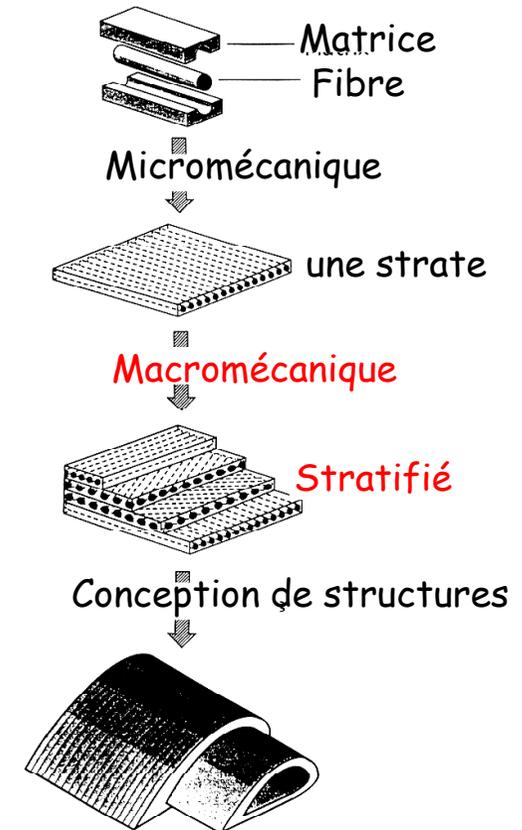


Macromécanique

- Micromécanique
 - Lois des mélanges
 - Halpin-Tsai
 - Renforts discontinus
 - Macromécanique
 - Anisotropie
 - Comportement d'une strate
 - Composites orthotropes sous contraintes planes
 - Comportement des stratifiés, symétries
 - Résistance et critères de rupture
 - Endommagement et mécanique de la rupture
- Technologie des composites (Master)**
- Comportement des structures sandwich
 - Composites textiles
 - Fatigue, design industriel, etc...

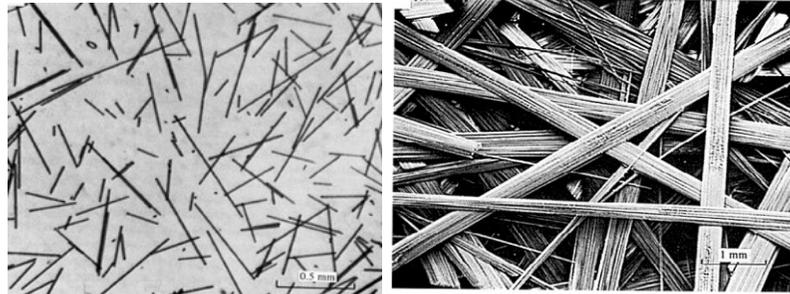
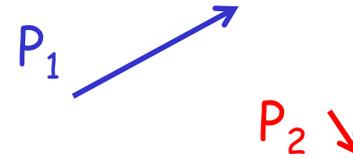


Anisotropie

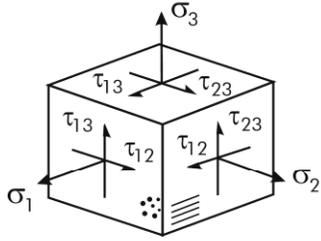
Matériaux homo ou hétérogènes



Matériaux iso ou anisotropes



Elasticité des matériaux anisotropes



Propriétés élastiques
du composite

Matériaux anisotropes

81 cstes

Linéaire et élastique

Loi de Hooke

Les contraintes et les
déformations sont
symétriques

36 cstes

Densité d'énergie de déformation

21 cstes

$$C_{ij} = C_{ji}$$

Symétries du matériau

Monoclinique

13 cstes

Orthotropie

9 cstes

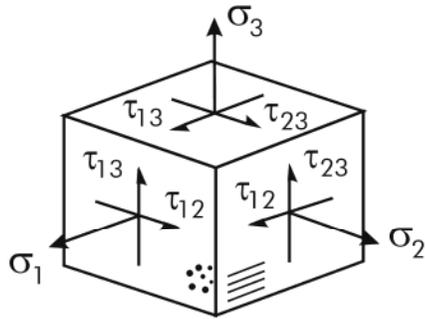
Transversalement isotrope

5 cstes

Isotrope

2 cstes

Anisotrope et élastique



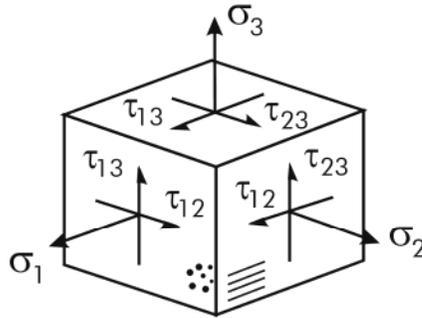
$$\sigma_{ij} = f(\varepsilon_{kl})$$

Hypo: Linéaire et élastique (81)

