Exercise 1 *W State*

Let's consider the $|W\rangle$ state:

$$|W\rangle = \frac{1}{\sqrt{3}} \left(|001\rangle + |010\rangle + |100\rangle\right) \tag{1}$$

Notice that it can be "factorized" as follow:

$$|W\rangle = \frac{1}{\sqrt{3}} |0\rangle \otimes \left(|01\rangle + |10\rangle\right) + \frac{1}{\sqrt{3}} |100\rangle \tag{2}$$

Now let $U = (X \otimes I) \cdot CNOT$, this operator gives the Bell-state: $U|00\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$. So we can rewrite with CU the control-U operator (with control on the first qubit and target on the last two qubits):

$$\begin{aligned} |W\rangle &= \sqrt{\frac{2}{3}} |0\rangle \otimes U|00\rangle + \frac{1}{\sqrt{3}} |100\rangle & (3) \\ &= \sqrt{\frac{2}{3}} (X \otimes I \otimes I)\left(|1\rangle \otimes U|00\rangle\right) + \frac{1}{\sqrt{3}} (X \otimes I \otimes I)|000\rangle & (4) \\ &= (X \otimes I \otimes I) CU \left(\sqrt{\frac{2}{3}} |100\rangle + \sqrt{\frac{1}{3}} |000\rangle\right) & (5) \end{aligned}$$

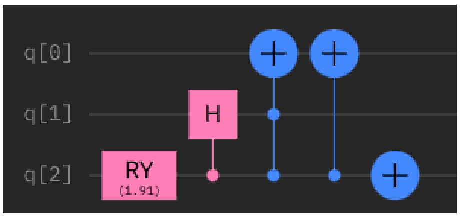
Finally, with the RY_θ operator with $\theta = 2\arccos\frac{1}{\sqrt{3}}$ we have:

$$RY_\theta |0\rangle = \sqrt{\frac{1}{3}} |0\rangle + \sqrt{\frac{2}{3}} |1\rangle \tag{6}$$

Hence:

$$|W\rangle = (X \otimes I \otimes I) CU \left(RY_\theta \otimes I \otimes I\right) |000\rangle \tag{7}$$

Here is a corresponding circuit on IBMQ:



Note that the solution is not unique. For instance this would work as well: