Homework 3 (Graded): due Monday, March 20 2023
CS-526 Learning Theory

Problems from *Understanding Machine Learning: From Theory to Algorithms* by Shai Shalev-Shwartz and Shai Ben-David:

1. Exercise 5 of Chapter 3.
2. Exercise 2 of Chapter 5. Note that here we expect just a qualitative answer, without any computations.
3. Exercise 3 of Chapter 5.
4. Exercise 2 of Chapter 6.
5. Exercise 8 of Chapter 6.

Problem 6. VC dimension of circles.
Consider the plane $\mathbb{R}^2$, equipped with the usual Euclidean norm $\|\cdot\|$. We denote as $B(\mathbf{y}, r) = \{\mathbf{x} \in \mathbb{R}^2 : \|\mathbf{x} - \mathbf{y}\| \leq r\}$ the closed disk of radius r centered in point $\mathbf{y} \in \mathbb{R}^2$. Let $\mathcal{H} = \{\mathbb{1}_{B(\mathbf{y},r)} : r \geq 0 \text{ and } \mathbf{y} \in \mathbb{R}^2\}$ be the hypothesis class that contains the indicator functions of all possible closed disks.

Let d be the VC dimension of $\mathcal{H}$. Try to figure out first what the value of d might be and then prove the correctness of your guess. For the latter, you need to do the following steps:

1. Show that for any $n \leq d$ there exist n distinct points in the plane shattered by $\mathcal{H}$.
   Hint: You can propose an instance of d points and for each labeling draw the valid circle.
2. Show that no set of n distinct points with $n > d$ can be shattered by $\mathcal{H}$.
   Hint: You should consider two cases: 1) one of the points is in the convex hull of the other points, and 2) none of the points is in the convex hull of the other points. A formal proof might be difficult. It will suffice if you give us a *convincing* argument.