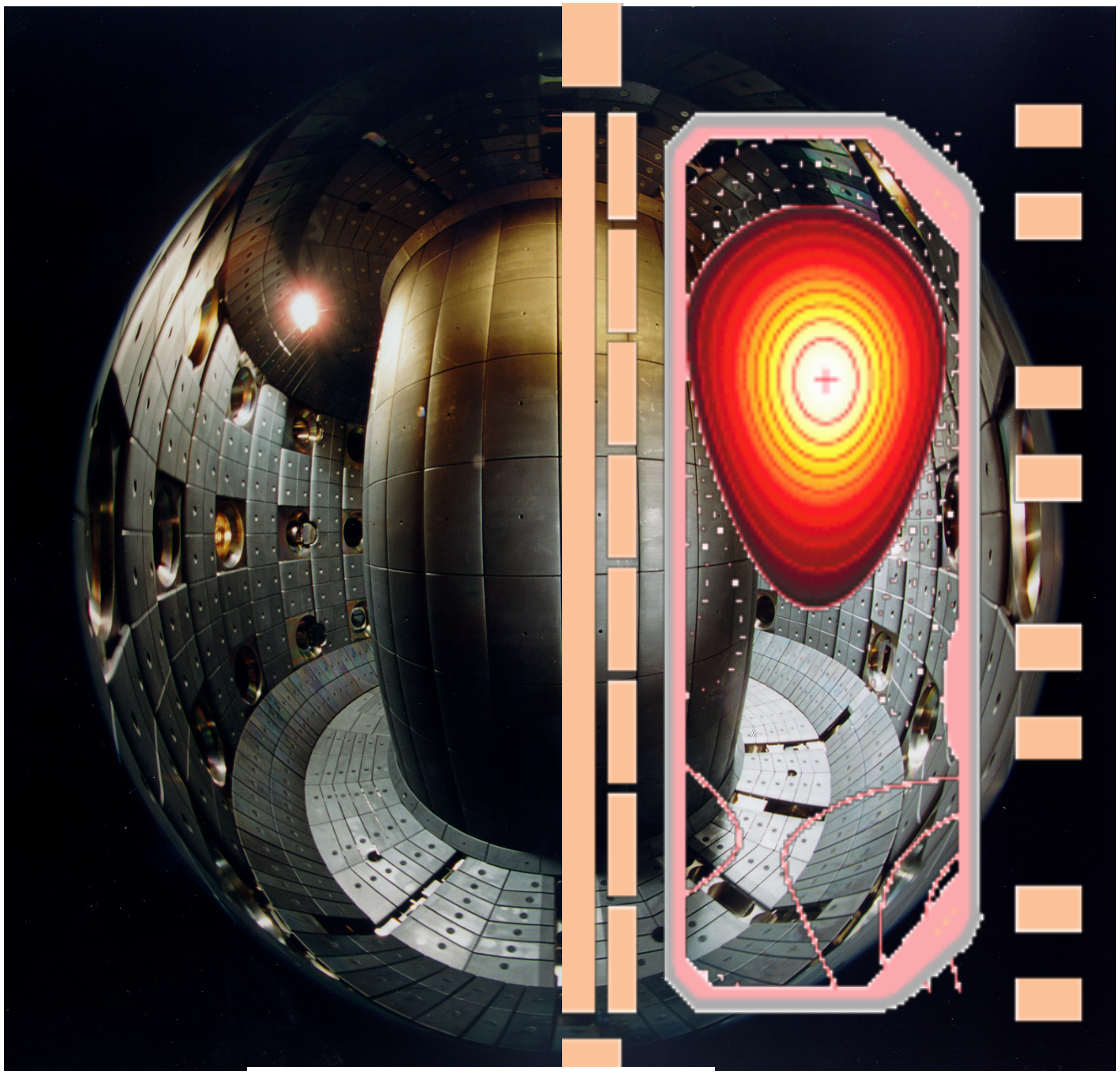
$R = 0.9\text{m};\;\; I_p \leq 1\text{MA};\;\; B_T \leq 1.54\text{T};\;\; 0.9 < \kappa < 2.8;\;\; -0.8 < \delta < 1$

