EPFL

4th year physics 30.11.2022

Exercises week 11 Autumn semester 2022

Astrophysics III: Stellar and galactic dynamics Exercises

Problem 1:

Derive the Jeans equations in spherical coordinates, starting from the collisionless Boltzmann equation. Assume a spherically symmetric system.

Hints:

• for the zeroth moment, you can express the zeroth moment of the Boltzmann equation

$$\frac{\partial \nu}{\partial t} + \sum_{i} \frac{\partial}{\partial x_{i}} (\nu \bar{v}_{i}) = 0, \quad \nu \equiv \int d\mathbf{v}^{3} f$$

first in vectorial notation, then in spherical coordinates.

- for the first moment, use the collisionless Boltzmann equation in spherical coordinates, and compute the first moment by multiplying with the radial velocity v_r and integrate over all velocity space.
- Finally, find the solution assuming the system is stationary, isotropic, and express the equation using velocity dispersion σ .
- Here are a few properties to keep in mind:

1)
$$f \to 0$$
 when $|v_i| \to \infty$ 2) $m \int f d^3 \mathbf{v} = \rho$ 3) $m \int v_i f d^3 \mathbf{v} = \rho \overline{v_i}$
4) $m \int v_i v_j f d^3 \mathbf{v} = \rho \overline{v_i v_j}$ 5) $\overline{v_i} \overline{v_j} + \sigma_{ij}^2 = \overline{v_i v_j}$

Problem 2:

Using the Jeans equations in spherical coordinates, determine the velocity dispersion analytically (assuming it is isotropic) for a Plummer sphere in equilibrium.

Problem 3:

Apply the results from the previous problems in the case of an N-body Plummer sphere. Verify the equilbrium nature of the system and integrate for several dynamical timescales to check.

First, create a snapshot using plummer.py and then use gtree to evolve your snapshot (use the help option).

Verify that your integral of motions are conserved with plotIntegrals.py.