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{12} \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \end{bmatrix} = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & \dots & \dots & \dots & \dots & \dots & \dots \\ C_{2211} & C_{2222} & C_{2233} & & & & & & \\ \dots & & \dots & & & & & & \\ \dots & & & \dots & & & & & \\ \dots & \dots & & & \dots & & & & \\ \dots & \dots & & & & \dots & & & \\ \dots & \dots & & & & & \dots & & \\ \dots & \dots & & & & & & \dots & \\ \dots & \dots & & & & & & & \dots \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \end{bmatrix}$$

Tenseurs symétriques

Les contraintes et les déformations sont symétriques (36)



$$\tau_{12} = \tau_{21} \quad \text{etc}$$

Notation contractée

$$\begin{aligned} \sigma_{11} &\gamma \sigma_1 && \text{etc} \\ C_{ijkl} &\gamma C_{ij} \end{aligned}$$

Densité d'énergie de déformation (21)

$$W = \frac{1}{2} \sigma_i \varepsilon_i$$

$$\frac{\partial^2 W}{\partial \varepsilon_i \partial \varepsilon_j} = C_{ij}$$

$$\sigma_i = C_{ij} \varepsilon_j$$

" " C_{ij} est symétrique

$$\frac{\partial^2 W}{\partial \varepsilon_j \partial \varepsilon_i} = C_{ji}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix}$$

6 x 6

Symétries des matériaux

Monocliniques (13)

Orthotropes (9)

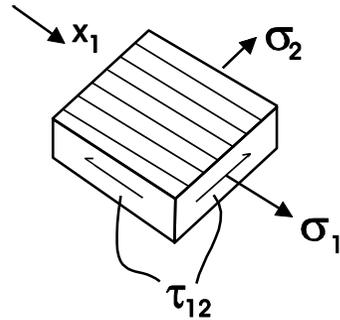
$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

Transversalement isotropes (5) $2 \leftrightarrow 3$ $C_{22} = C_{33}$ etc

Isotropes(2)

Constantes de l'ingénieur

$$\{\sigma\} = [C]\{\varepsilon\}$$



$$\{\varepsilon\} = [S]\{\sigma\}$$

**Test A: que σ_1
autres contraintes = 0**

$$\varepsilon_1 = \frac{\sigma_1}{E_1} \quad \nu_{ij} = -\frac{\varepsilon_j}{\varepsilon_i}$$

$$\varepsilon_2 = -\nu_{12}\varepsilon_1 = -\nu_{12} \frac{\sigma_1}{E_1}$$

$$\varepsilon_3 = -\nu_{13}\varepsilon_1 = -\nu_{13} \frac{\sigma_1}{E_1}$$

$$\gamma_{12} = \gamma_{23} = \gamma_{13} = 0$$

**Test B: que σ_2
autres contraintes = 0**

$$\varepsilon_2 = \frac{\sigma_2}{E_2}$$

$$\varepsilon_1 = -\nu_{21}\varepsilon_2 = -\nu_{21} \frac{\sigma_2}{E_2}$$

$$\varepsilon_3 = \dots$$

$$\gamma_{12} = \gamma_{23} = \gamma_{13} = 0$$

**Test C: que τ_{12}
autres contraintes = 0**

$$\gamma_{12} = \frac{\tau_{12}}{G_{12}}$$

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \gamma_{23} = \gamma_{13} = 0$$

Constantes de l'ingénieur

$$\varepsilon_1 = \frac{\sigma_1}{E_1}$$

$$\varepsilon_1 = -\nu_{21}\varepsilon_2 = -\nu_{21}\frac{\sigma_2}{E_2}$$

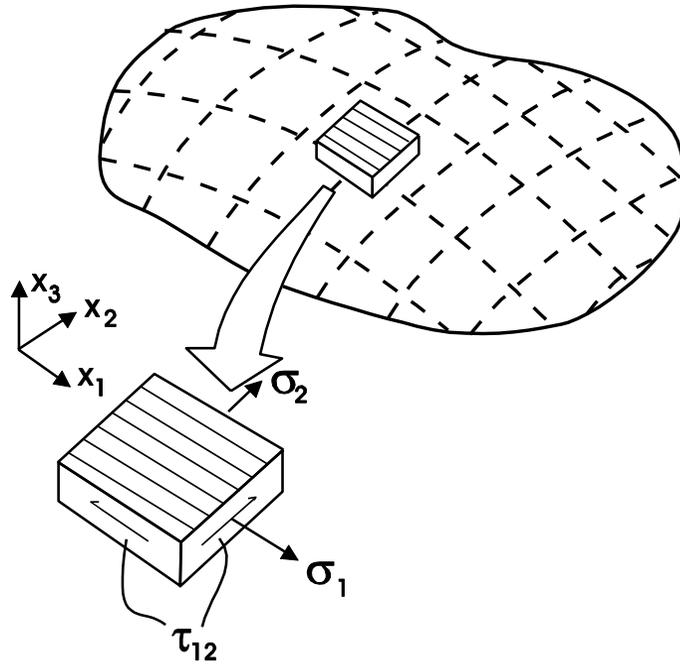
$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix}$$

$$\{\varepsilon\} = [S]\{\sigma\}$$

$$\gamma_{12} = \frac{\tau_{12}}{G_{12}}$$

$$S_{12} = S_{21} \text{ donc } -\frac{\nu_{21}}{E_2} = -\frac{\nu_{12}}{E_1} \quad \text{pour un matériau orthotrope}$$