Unique TCV feature: flexible plasma shapes



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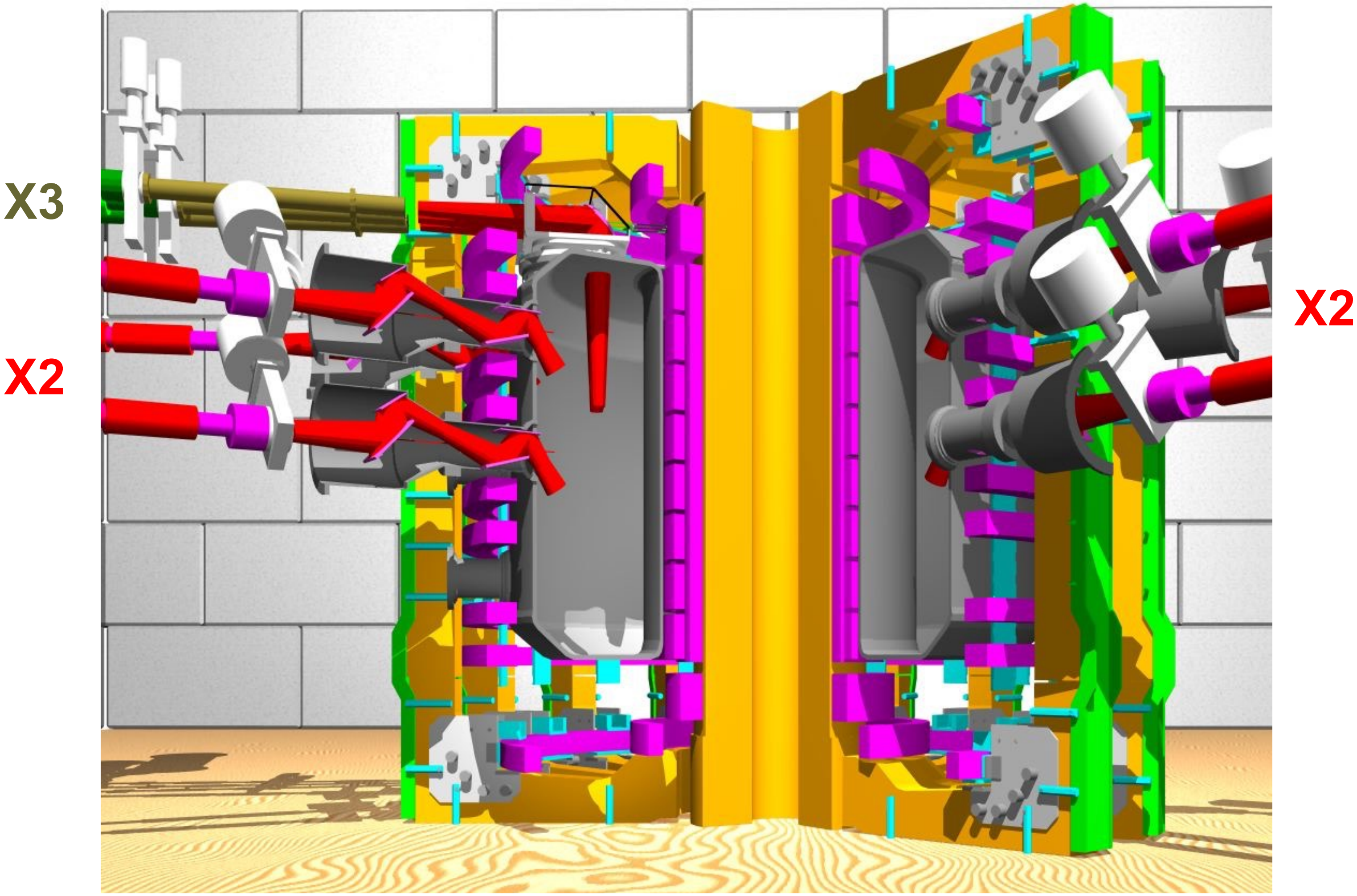
Unique TCV feature: EC heating and current drive systems

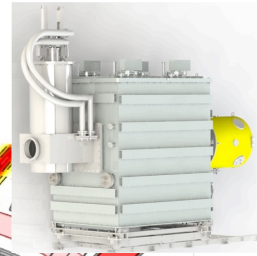
Second harmonic (X2) Third harmonic (X3)



