Mini-project 2022
Quantum Information Processing

In the following project, you will verify the results of the Bell Inequality where Eve prepares a 2-qubits state function for Alice and Bob.

Let's consider the CHSH operator:

$$\mathcal{B} = A \otimes B - A' \otimes B + A \otimes B' + A' \otimes B' \tag{1}$$

With the usual angles $\alpha, \alpha', \beta, \beta'$:

$$A = |\alpha\rangle \langle\alpha| - \left|\alpha^{\perp}\right\rangle \left\langle\alpha^{\perp}\right| \tag{2}$$
$$A' = |\alpha'\rangle \langle\alpha'| - \left|\alpha'^{\perp}\right\rangle \left\langle\alpha'^{\perp}\right| \tag{3}$$
$$B = |\beta\rangle \langle\beta| - \left|\beta^{\perp}\right\rangle \left\langle\beta^{\perp}\right| \tag{4}$$
$$B' = |\beta'\rangle \langle\beta'| - \left|\beta'^{\perp}\right\rangle \left\langle\beta'^{\perp}\right| \tag{5}$$

We take $|\alpha\rangle = \cos(\alpha)|0\rangle + \sin(\alpha)|1\rangle$ and $\left|\alpha^{\perp}\right\rangle = \left|\alpha + \frac{\pi}{2}\right\rangle$.

Measurement

Let's consider the state $|\psi\rangle = |0\rangle$, we want to measure it in the basis provided by the observable A.

1. Use the following gate $RY(\theta) = e^{-i\frac{\theta}{2}Y}$ to suggest (and explain!) a circuit to measure the probability $P(|\psi\rangle \to |\alpha\rangle)$.

2. Implement your circuit on a NISQ machine, and run it on their quantum devices (not "simulators") for $\alpha = \frac{-\pi}{4}$ and $\alpha = \frac{\pi}{8}$. Provide the histogram of your results.

3. From the former histograms, compute your experimental observation $\langle\psi| A |\psi\rangle$ and check if it matches the experimental expected value.

State Preparation

1. First we want to prepare the ideal Bell state with $|\psi_0\rangle = |B_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

   (a) Propose a design of a circuit to generate this state.

   (b) Implement your circuit on a NISQ machine and measure the outcome in the computational basis. Run the circuit on a quantum device with 1024 shots and report the percentage each outcome $(00, 01, 10, 11)$.

2. Now, let's assume that Eve tries to keep a qubit for herself and prepares the state $|\psi_1\rangle = \frac{1}{\sqrt{2}}(|000\rangle_{ABE} + |111\rangle_{ABE})$.

(a) Propose a design of a circuit to generate this state

(b) Implement your circuit on a NISQ machine and measure the outcome in the computational basis of (A, B) (only A and B do measurements). Run the circuit on a quantum device with 1024 shots and report the percentage each outcome $(00, 01, 10, 11)$.

3. Compare your results for $|\psi_0\rangle$ and $|\psi_1\rangle$. Do you see any difference? Compute (analytically) the density matrix ρ_{AB} in both cases.

CHSH Operator

Now let's consider the CHSH angles $\alpha = 0, \alpha' = \frac{-\pi}{4}, \beta = \frac{\pi}{8}, \beta' = \frac{-\pi}{8}$.

1. Start with the Bell state preparation $|\psi_0\rangle$.

   (a) Propose a circuit to measure the outcome of the observable $A \otimes B$.

   (b) Run the circuit on a quantum device on the IBM Composer. Report the outcomes and provide the experimental mean value of $\langle\psi_0| A \otimes B |\psi_0\rangle$.

   (c) Repeat the same steps for each operator $A \otimes B'$, $A' \otimes B$, $A' \otimes B'$, but only report your experimental mean value $\langle\psi_0| A' \otimes B |\psi_0\rangle$, $\langle\psi_0| A \otimes B' |\psi_0\rangle$, $\langle\psi_0| A' \otimes B' |\psi_0\rangle$.

   (d) Now, calculate your experimental value $\langle\psi_0| \mathcal{B} |\psi_0\rangle$ and comment on your result vs the expected theoretical value.

2. Now let's consider the initial state $|\psi_1\rangle$ and re-answer each question (a),(b),(c),(d) from the previous question. Note that what is computed this time is $\mathrm{Tr}(\rho_{AB}\mathcal{B})$.

3. *(Bonus)* Do the same experiments with a Qiskit notebook where the angles are sampled randomly. Check your results of your measurements $\langle\psi_0| \mathcal{B} |\psi_0\rangle$ and $\mathrm{Tr}(\rho_{AB}\mathcal{B})$ and provide a link or a file to your notebook. (If you provide a link, make sure you have given reading permissions to the file)