Matériaux orthotropes en contraintes planes (3=0)



$$[S_{ij}] = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

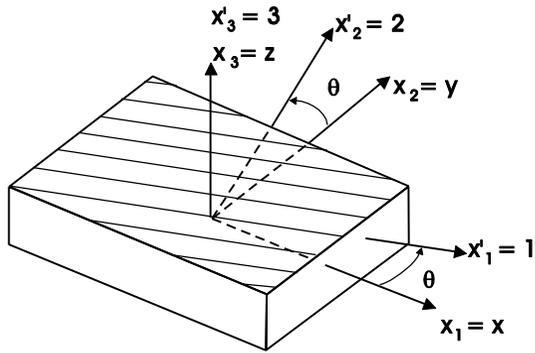
$$Q_{11} = \frac{E_1}{(1-\nu_{12}\nu_{21})}$$

$$Q_{22} = \frac{E_2}{(1-\nu_{12}\nu_{21})}$$

$$Q_{12} = \frac{\nu_{12}E_2}{(1-\nu_{12}\nu_{21})} = \frac{\nu_{21}E_1}{(1-\nu_{12}\nu_{21})}$$

$$Q_{66} = G_{12}$$

Importance de l'orientation des fibres



$$m = \cos, n = \sin$$

$$[T] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = [T]^{-1} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T]^{-1} [Q] \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = [T]^{-1} [Q] [T] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = [\bar{Q}] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

$$\bar{Q}_{11} = m^4 Q_{11} + 2m^2 n^2 (Q_{12} + 2Q_{66}) + n^4 Q_{22}$$

$$\bar{Q}_{21} = \bar{Q}_{12} = m^2 n^2 (Q_{11} + Q_{22} - 4Q_{66}) + Q_{12} (m^4 + n^4)$$

$$\bar{Q}_{22} = n^4 Q_{11} + 2m^2 n^2 (Q_{12} + 2Q_{66}) + m^4 Q_{22}$$

$$\bar{Q}_{16} = m^3 n (Q_{11} - Q_{12}) + mn^3 (Q_{12} - Q_{22}) - 2mn(m^2 - n^2) Q_{66}$$

$$\bar{Q}_{26} = mn^3 (Q_{11} - Q_{12}) + m^3 n (Q_{12} - Q_{22}) + 2mn(m^2 - n^2) Q_{66}$$

$$\bar{Q}_{66} = m^2 n^2 (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) + (m^4 + n^4) Q_{66}$$

Matériaux orthotropes en contraintes planes

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

$$\bar{S}_{11} = m^4 S_{11} + m^2 n^2 (2S_{12} + S_{66}) + n^4 S_{22}$$

$$\bar{S}_{21} = \bar{S}_{12} = m^2 n^2 (S_{11} + S_{22} - S_{66}) + S_{12} (m^4 + n^4)$$

$$\bar{S}_{22} = n^4 S_{11} + m^2 n^2 (2S_{12} + S_{66}) + m^4 S_{22}$$

$$\bar{S}_{16} = 2m^3 n (S_{11} - S_{12}) + 2mn^3 (S_{12} - S_{22}) - mn(m^2 - n^2) S_{66}$$

$$\bar{S}_{26} = 2mn^3 (S_{11} - S_{12}) + 2m^3 n (S_{12} - S_{22}) - mn(m^2 - n^2) S_{66}$$

$$\bar{S}_{66} = 4m^2 n^2 (S_{11} - S_{12}) + 4m^2 n^2 (S_{12} - S_{22}) - (m^2 - n^2)^2 S_{66}$$

De la théorie des stratifiés aux outils de conception

Théorie des stratifiés

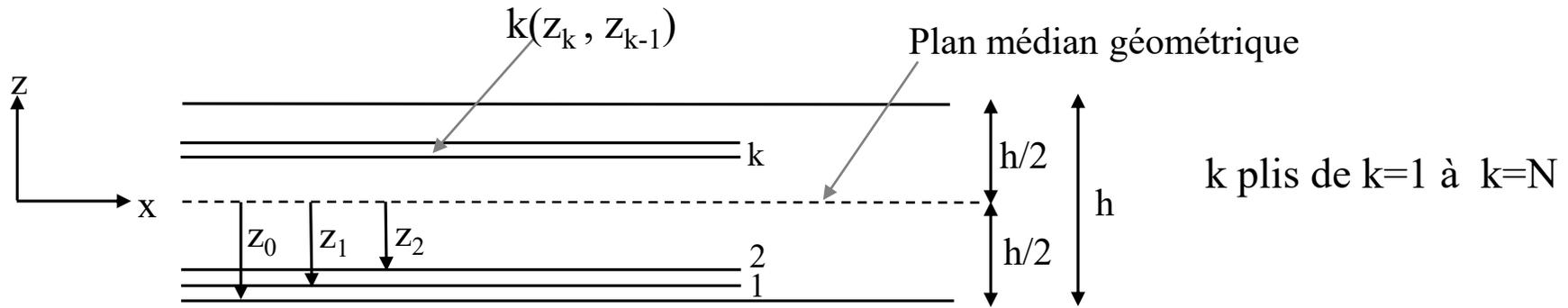
Symétrie des stratifiés

Effets de couplage

Propriétés effectives

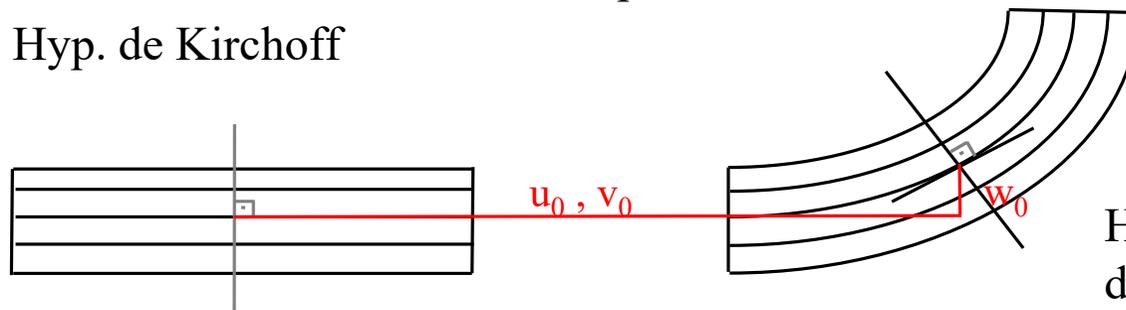
Guides pour la conception

Elasticité des stratifiés



(H) CLT: Classical Laminate Theory

- Linéaire élastique ε_x
- Orthotrope
- Membrane, contraintes planes ($\sigma_3, \varepsilon_3 = 0$) \equiv pas de déformation selon l'axe z , seulement déformation hors du plan
- Hyp. de Kirchoff



Hyp: Adhésion parfaite entre les plis, déformation selon $z = 0$
 si non plus en contrainte plane et des termes sup sont à ajouter dans les prochaines équations

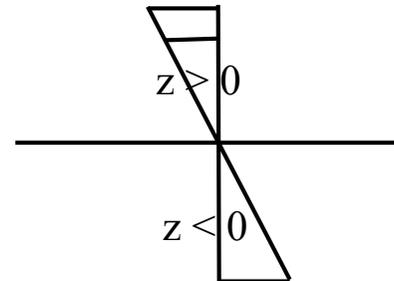
Elasticité des stratifiés

- Déplacement de tout point de cote z

$$u = u_0 - z \frac{\partial w_0}{\partial x}$$

$$v = v_0 - z \frac{\partial w_0}{\partial y}$$

$$w = w_0$$



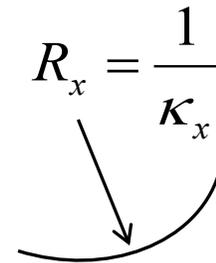
- Déformation

$$\varepsilon_x = \varepsilon_x^0 - z \frac{\partial^2 w_0}{\partial x^2}$$

$\underbrace{\hspace{10em}}_{-\kappa : \text{courbure}}$

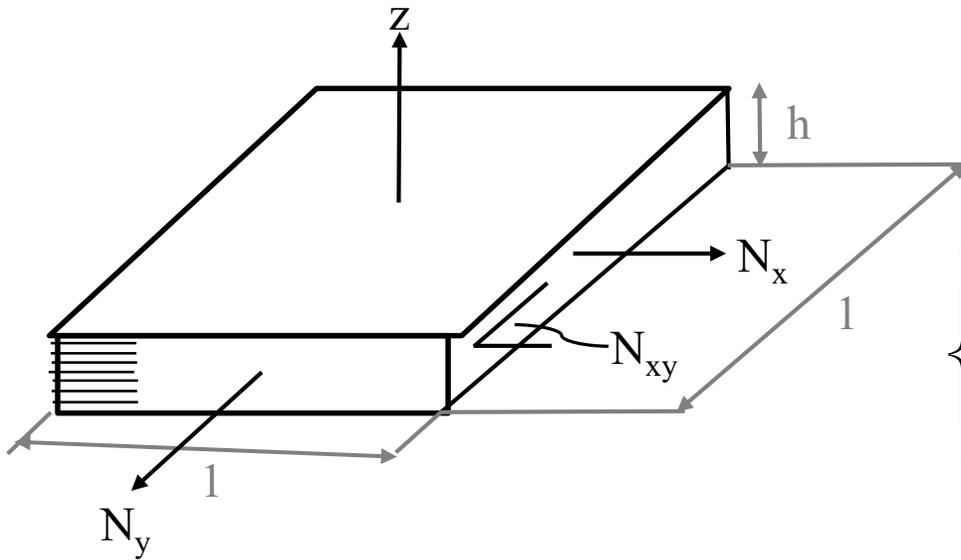
$$\varepsilon_y = \varepsilon_y^0 - z \frac{\partial^2 w_0}{\partial y^2}$$

