Nuclear Fusion and Plasma Physics - Exercises

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Problem Set 11 - 5 December 2022

Exercise 1 - A steady state Tokamak with copper coils

In this exercise we will attempt to design a Tokamak operating at steady-state using coils made out of copper (Cu). Imagine that you want to build a Tokamak with a major radius $R = 5$ m and a minor radius $a = 2$ m. The field at the center of the tokamak is 6 T. Assume that the 6 T field is produced with 20 coils.

a) What is the current in each coil?

b) Assume a current density in each coil of 5×10^7 A/m^2 (which is high!). Compute the coil cross section. If you have Cu at room temperature with resistivity $\rho = 1.68 \times 10^{-8}\ \Omega\,\text{m}$, what is the power loss?

c) Using the data on the curve of ρ versus T shown in Fig. 1, what is the power loss if you cool the coil down to 80 K (the temperature of (cheap) liquid N_2)?

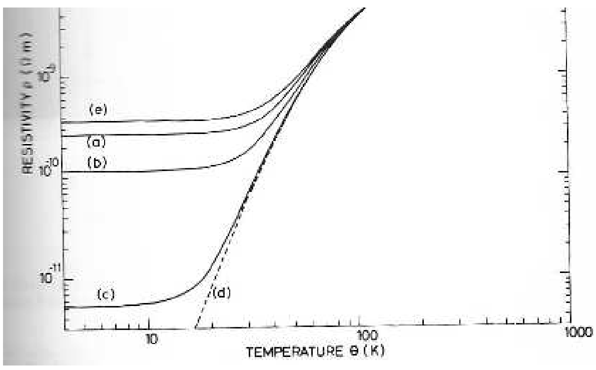

Figure 1: Variation of the resistivity of Copper as a function of the temperature. The different curves are for different types of material treatment.

Exercise 2 - Design of a SC solenoid

This exercise is based on Section 3.1 of the book "Superconducting magnets" by M. Wilson.

In this exercise we take an engineering approach to the design of a superconducting magnet. Consider the solenoid of length $2l$ illustrated in Fig. 2.

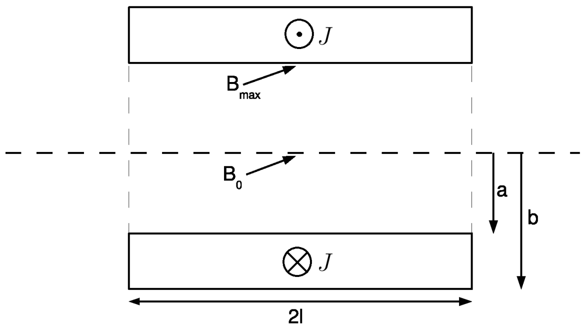

Figure 2: Parameter definitions of the solenoid. Note the location of the maximum field in the conductor.

The B field in the center is $B = aJF(\alpha, \beta)$ where J is the average overall current density, $\alpha = b/a$, $\beta = l/a$ and F is given by

$$F(\alpha, \beta) = \mu_0 \beta \ln \left\{ \frac{\alpha + \sqrt{(\alpha^2 + \beta^2)}}{1 + \sqrt{(1 + \beta^2)}} \right\}$$

Lines of constant $F(\alpha, \beta)$ are given in Fig. 3. The figure also has a curve showing the parameter values corresponding to a minimum volume design.

Assume that we wish to have a field of 6 T in a solenoid with bore diameter $2a = 150$ mm.

a) The current limits as a function of the field B of the superconductor are given by the upper line in Fig. 5. Calculate the current density by considering the limit of J as a function of B.

b) Calculate the coil parameters which give the minimum volume using Fig. 3.

c) From Fig. 4, find the maximum field in the solenoid, B_{w}.

d) Given the maximum field calculated in (c), can you run your magnet at the j_c found in (b)? What field will actually be achieved?

e) If you still want to have $B_0 = 6$ T, what should you do?