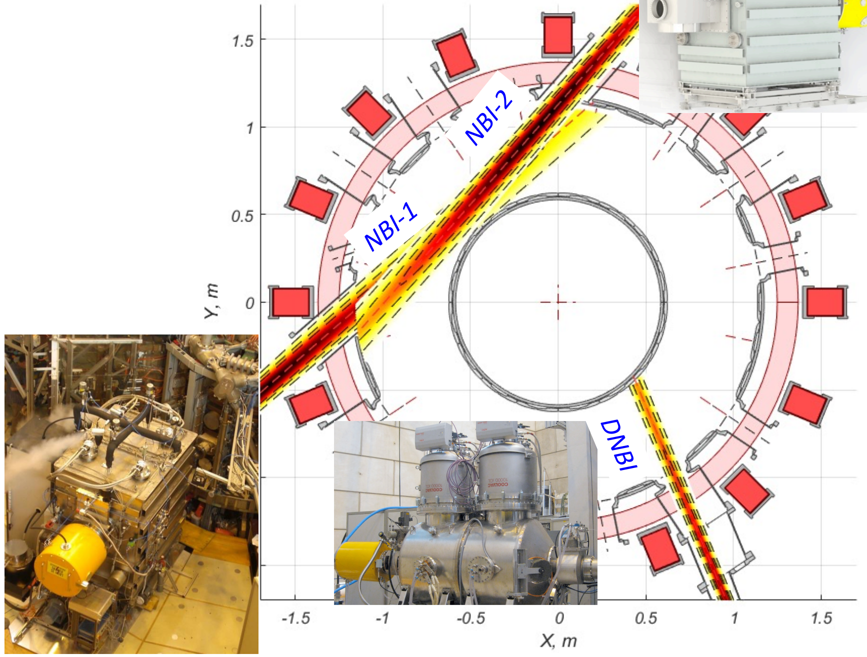
Plasma temperatures up to 12keV = 100 millions degrees K

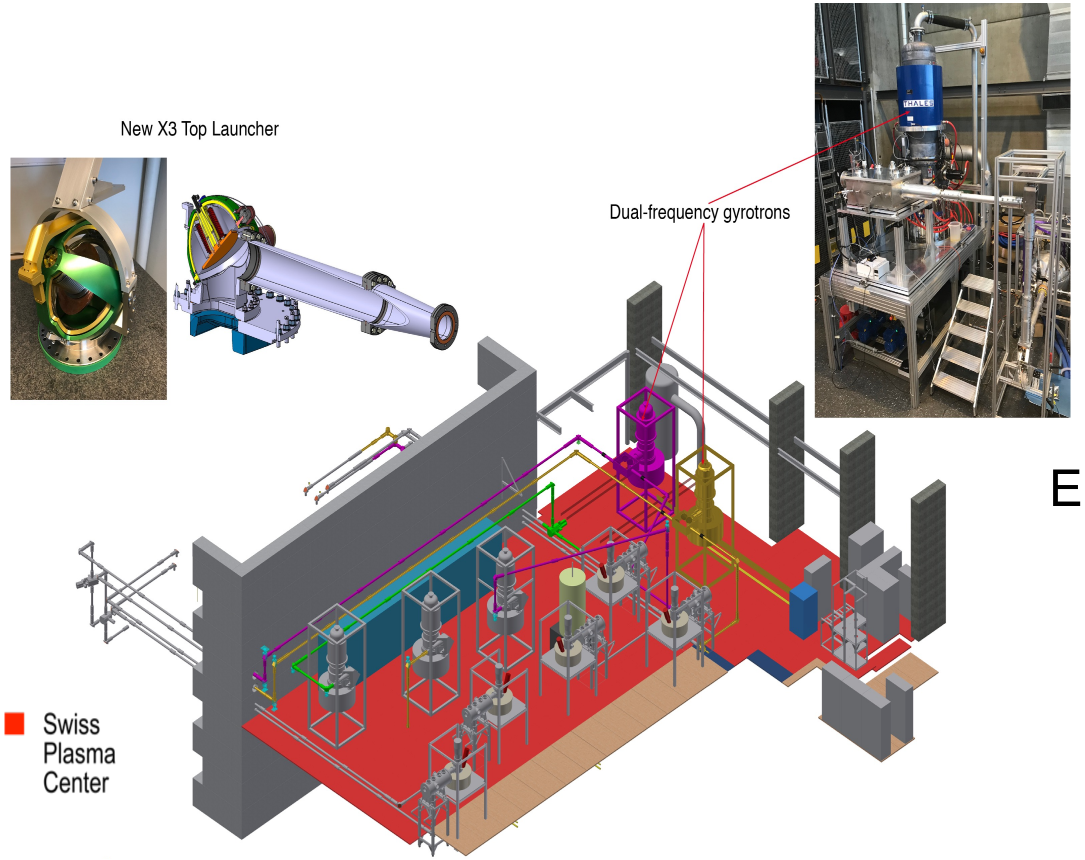
TCV heating upgrades (up to 6.5MW)

