

## Astrophysics III : Stellar and galactic dynamics

Exercises**Problem 1 :**

Derive the epicycle frequencies for the following potentials :

a) Point mass :

$$\Phi(r) = -\frac{GM}{r}$$

b) Plummer-Schuster :

$$\Phi(r) = -\frac{GM}{\sqrt{e^2 + r^2}}$$

c) Miyamoto-Nagai :

$$\Phi(R, z) = -\frac{GM}{\sqrt{R^2 + (a + \sqrt{b^2 + z^2})^2}}$$

**Problem 2 :**

Implement the previous computation in `plummer.py`, `miyamoto.py` and `pm.py`. Examine the precision of the predictions above by playing around with them on the numerical integration of the above potentials.

With `orbit.py`, you will obtain 3 figures :

- Figure 1 : the blue lines show the trajectories integrated over time in fixed coordinate planes. The green dashed line on the top-left panel shows the theoretical prediction.
- Figure 2 : the blue line shows the integrated trajectories versus time for the three cartesian coordinates in the rotating frame. The red dashed line shows these for a circular orbit.
- Figure 3 : the blue line shows the trajectories integrated over time in rotating coordinate planes. The red dashed line shows again these trajectories for a circular orbit.

These scripts also print in your terminal the ratio between the frequencies. You can compare these to the theoretical values, that you can find using the `plotfreq.py` script, i.e :

```
./plotfreq.py --potentialfile pm.py
```

Compare the theoretical predictions to the results of the integration from `orbit.py`.

**Problem 3 :**

Show that the following relations hold :

$$\begin{aligned}A(R) &\equiv \frac{1}{2} \left( \frac{v_c}{R} - \frac{dv_c}{dR} \right) = -\frac{1}{2} R \frac{d\Omega}{dR} \\B(R) &\equiv -\frac{1}{2} \left( \frac{v_c}{R} + \frac{dv_c}{dR} \right) = - \left( \Omega + \frac{1}{2} R \frac{d\Omega}{dR} \right) \\ \Omega &= A - B \\ \kappa^2 &\equiv \left( R \frac{d(\Omega^2)}{dR} + 4\Omega^2 \right) = -4B(A - B) = -4B\Omega\end{aligned}$$

**Problem 4 :**

Derive the relation between  $R$  and  $\dot{R}$  in the  $z = 0$  plane (approximation of the third integral), if we assume that the total angular momentum is conserved in an axisymmetric potential.