$$\gamma_{xy} = \gamma_{xy}^0 - 2z \frac{\partial^2 w_0}{\partial x \partial y}$$



$$\{\varepsilon\} = \{\varepsilon^0\} + z\{\kappa\}$$

Elasticité des stratifiés



$$S = h \cdot 1 = h$$

largeur unitaire

$$\sigma = \frac{N}{S} = \frac{N}{h}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \overline{Q} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

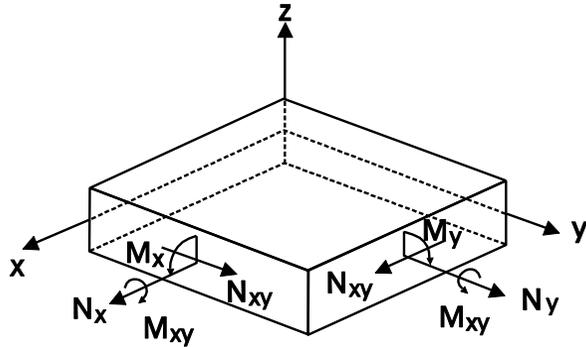
$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \mathbf{K}_x \\ \mathbf{K}_y \\ \mathbf{K}_{xy} \end{Bmatrix}$$

$$\rightarrow [\sigma] = [\overline{Q}] (\{\varepsilon^0\} + z \{\mathbf{K}\})$$

Elasticité des stratifiés

$$\begin{aligned}
 \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} &= \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} dz = \int_{-h/2}^{h/2} [\bar{Q}] ([\varepsilon^0] + z[\kappa]) dz \\
 &= \sum_{k=1}^N \int_{z_{k-1}}^{z_k} [\bar{Q}]_k ([\varepsilon^0] + z[\kappa]) dz \\
 &\quad \neq f(z) \\
 &= \sum_{k=1}^N [\bar{Q}]_k [\varepsilon^0] (z_k - z_{k-1}) + [\bar{Q}]_k [\kappa] \frac{z_k^2 - z_{k-1}^2}{2} \\
 \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} &= [A][\varepsilon^0] + [B][\kappa]
 \end{aligned}$$

Elasticité des stratifiés



$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix}$$

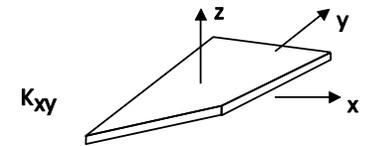
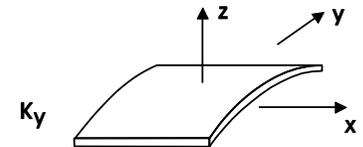
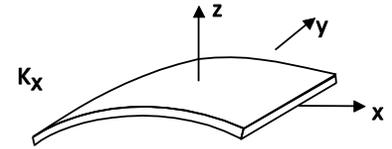
$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

$$A_{ij} = \sum_{k=1}^N (\overline{Q}_{ij})_k (z_k - z_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^N (\overline{Q}_{ij})_k (z_k^2 - z_{k-1}^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^N (\overline{Q}_{ij})_k (z_k^3 - z_{k-1}^3)$$



Couplages

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

Elasticité des stratifiés

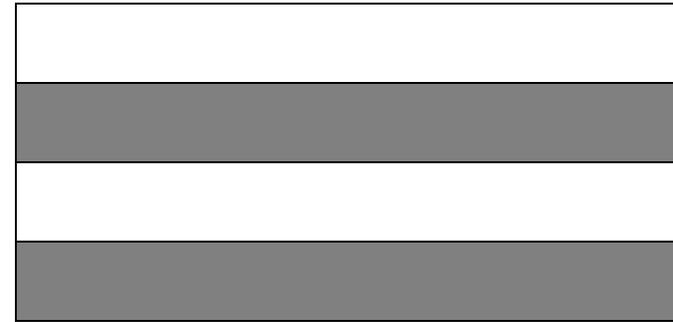
$$\begin{bmatrix} \varepsilon^0 \\ \kappa \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} N \\ M \end{bmatrix}$$

$$[a] = [A^*] - [B^*][D^*]^{-1}[C^*]$$

$$[b] = [B^*][D^*]^{-1} \quad [c] = -[D^*]^{-1}[C^*] \quad [d] = [D^*]^{-1}$$

$$[A^*] = [A]^{-1} \quad [B^*] = -[A]^{-1}[B] \quad [C^*] = [B][A]^{-1} \quad [D^*] = [D] - [B][A]^{-1}[B]$$

