Introduction to Differentiable Manifolds	
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Exercise Series - Homework 3	**2022–12–6**

Exercise .1. (**Week 8**) Compute the flows of the following vector fields.

(a) On the plane $\mathbb{R}^2$, the "angular" vector field $X = x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}$.

(b) A constant vector field X on the torus $\mathbb{T}^n$. (What is a constant vector field on the torus?)

Exercise .2 (Week 10). Consider the following 1-form on $M = \mathbb{R}^3$:

$$\omega = \frac{-4z\,\mathrm{d}x}{(x^2+1)^2} + \frac{2y\,\mathrm{d}y}{y^2+1} + \frac{2x\,\mathrm{d}z}{x^2+1}$$

(a) Set up and compute the line integral of ω along the line going from $(0,0,0)$ to $(1,1,1)$

(b) Consider the smooth map $\Psi : W \to \mathbb{R}^3$ given by $(r,\varphi,\theta) \in W := \mathbb{R}^+ \times (0,2\pi) \times (0,\pi)$:

$$\Psi(r,\varphi,\theta) = (r\cos\varphi\sin\theta, r\sin\varphi\sin\theta, r\cos\theta) \in \mathbb{R}^3.$$

Compute $\Psi^*\omega$.

Exercise .3 (Week 11). For a point $p \in \mathbb{R}^3$ and vectors $v, w \in \mathrm{T}_p\mathbb{R}^3 \equiv \mathbb{R}^3$ we define $\omega|_p(v,w) := \det(p \mid v \mid w)$. Show that ω is a smooth differential 2-form on $\mathbb{R}^3$, and express ω as a linear combination of the elementary alternating 2-forms determined by the standard coordinate chart (x^0, x^1, x^2).