Astrophysics III: Stellar and galactic dynamics

Exercises

Problem 1:

Derive the equations of motion of a particle in a potential Φ inside a uniformly rotating reference frame $\vec{\Omega}$, the Hamiltonian of which is:

$$H(q,p) = \frac{1}{2}\vec{p}^2 + \Phi(\vec{q}) - \vec{\Omega}\cdot(\vec{q}\times\vec{p}) \tag{1}$$

Problem 2:

Using surfaces of section, explore the following potentials with the script mapping.py (use the help for more information and look at the beginning of the file for some examples):

a) Plummer-Schuster:

$$\Phi(r) = -\frac{GM}{\sqrt{e^2+r^2}}$$

b) Miyamoto-Nagai:

$$\Phi(R,z) = -\frac{GM}{\sqrt{R^2+(a+\sqrt{b^2+z^2})^2}}$$

b) Harmonic:

$$\Phi(x,y,z) = \frac{1}{2}\omega_x^2x^2 + \frac{1}{2}\omega_y^2y^2 + \frac{1}{2}\omega_z^2z^2$$

The aim of this problem is to understand what happens when stacking potentials on top of each other. You can add a potential by giving it a nonzero total mass as a command line argument. You can start by stacking some potentials and plotting them.

In order to get surface of section, remove the option `--plotpotential`. Depending on the problem, you will get one or two figures.

- Figure 1 displays the phase space x - vx of your orbit
- If your orbit is circular or quasi-circular, Figure 2 will display the trajectory in the x-y plane.

Try various potentials and various initial conditions for your test particle. After a few runs, try to predict the shape of the phase space and orbits once the parameters are chosen, and see if your predictions are close to the plots.

Problem 3:

Using surfaces of sections, explore numerically the phase space of the logarithmic potential:

$$\Phi(x, y) = \frac{1}{2} V_0^2 \ln \left(R_\mathrm{c}^2 + x^2 + \frac{y^2}{q^2} \right)$$

q here is an arbitrary scalar, not a coordinate!
a) in a non-rotating reference frame,
b) in a uniformly rotating reference frame. In this case, determine analytically the Lagrange positions along the x axis.