Master in Financial Engineering at EPFL

Optimization Methods

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Outline of the course

- *Topic 1: Static optimization.* Optimization programs with and without constraints. Examples of optimization problems in economic theory. Existence of solutions. Unconstrained optima: first-order and second-order conditions. Equality constraints and the Theorem of Lagrange. The Constraint Qualification. The Lagrange multipliers and their interpretation. Using the Theorem of Lagrange. Two examples from economics: Consumer and Producer Theory. One example from finance: optimal portfolios and mean-variance analysis. Inequality constraints and the Theorem of Kühn-Tucker. The Constraint Qualification. The Kühn-Tucker multipliers. Using the Theorem of Kühn-Tucker. An illustration from Consumer and Producer Theory. The general case: mixed constraints. Algorithms for univariate and multivariate nonlinear optimization (binary search, golden section search, Newton's method, steepest descent).

- *Topic 2: Application to the choice under uncertainty.* The investor's risk attitudes. Mean-variance criterium and expected utility criterium. Risk premium and certainty equivalent. Arrow–Pratt coefficient of risk aversion and corresponding utility functions. The investor's optimal portfolio with one risky asset. Optimal portfolio and wealth effect. Optimal portfolio with multiple risky assets. Portfolio expected return and variance. Portfolio diversification. The impact of different risk attitudes in portfolio choice.

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- *Topic 3: Application to the Estimation of the term structure of interest rates.* The term structure of interest rates. Forward rates and the forward term structure of interest rates. Interpolation techniques and estimation methods: Ordinary Least Squares (OLS) vs Nonlinear Least Squares.

Prerequisites

Basic concepts of linear algebra and probability.

Course timetable

The course timetable is: 25/09, 02/10, 09/10, 16/10, 30/10. Time schedule of the first four lessons is 09:15-12:00. The last lesson is from 09:15-11:00. The classroom is EXTRA 126.

Exercise sessions

Thomas Geelen is the teaching assistant of the course and he leads the exercise sessions that take place every Tuesday after the course (e-mail: thomas.geelen@epfl.ch). Time schedule: 15:15-17:00. Classroom EXTRA 126.

Grading

The grading will be based on exercises (30%), and (70%) final exam. Exercises will be posted on Tuesday and will be due on the next Tuesday. Please handle-in the exercises to Thomas at the beginning of the exercise session. Groups of max 6 persons are allowed.

The final exam is closed-books and closed-notes, and it will take place on November 13, 2017 from 09:15 to 12:15 (classroom EXTRA 126).

Learning outcome of the course

At the end of this course the student should be able to:

1. Describe and solve optimization programs with and without equality or inequality constraints. Describe algorithms adopted to solve such a univariate and multivariate optimization problems. Present applications to economics and finance.

2. Discuss the expected utility criterium and the mean-variance criterium. Show how the different risk attitudes of the investor can be characterized via the shape of the utility function. Discuss the investor's optimal portfolio, the impact of risk aversion and the wealth effect. Compute the expected return and variance of a portfolio in a static market model with a finite number of assets. Apply basic linear algebra to study the properties of the covariance matrix of the assets in the market. Show how to reduce the portfolio variance by the means of portfolio diversification.

3. Present the problem of interpolation and estimation of the term structure of interest rates.

References and material for the course

Slides. There is no required book for the course. Anyway, useful references are:

– Dixit,, A. K., "Optimization in economic theory", Oxford University Press, second edition.

– LeRoy, S. F., and J. Werner, "Principles of Financial Economics", Cambridge University Press.

– Luenberger, D G., "Linear and Nonlinear Programming", Kluwer Academic Publisher, second edition.

– Claus Munk, "Financial Asset Pricing Theory", Oxford University Press.

– W. Rudin, "Principles of Mathematical Analysis", McGraw-Hill Education, third edition.

– C. P. Simon and L.E. Blume, "Mathematics for Economists", W. W. Norton and Company.

– R. K. Sundaram, "A First Course in Optimization Theory", Cambridge University Press.