Exercice



$$[A] =$$

$$[A] =$$

$$[B] =$$

$$[B] =$$

$$[D] =$$

$$[D] =$$

Notations et symétries des stratifiés

$$[0/0/0/0/0/0] = [0_6]$$

$$[0/90/90/0] = [0/90]_x$$

$$[0/90/0] = [0/\overline{90}]_x$$

$$[+45/-45/-45/45] = [\pm 45]_x$$

$$[30/-30/30/-30/-30/30/-30/30] = [\pm 30]_{2x}$$

$$[30/-30/30/-30/30/-30/30/-30] = [\pm 30]_4$$

$$[0/45/-45/-45/45/0] = [0/\pm 45]_x$$

$$[0/0/45/-45/0/0/0/0/-45/45/0/0] = [0_2/\pm 45/0_2]_x$$

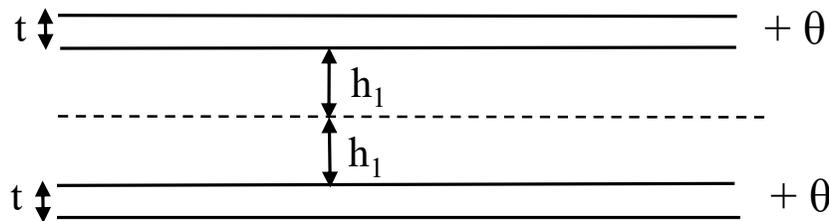
$$[0/15/-15/15/-15/0] = [0/\pm 15/\pm 15/0] = [0/(\pm 15)_2/0]$$

$$[0^K/0^K/45^C/-45^C/90^C/-45^C/45^C/0^K/0^K] = [0_2^K/\pm 45^C/\overline{90^C}]_x$$

Symétries des stratifiés

- Stratifiés symétriques

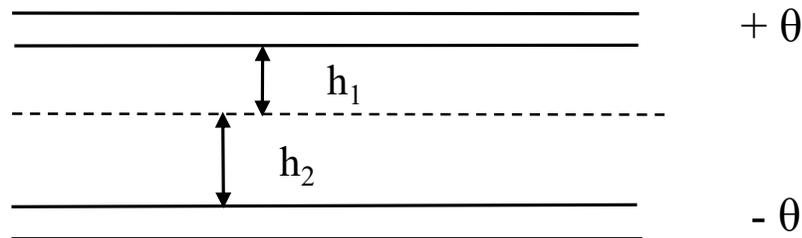
Pas de couplage entre extension et flexion



$$B_{ij} = 0$$

- Stratifiés balancés

Pas de couplage entre extension et cisaillement



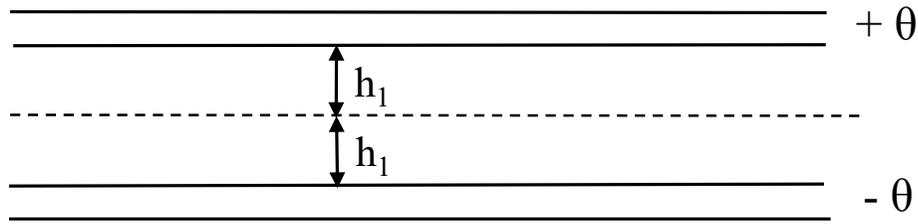
$$A_{16} = A_{26} = 0$$

- Balancé + symétrique

$$A_{16} = A_{26} = 0 \quad B_{ij} = 0$$

Symétries des stratifiés

- Antisymétriques



$$D_{16} = D_{26} = 0$$

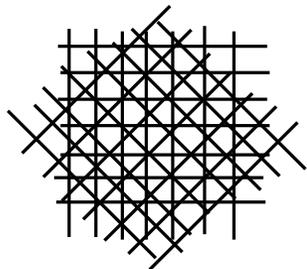
Pas de couplage entre flexion et torsion