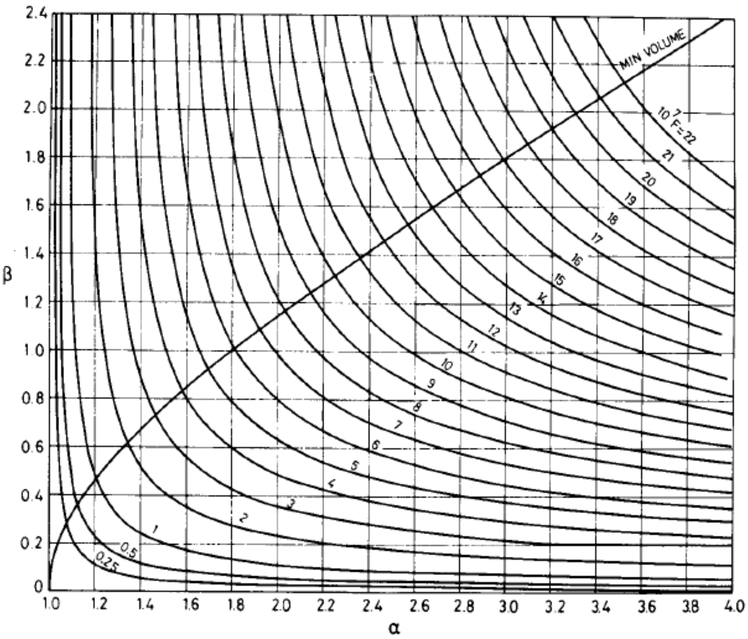

Figure 3: Function F, relating the central field in a simple solenoid to its radius, current density and shape factors α and β.

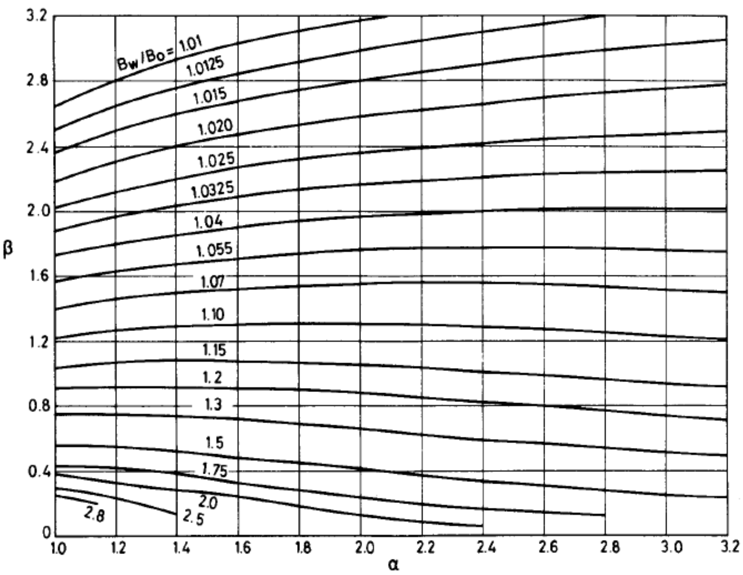

Figure 4: Ratio of maximum to central field B_{w}/B_0 in a simple solenoid as a function of the shape factors α and β.

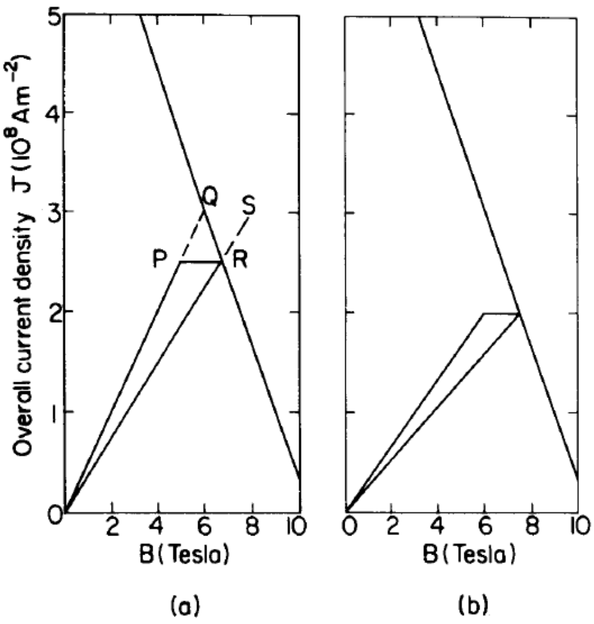

Figure 5: Load lines and current density limit for the solenoid. The different load lines illustrate the effect of the maximum field which sets the limit of the current density.