To improve plasma performance up to reactor relevant pressures and investigate burning plasma physics



Neutral Beam Heating
1MW 30keV + 1MW 50keV

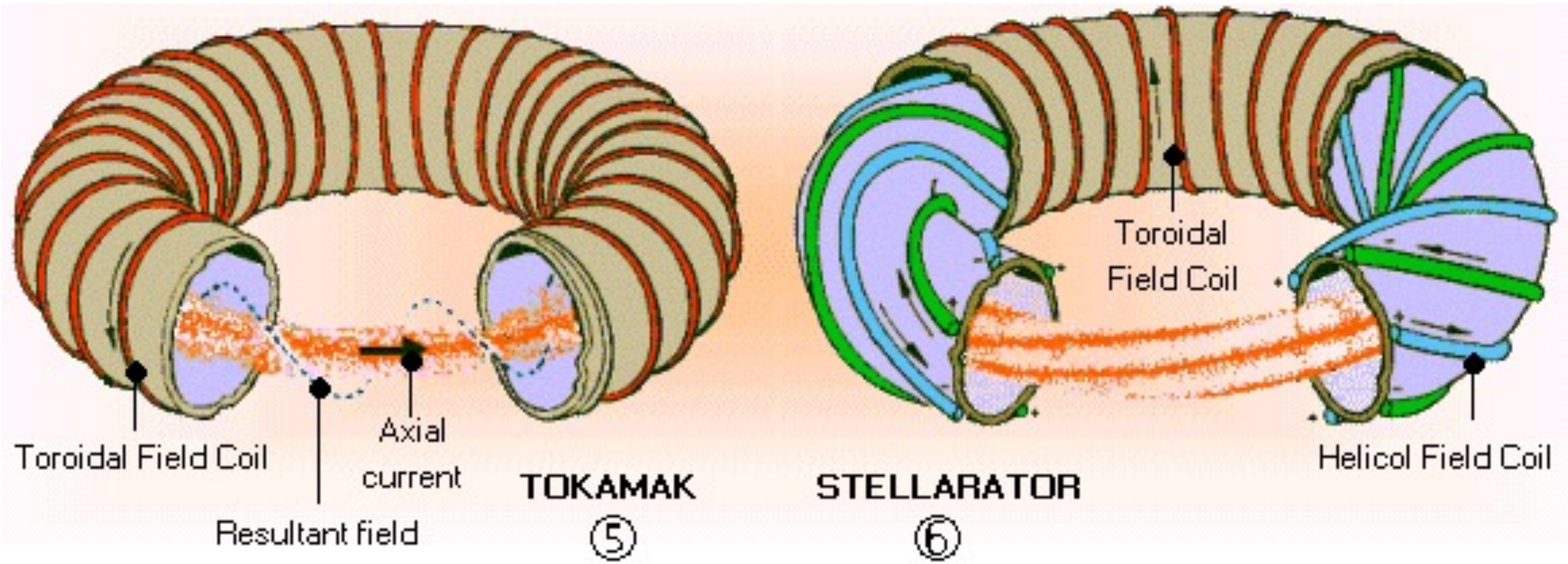




Electron Cyclotron Heating - Gyrotrons
2x 0.75MW / 83GHz
2x 1MW / 84&126GHz

