

APA quiz 3

1. Let $A, B \in \mathcal{F}$ be two independent events.

Which of the following statements are correct?

a) $P(A \cup B) = 1 - P(A^c)P(B^c)$

b) $P(A \cap B) \leq \max\{P(A)^2, P(B)^2\}$

c) $P(A \cup B) \leq P(A) + P(B)$

d) $P(A \cup B) = P(A) + P(B)$

e) $P(A \cap B) \leq \frac{1}{4}$

f) $P(A \cup B) \geq P(A) \cdot P(B^c) + P(A^c) \cdot P(B)$

2. Let X, Y, Z be 3 random variables defined on $(\Omega, \mathcal{F}, \mathbb{P})$.
Which of the following statements are correct?

a) if $X - Y \perp Y$, then $X \perp 2Y$

b) if $X - Y \perp Z$, then $X^2 - Y^2 \perp Z^2$

c) if $X - Y \perp Y$, then $(X - Y)^2 \perp Y^2$

d) it is impossible that $X - Y \perp Y$

e) if $X \perp Y$, then $X^2 \perp Y^3$

f) if $X \perp Y$ and $Y \perp Z$, then $X \perp Z$

3. Assume X_1, X_2, X_3 are independent and identically distributed with $P(\{X_i = +1\}) = P(\{X_i = 0\}) = \frac{1}{2} \quad \forall 1 \leq i \leq 3$

Let $Y = X_1 + X_2 + X_3$. Which of the following statements are correct?

a) $P(\{Y = +3\}) = \frac{1}{8}$

b) $P(\{Y = +2\}) = \frac{1}{4}$

c) $P(\{Y = +1\}) = \frac{1}{2}$

d) $P(\{Y \leq 0\}) = 0$

e) $X_1 + X_2$ and $X_2 + X_3$ are independent

f) The cdf of Y has 3 jumps.

4. Let X_1 be a random variable with $P(\{X_1 = +1\}) = P(\{X_1 = -1\}) = \frac{1}{2}$ and X_2 be a continuous $N(0, \sigma^2)$ random variable ($\sigma > 0$). Assume also $X_1 \perp\!\!\!\perp X_2$ and let $Y = X_1 + X_2$. Which of the following statements are correct?

- a) Y is a continuous random variable
- b) Y is a Gaussian random variable
- c) Y and $-Y$ are identically distributed
- d) Fix $t > 0$ large: $P(\{Y \geq t\})$ increases when σ increases
- e) $P(\{-1 \leq Y \leq +1\})$ increases when σ increases.
- f) Fix $\varepsilon > 0$ small: $P(\{-\varepsilon \leq Y \leq +\varepsilon\})$ decreases when σ decreases.