Nuclear Fusion and Plasma Physics - Exercises

Prof. A. Fasoli - Swiss Plasma Center / EPFL

Solutions to Problem set 12 - 12 December 2022

Exercise 1 - Control of plasma burn

(a) *Does a burning ITER plasma risk undergoing a thermal instability?*

Thermal stability can be determined from the energy balance equation:

$$\frac{d(3\,nT)}{dt} = \frac{P_{in}}{V} + \frac{1}{4}n^2 \langle\sigma v\rangle E_\alpha - \frac{3\,nT}{\tau_E(n,T)}. \tag{1}$$

As ignition is approached, we can neglect P_{in} (i.e. set $P_{in} = 0$) because the α particles become the dominant source of heating. Keeping in mind that we have assumed $n = const$, we then have

$$3n\frac{dT}{dt} \approx \frac{1}{4}n^2 \langle\sigma v\rangle E_\alpha - \frac{3nT}{\tau_E(T)} \tag{2}$$

or

$$\frac{dT}{dt} = \frac{1}{12}n \langle\sigma v\rangle E_\alpha - \frac{T}{\tau_E(T)} \tag{3}$$

In equilibrium ($\frac{dT}{dt} = 0$), the solution is

$$\frac{nE_\alpha\tau_E}{12} = \frac{T_0}{\langle\sigma v\rangle} \tag{4}$$

where T_0 is the equilibrium temperature.

Consider now a small variation of temperature ΔT such that $T = T_0 + \Delta T$:

$$\begin{aligned}
\frac{d\Delta T}{dt} &= \frac{1}{12}nE_\alpha\frac{d\langle\sigma v\rangle}{dT}\Delta T - \frac{\Delta T}{\tau_E} - T\frac{d\tau_E^{-1}}{dT}\Delta T \qquad (5)\\
&= \frac{1}{12}nE_\alpha\frac{d\langle\sigma v\rangle}{dT}\Delta T - \frac{\Delta T}{\tau_E} + \frac{T}{\tau_E^2}\frac{d\tau_E}{dT}\Delta T \qquad (6)\\
&= \frac{1}{\tau_E}\left[\frac{1}{12}nE_\alpha\tau_E\frac{d\langle\sigma v\rangle}{dT} - 1 + \frac{T}{\tau_E}\frac{d\tau_E}{dT}\right]\Delta T. \qquad (7)
\end{aligned}$$

If this variation occurs near equilibrium, we can substitute the equilibrium solution to obtain the final expression:

$$\frac{d\Delta T}{dt} = \frac{1}{\tau_E}\left[\frac{T}{\langle\sigma v\rangle}\frac{d\langle\sigma v\rangle}{dT} - 1 + \frac{T}{\tau_E}\frac{d\tau_E}{dT}\right]\Delta T \tag{8}$$

The stability of the solution is thus determined by the sign of $\left[\frac{T}{\langle\sigma v\rangle}\frac{d\langle\sigma v\rangle}{dT} - 1 + \frac{T}{\tau_E}\frac{d\tau_E}{dT}\right]$ (negative = stable, positive = unstable). The stability criterion hence becomes

$$\frac{T}{\tau_E}\frac{d\tau_E}{dT} < 1 - \frac{T}{\langle\sigma v\rangle}\frac{d\langle\sigma v\rangle}{dT} \tag{9}$$

We now use the scaling law for τ_E, considering only factors of T,

$$\tau_E \sim \frac{1}{T}(T^{0.5})^{-0.7}(T)^{-0.9}(T^{-2})^{-0.01} = T^{-1-0.35-0.9+0.02} = T^{-2.23}, \tag{10}$$

to estimate

$$\frac{T}{\tau_E}\frac{d\tau_E}{dT} \approx \frac{T}{\alpha T^{-2.3}}\alpha(-2.3\,T^{-3.3}) = -2.3. \tag{11}$$

Similarly,

$$\frac{T}{\langle\sigma v\rangle}\frac{d\langle\sigma v\rangle}{dT} \approx \frac{T}{AT^2}\frac{d(AT^2)}{dT} = 2. \tag{12}$$

This implies that the criterion

$$-2.3 < 1 - 2 = -1 \tag{13}$$

is satisfied and **the burn should be stable**.

(b) *What measure can you think of to control the burn and prevent instability?*

- Reduce heating power (if any).
- Gas and/or impurity injection.
- Reduce magnetic field/turn off the coils.

Exercise 2 - On the use of Tritium in ITER and fusion reactors

(a) - Tritium burn up fraction for a reactor:

$$f_B = \frac{1}{1 + \frac{1}{\tau_T\, n_T <\sigma v>_{DT}}} = \frac{1}{1 + \frac{1}{2[\text{s}]\cdot 2{\times}10^{20}[\text{m}^{-3}]\cdot 2.475{\times}10^{-22}[\text{m}^3/\text{s}]}} \approx 9\% \tag{14}$$

- Tritium burn up fraction for ITER:

$$f_B = \frac{1}{1 + \frac{1}{\tau_T\, n_T <\sigma v>_{DT}}} = \frac{1}{1 + \frac{1}{1[\text{s}]\cdot 1{\times}10^{20}[\text{m}^{-3}]\cdot 1.1{\times}10^{-22}[\text{m}^3/\text{s}]}} \approx 1.1\% \tag{15}$$

(b) - Tritium mass burn rate for a reactor (assuming that the efficiency in the conversion from thermal to electrical power is about 40%):

$$\frac{dM}{dt} \approx 56 \times \text{fusion power}(\text{GW}_{\text{thermal}}) \text{ [kg/year]} = 56 \times 1/0.4 = 140 \text{ [kg/year]} \quad (16)$$

- Tritium mass burn rate for ITER (assuming that ITER will produce only 10% of the power of a reactor):

$$\frac{dM}{dt} \approx 56 \times \text{fusion power}(\text{GW}_{\text{thermal}}) \text{ [kg/year]} = 56 \times 0.1 \times 1/0.4 = 14 \text{ [kg/year]} \quad (17)$$

(c) - Tritium inventory mass for a reactor:

$$M_0 \approx \frac{t_p \frac{dM}{dt}}{\eta_f f_B} = \frac{1 \times 140}{365 \times 0.5 \times 0.09} \approx 8.5 \text{ kg} \quad (18)$$

- Tritium inventory mass for ITER:

$$M_0 \approx \frac{t_p \frac{dM}{dt}}{\eta_f f_B} = \frac{1 \times 14}{365 \times 0.2 \times 0.011} \approx 17.4 \text{ kg} \quad (19)$$