MCAA lecture 2: quiz solutions

1) Only statement d) is correct, as long as:

a') i is transient iff $\mathbb{P}(X_n \neq i \ \forall n \geq 1 \mid X_0 = i) > 0$

b') i is transient iff $\mathbb{P}(\exists n \geq 1 \text{ s.t. } X_n = i \mid X_0 = i) < 1$

c') i is recurrent iff $\mathbb{P}(\exists n \geq 1 \text{ s.t. } X_n = i \mid X_0 = i) = 1$

e') i is recurrent iff $\mathbb{P}(T_i = +\infty \mid X_0 = i) = 0$

(iff $\mathbb{P}(T_i < +\infty \mid X_0 = i) = 1$)

2) Only b) & d) can influence the rec./trans. of some states;
a) & c) can't (as long as the graph remains a valid transition graph, of course).

3) a) $\mathbb{P}(X \geq n) = \sum_{m \geq n} 2^{-m} = 2^{-n+1}$ for $n \geq 1$

and $\mathbb{E}(X) = \sum_{n \geq 1} n \cdot \mathbb{P}(X = n) = \sum_{n \geq 1} \mathbb{P}(X \geq n) = 2 < +\infty$

(useful trick!)

b) $\mathbb{P}(X \geq n) = \sum_{m \geq n} \frac{C}{m^2} = \Theta\left(\frac{1}{n}\right)$

and $\mathbb{E}(X) = \sum_{n \geq 1} \underbrace{n \cdot \mathbb{P}(X = n)}_{\Theta\left(\frac{1}{n}\right)} = +\infty$ (harmonic series)

c) This question is a trap: it is impossible that $\mathbb{P}(X = n) = \frac{C}{n}$ for some $0 < C < +\infty$, as this would imply $\sum_{n \geq 1} \mathbb{P}(X = n) = C \cdot \sum_{n \geq 1} \frac{1}{n} = +\infty$

(again, harmonic series)