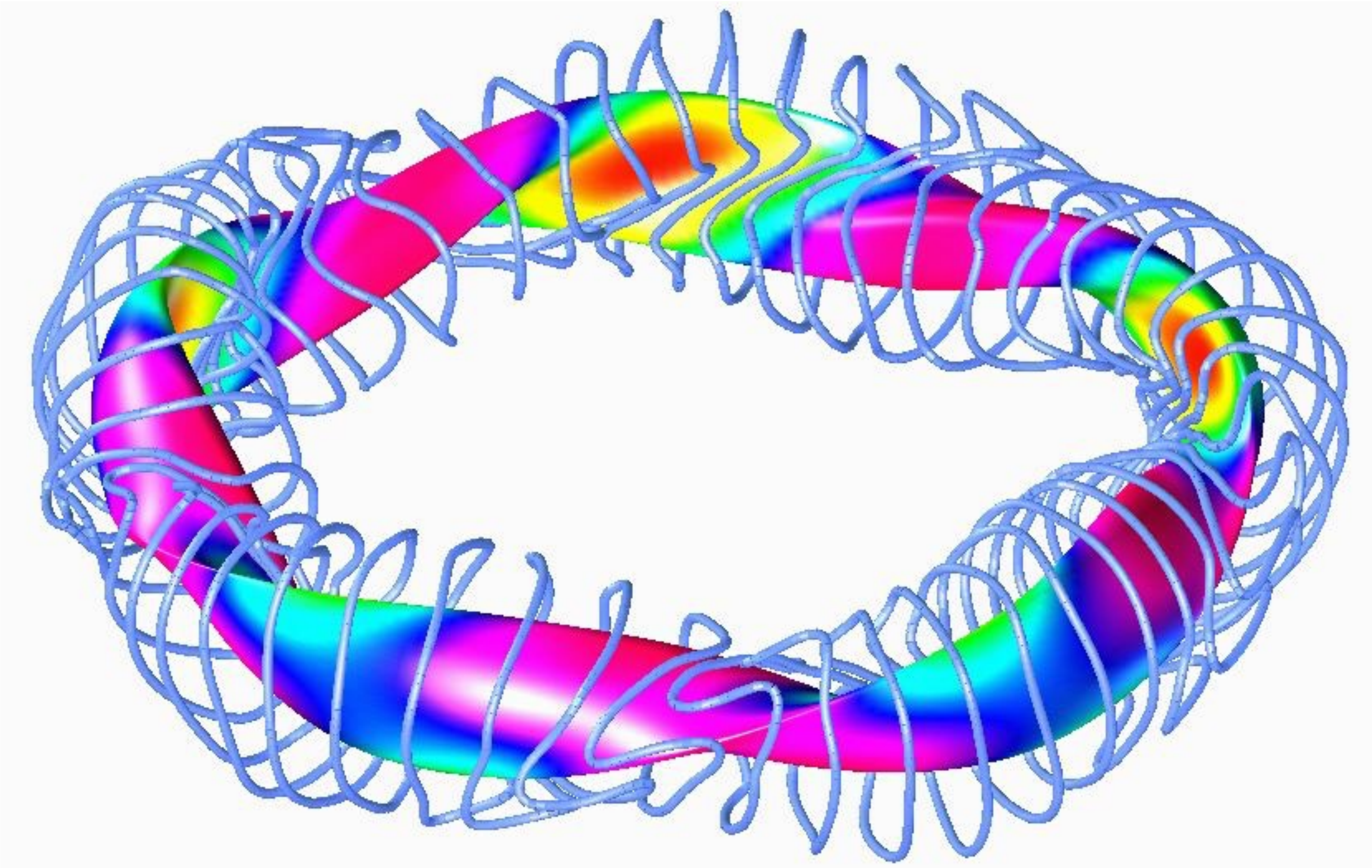
EPFL

Tokamak vs. Stellarator



EPFL

W7-X stellarator in Germany



Swiss
Plasma
Center

Record triple product - $n\tau_E T$

Tokamak (JT60, Japan, 1996)

1.5×10^{21} keV s m^{-3}

Stellarator (W7-X, Germany, 2018)

6.4×10^{19} keV s m^{-3}