- Croisé, ``crossply laminate``

ssi à 0° et 90°

$$B_{12} = B_{16} = B_{26} = 0$$

- $(0, +45, -45, 90)$



Quasi-isotrope

$$A_{11} \approx A_{22}$$

$(0, \pm 60)$

Stratifiés symétriques balancés

Laminate	A_{11} (MN/m)	A_{22} (MN/m)	G_{xy} (GPa)
$[90_8]_s$	11.09	153.28	2.29
$[0/90_7]_s$	28.86	135.51	2.29
$[0_2/90_6]_s$	46.64	117.73	2.29
$[0_3/90_5]_s$	64.41	99.96	2.29
$[0_4/90_4]_s$	82.18	82.18	2.29
$[0_5/90_3]_s$	99.96	64.41	2.29
$[0_6/90_2]_s$	117.73	46.64	2.29
$[0_7/90]_s$	135.51	28.86	2.29
$[0_8]_s$	153.28	11.09	2.29
$[\pm 45/90_6]_s$	20.21	126.85	6.62
$[\pm 45/0/90_5]_s$	37.98	109.08	6.62
$[\pm 45/0_2/90_4]_s$	55.76	91.31	6.62
$[\pm 45/0_3/90_3]_s$	73.53	73.53	6.62
$[\pm 45/0_4/90_2]_s$	91.31	55.76	6.62
$[\pm 45/0_5/90]_s$	109.08	37.98	6.62
$[\pm 45/0_6]_s$	126.85	20.21	6.62
$[\pm 45_2/90_4]_s$	29.33	100.43	10.95
$[\pm 45_2/0/90_3]_s$	47.11	82.65	10.95
$[\pm 45_2/0_2/90_2]_s$	64.88	64.88	10.95
$[\pm 45_2/0_3/90]_s$	82.65	47.11	10.95
$[\pm 45_2/0_4]_s$	100.43	29.33	10.95
$[\pm 45_3/90_2]_s$	38.45	74.00	15.27 F
$[\pm 45_3/0/90]_s$	56.23	56.23	15.27 F
$[\pm 45_3/0_2]_s$	74.00	38.45	15.28 F
$[\pm 45_4]_s$	47.58	47.58	19.60 F

Dimensionnement : propriétés effectives

Efforts connus

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}$$

h = épaisseur du stratifié

Contraintes globales moyennes (fictives)

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = 1/h \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = 1/h \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}$$

$$\frac{1}{h} A_{ij} = \sum_{k=1}^N (\overline{Q}_{ij})_k \frac{\text{épaisseur}_k}{h} = \sum_{k=1}^N (\overline{Q}_{ij})_k \text{pourcentage}_k$$

Inverser pour obtenir des modules effectifs... et les déformations

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = h \cdot \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}^{-1} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & 0 \\ -\frac{\nu_{yx}}{E_y} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

Propriétés effectives

Si symétrique et balancé

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} + \begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{22} & 0 \\ 0 & 0 & a_{66} \end{bmatrix} \left\{ \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} + \begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix} \right\}$$

Si seulement charge mécanique
et déformation uniforme
sur l'épaisseur

$$\varepsilon_x^0 = a_{11} N_x = \varepsilon_x$$

$$\sigma_x = \frac{N_x}{h}$$

$$\varepsilon_x = a_{11} h \sigma_x$$

$$E_x = \frac{\sigma_x}{\varepsilon_x} = \frac{1}{h a_{11}}$$

$$E_x = \frac{A_{11}A_{22} - A_{12}^2}{h A_{22}}$$

Propriétés effectives de l'ingénieur
à intégrer aux équations de la résistance des matériaux

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & 0 \\ -\frac{\nu_{yx}}{E_y} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

