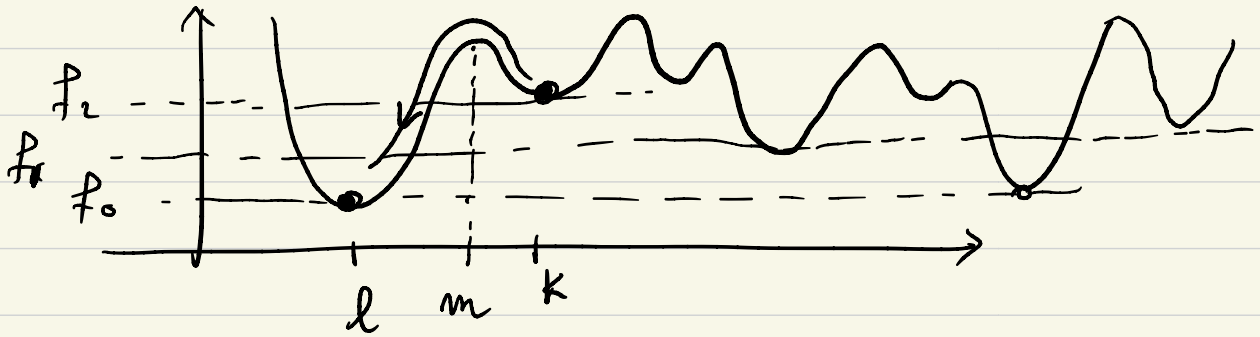
Quiz 9 discussion.

I. Function Minimisation.



a) Probability of shortest path from $k \to l$:

$$\prod_{i:k\to l} a_{i,i-1} = \underbrace{\prod_{i:k\to m-1} a_{i,i-1}}\ \underbrace{\prod_{i:m\to l} a_{i,i-1}}_{\to 1}$$

$$\frac{\pi(i-1)}{\pi(i)} = e^{-\beta(f(i-1)-f(i))}$$

$$= e^{-\beta\Delta} \qquad \Delta = f(m) - f(k).$$

b) We ask that $\pi(i \in \text{global minima}) \simeq 1-\epsilon$; $0 < \epsilon \ll 1$.

$$\Rightarrow \frac{N_0 e^{-\beta f_0}}{N_0 e^{-\beta f_0} + N_1 e^{-\beta f_1} + N_2 e^{-\beta f_2}} \simeq 1-\epsilon \quad \text{and for } \beta \text{ large this}$$

is approximated by $\dfrac{1}{1 + \frac{N_1}{N_0} e^{-\beta(f_1-f_0)} + \dots} \simeq 1-\epsilon$

since $f_1 - f_0 > f_2 - f_0 > f_3 - f_0 > \dots$ $\Rightarrow$

$$\Rightarrow \quad 1 - \frac{N_1}{N_0} e^{-\beta(f_1 - f_0)} \simeq 1 - \epsilon \quad \Rightarrow \quad \epsilon \simeq \frac{N_1}{N_0} e^{-\beta(f_1 - f_0)}$$

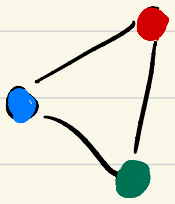
$$\Rightarrow \quad \beta(f_1 - f_0) + \log \frac{N_0}{N_1} \simeq \log(1/\epsilon)$$

$$\Rightarrow \quad \boxed{\beta \simeq \frac{1}{f_1 - f_0} \log\left(\frac{N_1}{N_0 \epsilon}\right).}$$

II. Coloring.

Number of colors. Max deg of vertices

1) Proper coloring always exists if $q = \Delta + 1$

2) 

$\Delta = 2$, $q = 3$ } one can move out of this configuration with the proposal move.

$\Rightarrow$ Base chain is <u>not</u> irreducible.

3) In the above example $q = 4$ makes the proposal chain irreducible.

In general we will discuss next time that $q \geq \Delta + 2$ suffices to have an irreducible proposal chain on any graph.