(1) Recall importance sampling

We want to compute $\mathbb{E}_\pi(f(x))$. We take φ an easy to sample from distr and consider the estimator

$$\frac{1}{M}\sum_{k=1}^{M} f(x_k)\,w(x_k) \qquad \text{where } x_k \overset{iid}{\sim} \varphi.$$

$$\text{and } w(x) = \frac{\pi(x)}{\varphi(x)}.$$

Recall from the class that the variance of the estimator is

$$\frac{1}{M}\,\mathrm{Var}_\varphi\big(f(x)\,w(x)\big).$$

Questions

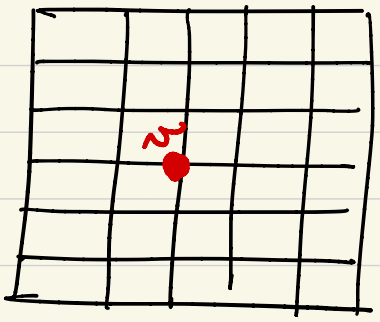
a) For what choice of φ is the variance minimized ?

b) In practice however can we really use this choice ?

(2) MCMC for the Ising model.

- Consider the $\underbrace{N \times N \times N \cdots \times N}_{d \text{ times}}$ grid $\subset \mathbb{Z}^d$.

and the Ising distribution



$d = 2$

$$\pi(\underline{s}) = \frac{e^{-\beta \sum_{k,\ell \in \epsilon} J_{k\ell} s_k s_\ell}}{Z} \quad ; \quad \underline{s} = (s_1, s_2, \ldots, s_{N^d}).$$

$$Z = \sum_{\underline{s} \in \{-1,+1\}^{N^d}} e^{-\beta \sum_{k,\ell \in \epsilon} J_{k\ell} s_k s_\ell}.$$

- At each MCMC step we propose the move (base chain)

$\underline{s} \to \underline{s}'$ where we select at random a vertex v and flip its spin $s'_v = -s_v$ while other spins remain the same.

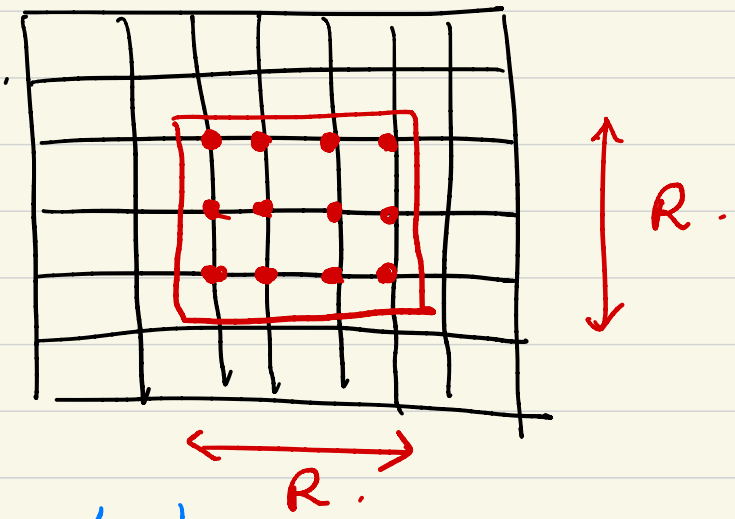
Question: The computational cost of computing the acceptance probability at each step

$$a_{\underline{s}\underline{s}'} = \min\left(1; \frac{\pi(\underline{s}')}{\pi(\underline{s})}\right) \text{ is } \begin{cases} \to \text{of order } \exp(N^d) \ ? \\ \to \text{of order } N^{2d} \ ? \\ \to \text{of order } d \ ? \end{cases}$$

(3) Proposal moves flipping clusters of spins. (Ising again)

- Suppose now the proposed move is to flip a whole cluster of spins in a region of size $R \times R$ in the grid

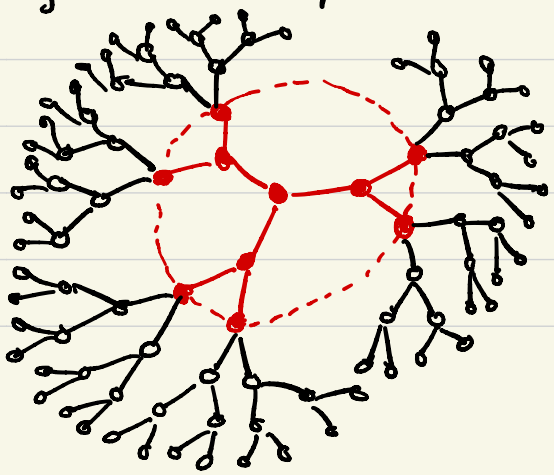
($d = 2$ here)



Question: the cost of computing the acceptance prob at each step

is → of order R^2 ?
→ of order R ?

Food for thought: consider the situation on a tree of degree Δ (say $\Delta = 3$) : • The proposed move flips in balls of radius $R = \alpha \log N$; $\alpha \ll 1$.



• There N vertices in total.

Cost of computing acceptance prob ?
→ is it order $R \sim \log N$?
→ is it a power of N ?