$$\mathrm{Hom}_K(V,W) = \{\varphi : V \to W \ \ K\text{-lin}\}$$

$$\varphi + \psi : v \longrightarrow \varphi(v) + \psi(v)$$

$\varphi + \psi$ et K-lin

$$\begin{aligned}(\varphi+\psi)(\lambda v + v') &= \varphi(\lambda v + v') + \psi(\lambda v + v') \\ &= \lambda\varphi(v) + \varphi(v') + \lambda\psi(v) + \psi(v') \\ &= \lambda(\varphi(v) + \psi(v)) + \varphi(v') + \psi(v')\end{aligned}$$

$$0_{\mathrm{Hom}_K(V,W)} = \underline{0}_W$$

$k \in K \quad (k.\varphi) : V \longrightarrow W$

$\qquad\qquad\qquad\qquad v \longrightarrow k.\varphi(v)$

$k.\varphi$ et K-linéaire (K-est commutatif)

$$(k.\varphi)(\lambda v + v') = k.\varphi(\lambda v + v')$$
$$= k.(\lambda \varphi(v) + \varphi(v'))$$

$$= k.\left(\lambda\varphi(v)+\varphi(v')\right) = k.\lambda.\varphi(v) + k\varphi(v')$$

$$= \lambda.\left(k.\varphi(v)\right) + k.\varphi(v')$$

$$= \lambda.(k.\varphi)(v) + (k.\varphi)(v')$$

$$K \times \mathrm{Hom}_K(V, W) \longrightarrow \mathrm{Hom}(V, W)$$

$$(k, \varphi) \longmapsto k.\varphi.$$

Si $W = K = K.1_K$

$x = x.1_K$

$\mathrm{Hom}_K(V, K) = V^* =$ dual de V

$$A/I = \{ a \pmod{I} = a + I \}$$

$$0_{A/I} = 0 \pmod{I} = 0_A + I = I$$

$$I = A \quad A/A = \{ \underset{A}{\underset{\shortparallel}{a+A}} \quad a \in A \} = \{A\}$$

$$I = \{0_A\} \quad A/\{0\} = \{ a + \{0\} \quad a \in A \} \simeq A$$

$$= \{ \{a\} \quad a \in A \}$$

$a+b=c$

$a+b(I) = a+b+I$

$\mathbb{Z}/q\mathbb{Z}$

$a+A \subset A$

$A \subset a+A$

$a' = a+a'-a$

$a' \in a+A$

$A = A \qquad A/A = \{a+A,\ a \in A\} = \{A\}$

A

$I = A/I$

$$\mathbb{Z}/2\mathbb{Z} = \left\{ \begin{array}{c} a+2\mathbb{Z} \\ a\in\mathbb{Z} \end{array} \right\} = \{ 2\mathbb{Z}, 1+2\mathbb{Z} \}$$

$a+2\mathbb{Z}$ = entiers impairs si a st imp

$a+2\mathbb{Z}$ = entier pairs si a et pair

$$\mathbb{Z}/3\mathbb{Z} = \{ 3\mathbb{Z}, 1+3\mathbb{Z}, 2+3\mathbb{Z} \}$$

$$|A| \geq |A/I|$$

π_I : $\bullet(\mathrm{mod}\, I)$: $a \in A \longrightarrow a(\mathrm{mod}\, I) = a + I \in A/I$

$\bullet(\mathrm{mod}\, I)$ est surjective.

un elt de A/I est de la forme

$a + I \quad a \in A$

et $a(\mathrm{mod}\, I) = a + I \mid \pi_I^{(-1)}(a \,\mathrm{mod}\, I) = \{a + I\}$

$= \{a' \text{ tq } a' - a \in I\}$

Frob_p. K car$(K) = p$

- si $n \in \mathbb{Z}$ $n_K = n.1_K = 0_K$ ssi $p \mid n$

$\ker(\mathrm{Can}_K(\mathbb{Z} \to K)) = p\mathbb{Z}$ p premier (ou 0)

- $x \in K \longrightarrow x^p \in K$

$\underbrace{x.x.\cdots.x}_{p \text{ fois}}$ ⚠ (pas $\underbrace{x+\cdots+x}_{p \text{ fois}} = px = 0_K$)

Comme $\mathrm{Car}(K) = p$

- $x \longrightarrow x^p$ est un morphisme d'anneau

$$(x.y)^p = x^p . y^p \quad (K \text{ commutif})$$

$$(x+y)^p = x^p + y^p$$

$$\mathrm{Frob}_p : \begin{array}{l} K \longrightarrow K \\ x \longrightarrow x^p \end{array}$$

$$(x+y)^p = x^p + y^p$$

$$= x^p + C_p^1 x^{p-1} \cdot y + C_p^2 x^{p-2} \cdot y^2 + \dots + C_p^{p-1} x y^{p-1}$$

$$+ y^p$$

$$C_p^1 = \frac{p!}{1!(p-1)!} = p \qquad p \cdot x^{p-1} \cdot y = 0_K$$

Si $k \in [1, p-1]$ $\quad C_p^k \in p\mathbb{Z} \quad C_p^k x^{p-k} y^k = 0$

$\mathbb{Z}/5\mathbb{Z}$ $\quad \mathrm{Car}(\mathbb{Z}/5\mathbb{Z}) = 5$

$0_{\mathbb{Z}/5\mathbb{Z}} = 5\mathbb{Z}$ $\quad n.(1 \pmod 5)$

$= 1 \pmod 5 + \dots + 1 \pmod 5$

$\mathbb{Z}/5\mathbb{Z}$

$n.1_{\mathbb{Z}/5\mathbb{Z}} \underset{\mathbb{Z}/5\mathbb{Z}}{=} n.(1+5\mathbb{Z}) \underset{\mathbb{Z}/5\mathbb{Z}}{=} n+5\mathbb{Z}$

((⚠ en tant que ensemble $n.(1+5\mathbb{Z}) = n+5n\mathbb{Z}$

$\neq$

$n+5\mathbb{Z}$))

$n.1 \bmod 5 = n \pmod 5$

$n \pmod 5 = 0 \pmod 5$ ssi $5 \mid n$

$\mathrm{Car}\left(\mathbb{Z}/p\mathbb{Z}\right) = p.$