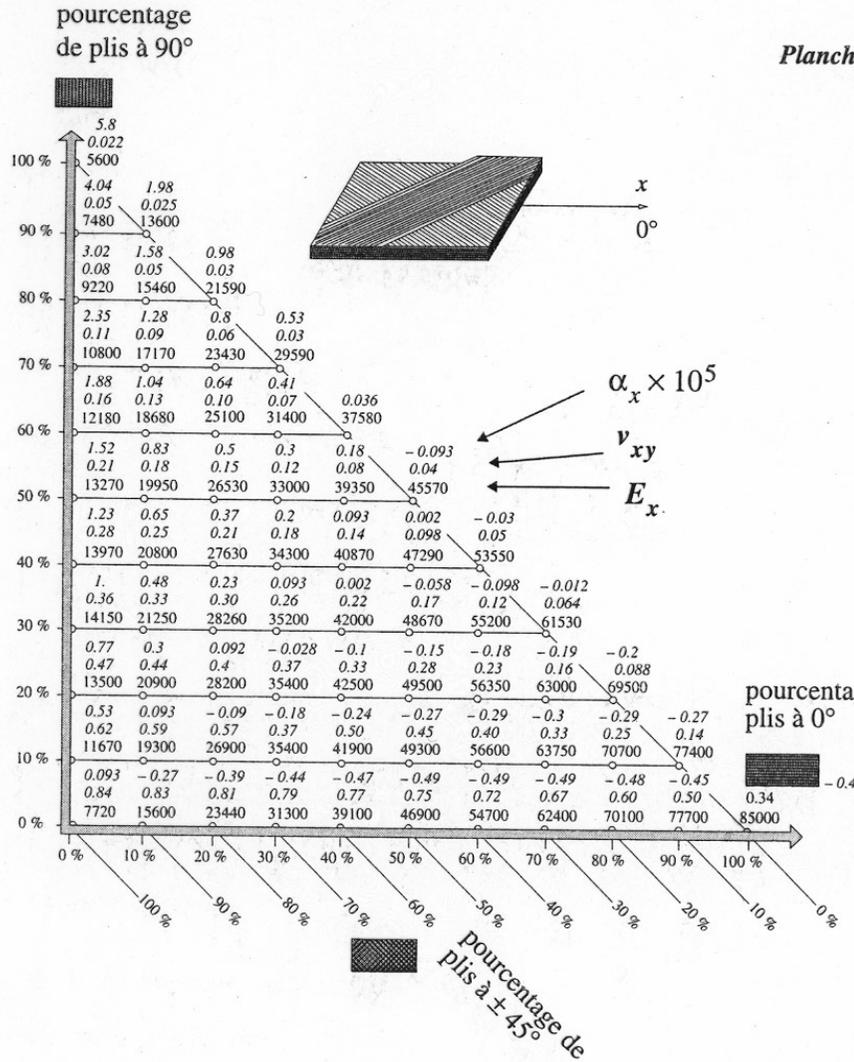
Stratifié kevlar/époxyde

Guides de conception exp D. Gay

$V_f = 60\%$ de fibres en volume

épaisseur du pli : 0.13 mm

caractéristiques du pli : cf. paragraphe 3.3.3.

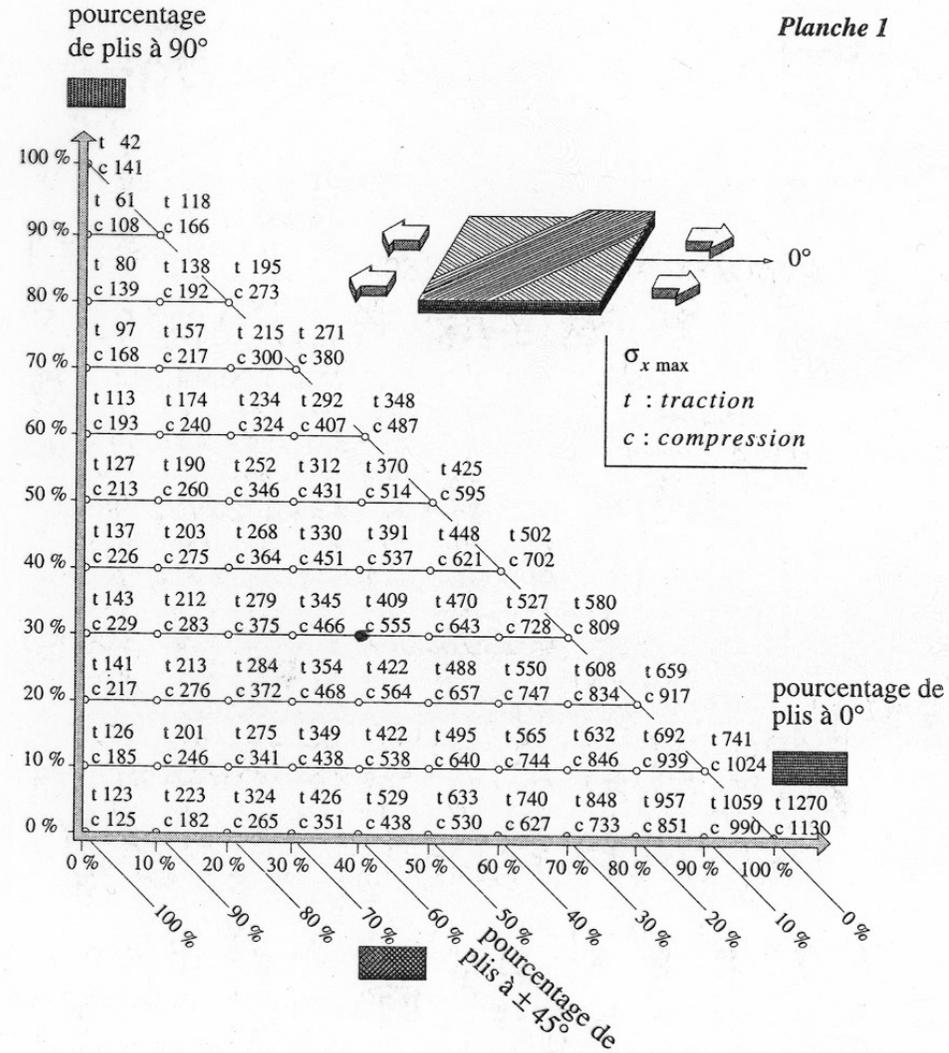


Stratifié carbone/époxyde

$V_f = 60\%$ de fibres en volume

épaisseur du pli : 0.13 mm

caractéristiques du pli : modules, contraintes de rupture : cf. paragraphe 3.3.3.



OSU Laminate

ABD Calculator

EsaComp

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